MATHEMATICS
GRADES 10–12

INTRODUCTION

The Mathematics Grades 10–12 Program of Studies has been derived from *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol*, January 2008 (the Common Curriculum Framework). The program of studies incorporates the conceptual framework for Grades 10–12 Mathematics and most of the general outcomes and specific outcomes that were established in the Common Curriculum Framework. (Note: Some of the outcomes for Mathematics 20-2 and 30-2 in this program of studies are different from the outcomes for Foundations of Mathematics in the Common Curriculum Framework.)

BACKGROUND

The Common Curriculum Framework was developed by seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions.

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

Students are curious, active learners with individual interests, abilities, needs and career goals. They come to school with varying knowledge, life experiences, expectations and backgrounds. A key component in developing mathematical literacy in students is making connections to these backgrounds, experiences, goals and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

FIRST NATIONS, MÉTIS AND INUIT PERSPECTIVES

First Nations, Métis and Inuit students in northern and western Canada come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural and isolated communities. Teachers need to understand the diversity of students' cultures and experiences.

First Nations, Métis and Inuit students often have a holistic view of the environment—they look for connections in learning and learn best when mathematics is contextualized. They may come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word, so oral communication and practical applications and experiences are important to student learning and understanding. By understanding and responding to nonverbal cues, teachers can optimize student learning and mathematical understanding.

A variety of teaching and assessment strategies help build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students.

Research indicates that when strategies go beyond the incidental inclusion of topics and objects unique to a culture or region, greater levels of understanding can be achieved (Banks and Banks, 1993).

AFFECTIVE DOMAIN

A positive attitude is an important aspect of the affective domain and has a profound effect on learning. Environments that create a sense of belonging, support risk taking and provide opportunities for success help students to develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, to participate willingly in classroom activities, to persist in challenging situations and to engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains and to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and to assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

GOALS FOR STUDENTS

The main goals of mathematics education are to prepare students to:

- solve problems
- communicate and reason mathematically
- make connections between mathematics and its applications
- become mathematically literate
- appreciate and value mathematics
- make informed decisions as contributors to society.

Students who have met these goals:

- gain an understanding and appreciation of the role of mathematics in society
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical problem solving

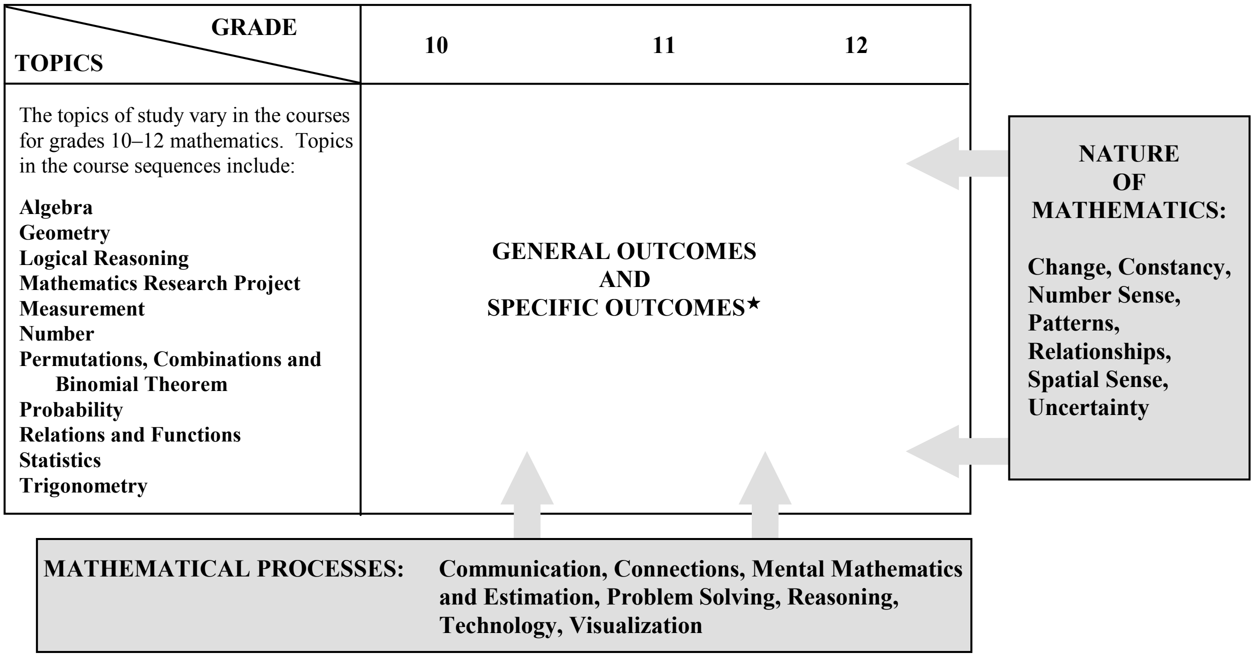
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics.

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

CONCEPTUAL FRAMEWORK FOR GRADES 10–12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



* Achievement indicators for the prescribed program of studies outcomes are provided in the companion document *The Alberta 10–12 Mathematics Program of Studies with Achievement Indicators*, 2008.

Mathematical Processes

The seven mathematical processes are critical aspects of learning, doing and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education.

This program of studies incorporates the following interrelated mathematical processes. They are to permeate the teaching and learning of mathematics.

Students are expected to:

- *Communication* [C] — use *communication* in order to learn and express their understanding
- *Connections* [CN] — make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
- *Mental Mathematics and Estimation* [ME] — demonstrate fluency with *mental mathematics and estimation*
- *Problem Solving* [PS] — develop and apply new mathematical knowledge through *problem solving*
- *Reasoning* [R] — develop mathematical *reasoning*
- *Technology* [T] — select and use *technology* as a tool for learning and for solving problems
- *Visualization* [V] — develop *visualization* skills to assist in processing information, making connections and solving problems.

All seven processes should be used in the teaching and learning of mathematics. Each specific outcome includes a list of relevant mathematical processes. The identified processes are to be used as a primary focus of instruction and assessment.

COMMUNICATION [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning by using mathematical terminology.

Communication can play a significant role in helping students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

CONNECTIONS [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p. 5).

MENTAL MATHEMATICS AND ESTIMATION [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves using strategies to perform mental calculations.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility in reasoning and calculating.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities, usually by referring to benchmarks or referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgements and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

PROBLEM SOLVING [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you ...?* or *How could you …?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks.

Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open- ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate his or her knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem, they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

REASONING [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and to justify their mathematical thinking. Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ….*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

TECHNOLOGY [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

VISUALIZATION [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.