Occasioning flow in the mathematics classroom: optimal experiences in common places

by

Ioana Mihaela Chiru

B. Ed., Simon Fraser University, 2009

Licence in Mathematics, University of Bucharest, Romania, 1992

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Name: **Ioana Mihaela Chiru**

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Examining Committee:

Chair: **Rina Zazkis,** Professor

Peter Liljedahl
Senior Supervisor
Associate Professor

Nathalie Sinclair
Supervisor
Professor

David Pimm
Internal/External Examiner
Adjunct Professor

Date Defended/Approved: June 28, 2017

Ethics Statement

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Abstract

This research looks at several mathematics high-school classrooms through the lens of Csíkszentmihályi's flow theory: optimal matching of skills and challenge, clear goals and feedback, loss of temporal awareness, intense concentration, a sense of control, merging of action and awareness, loss of self-consciousness and autotelic experience. The study focuses on creating and maintaining the flow experience in students. In order to uncover successful pedagogical interventions, the students are surveyed through questionnaires and interviews. The study discusses the crucial role of collaboration and of mathematical tasks in occasioning the flow experience, how students differ in experiencing flow, and how they learn to seek and re-create the flow experience. The study also examines the students' unfavourable perception of textbooks, the students' negative experiences of boredom and apathy, and the precarious relationship between teacher flow and student flow.

Keywords: flow; mathematics classroom; collaboration; boredom; task complexity; teacher flow

To my family, who keeps me in the flow of their love

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My students, who have taught me so much – may you find your flow, wherever, whenever.

Table of Contents

List of Tables

List of Figures

List of Acronyms

ESM	Experience Sampling Method
PFE	Proxies for Engagement
MAA	Merging of action and awareness
LSC	Loss of self-consciousness

Chapter 1. Introduction

Oh chestnut-tree, great-rooted blossomer,
Are you the leaf, the blossom or the hole?
O body swayed to music, O brightening glance,
How can we know the dancer from the dance?

William B. Yeats, 1927 - *Among School Children*

One of my earliest memories revolves, in concentric circles, around a mystery. One summer day, when I was about 9 years old, my younger brother and I had enjoyed reading a Sherlock Holmes story at a friend's house, and were all agog to read more. We were happy therefore to find at home a small book by Arthur Conan Doyle, belonging to our father, from his young days. There was a small problem, however: the book was in English, with a Russian dictionary. It was a book meant for Russians who wanted to learn English, and we knew neither English, nor Russian. However, this did not deter us. We unearthed an English-Romanian dictionary and we started at the beginning:

"the" – I wrote down in a little notebook what it meant

"sign"

"of"

"the" – no need to write it again, we already had it

"four"

And we kept going, looking up in every single word: "and", "though", "he" … until our parents came home in the evening. I remember they were extremely displeased that we "wasted" time and did not go outside to play in the sun. In that whole day, we translated no more than two short pages, and we got a good scolding for it, but we felt we accomplished something worthwhile, and we could hardly wait for the next day to continue the same finicky, gruelling work. It was not only the beginning of my love for the English language, but also one of my first remembered encounters with this unusual experience which made me lose track of time and demonstrate such uncharacteristic patience.

Thinking about mathematics, solving mathematics problems, even practicing mundane mathematical skills – in short, anything that had to do with math – gave me the same kind of unreasonable thrill: it was the thing that that caused me greatest frustration, but also the greatest satisfaction. Amid the greige of everyday life, mathematics brought moments of luminous enjoyment, instances of excitement, of curiosity and perseverance. I didn't know what ***it*** was called, but sought to recreate ***it*** as much and as often as possible.

And then I became a mathematics teacher.

In retrospect, I was, of course, quite naive. I expected my students to feel the same way as I did. I expected that all I would have to do is straighten out a few kinks here and there, give wise words of encouragement a few times, steadfastly profess my love of mathematics, and things would work out: the students would enjoy mathematics, just like I did. However, after one year of teaching, I knew something was amiss with my best laid plans.

To confuse me even more, overall, I've taught nice children, who made efforts to learn. They were reasonably interested during class; they did their homework – most of the time. They were polite and did not show outward signs of rejection of mathematics. But after a while, as I got to know the students better, and they talked to me about their lives, I could not fool myself anymore: they were "good students" because they had to, because they were compliant, because their parents had told them it was important, because they wanted to have only A's. This was not what I wanted, not what I had signed up for. I felt frustrated and powerless, as if I had changed, and had become assimilated by a cold machine which played havoc with my high-minded ideals of enjoyment in mathematics. Meanwhile, the students kept mentioning some classes, such as drama, PE, and art, where they were having "fun". I researched them, spoke with the teachers: yes, they said, their classes had to be enjoyable, otherwise who would take them? They pointed out that mathematics is something that all students have to do, and thus, they argued, it's not in the nature of things for mathematics teachers to worry about frivolous matters such as "fun".

At the end of the first year I wrote a poem, which I reproduce here. Re-reading it reminds me of the frustration and dissatisfaction I was feeling at the time.

> If June is happy, why am I sad?
>
> The children sit down in their silent array
> The machine has determined it is the right way
> The machine put me in charge of the grinding
> I rise to the challenge: I'm very obliging.
>
> The assets are checked, it is grades o'clock
> I answer the summons, I survey the stock
> The children file out, wishing me a great day
> Too late for my game, too late for my play.

The following year I applied some of the ideas imparted by the Fine Arts teachers I had spoken to (pre-emptively, I confess my belief that mathematics is the Finest Art of all). An 'equations snakes and ladders' here, a 'proportional reasoning bingo' there, produced mixed and, sometimes, undesirable results. When they were not beleaguered by worries about grades, many students were intensely focussed on winning the *game*, while a not insignificant minority suggested that all that playing of games was a waste of time. I soon had to contend with the fact that my approach gave rise, in the first group, to an undercurrent of trivialization of mathematics, while for the others it was as if mathematics had to be unpleasant in order for them to take it seriously. Perhaps both groups of students were feeling the same thing: an insidious Mary-Poppins-isation of the classroom, whereby the games were "a spoonful of sugar to make the ~~medicine~~ mathematics go down". Reflecting on this, it started to dawn on me that I was not doing the students a service, and that I had to find another way. I gave up on "gamification" and did the only sensible thing left to me: I went back to school myself.

My first course at SFU made me relive my childhood days: I was doing mathematics for the sheer pleasure of doing it, did not feel tired or bored, never worried about what I would have to do next, while time passed – all by itself – in the distant background. That was precisely what I wanted for my students, and, fully aware of the distinctions between a group of teenagers and a group of self-selected adults, I made experimental tweaks to my practice, in a tentative attempt to encourage and notice this particular experience in the students. Looking back, I did not know exactly what or how to tweak, what or how to encourage. I just had a general idea that I was looking for what is known in teacher jargon as "engagement", and, if that wasn't ambitious enough, I also wanted to see "enjoyment". And I also wanted to see mathematics: beautiful, surprising, arresting. Noticing this ***it*** was equally hesitant and fraught with confusion. It so happened that sometimes – unless I was deluding myself – students seemed different, as if they inhabited fragile bubbles of excitement, curiosity and wonder, inside which they became so engaged in mathematics that they forgot about everything else. At those times, there was a spark in their eyes, a tingle in their voices, a playfulness in their exchanges – there was something there, a *je ne sais quoi,* which I was unable to name until I read about Csíkszentmihályi and flow.

As I was reading, through my mind flashed a memory: the image of a former student, whiteboard marker in hand, looking into my eyes, fervently waiting for a new problem, with the fierce concentration of an animal waiting for the right moment to pounce on its prey. I had always treasured the scene, as I had found it amusing at the time. However, flow theory bestowed on it new meaning and significance, and captured the essence of the student's experience with shining precision: the concentration, the calm conviction that nothing I could throw at her was impossible, the narrow focus of attention, the oneness with the tool, the doing for the sake of doing. For me, as a teacher, encountering flow theory meant, first and foremost, the recognition that what I was looking for was possible: I had a name, a place, a road map, a measuring stick. Thus, it became possible to transform the desultory, fumbling pedagogical acupuncture that was "my practice" into something which – while not always successful – has a clear, compelling purpose and is true to my convictions about how life should be lived, how mathematics should be learned.

Secondly, flow theory gave a name and scientific gravitas to an experience that was previously described only through poetic metaphors. And while poets might have gotten there first, the disciplined imagination of a scientist is needed to probe, untangle and organize the multiple helix of the flow experience: clear goals and feedback, a sense that one's skills are balanced with the challenge presented, which leads to the feeling that one is in complete control, intense concentration on the task at hand, the disappearance of self-centredness, the distortion of temporal experience, and the high intrinsic value of the experience, so much so that people are willing to perform the flow-inducing activity "for its own sake, with little concern for what they will get out of it" (Csíkszentmihályi, 1990, p. 71).

This thesis is the result of my journey exploring flow in my classroom. Chapter 2 will be dedicated to a review of the literature on flow, and on fostering an environment auspicious for flow, while chapter 3 elaborates on the setting and methodology of the study. Chapter 4 comprises vignettes of students' experiences, succeeded, in chapters 5 and 6, by an in-depth analysis and discussion of themes emerging from these experiences. Chapter 7 will present the conclusion of the study and final considerations on occasioning flow in my classroom.

Chapter 2. Literature review

> *For in every action, whether caused by necessity or free will, the main intention of the agent is to express his own image; thus it is that every doer, whenever he does, enjoys the doing; because everything that is desires to be, and in action the doer unfolds his being, enjoyment naturally follows, for a thing desired always brings delight…*
>
> Dante Alighieri, 1317 - *De Monarchia*

In the introduction to his Collected Works (2014a), Csíkszentmihályi muses about how his research curiosity was piqued when he observed how people – himself included – feel most “alive” while engaged in hobbies and “unimportant” activities, such as rock climbing, hiking, or playing chess, while feeling bored and dissatisfied during the rest of their lives. He also puzzled over the yet-unexplained behaviour of artists at work, and could not understand their steadfast dedication to a craft which most of the time brought them uncertain money and little fame. He writes:

> In a world supposedly ruled by the pursuit of money, power, prestige, and pleasure, it is surprising to find certain people who sacrifice all those goals for no apparent reason: people who risk their lives climbing rocks, who devote their lives to art, and who spend their energy playing chess. By finding out why they are willing to give up material rewards for the elusive experience of performing enjoyable acts, we hope to learn something that will allow us to make everyday life more meaningful. At present, most of the institutions that take up our time—schools, offices, factories—are organized around the assumption that serious work is grim and unpleasant. Because of this assumption, most of our time is spent doing unpleasant things. By studying enjoyment, we might learn how to redress this harmful situation. (1975a, p. 1)

Gradually, he became convinced that the theories of human motivation current at the time, based on the *deficit model,* according to which humans engage in activities in order to satisfy basic needs, did not accurately describe the experiences he was witnessing, and that what was needed was a “phenomenology of play” (as cited in Engeser, 2012, p. vi). He thus proposed a new framework of understanding behaviour, which centred on the related concepts of play and enjoyment, model that he called “flow theory” (Csíkszentmihályi, 1975b). In the beginning, he thought of flow theory as “an interesting but marginal diversion, with little to contribute to psychology or the social sciences”

(2014a, p. xix). This chapter is dedicated to exploring the journey of this anything-but-marginal “diversion”, after almost 50 years of research.

2.1. Theoretical beginnings

Csíkszentmihályi’s original qualitative research consisted of interviews conducted with chess players, rock climbers, dancers, surgeons and composers of modern music (1975a, 1975b). Starting from what he considered an axiom: that a person needs opportunities for action, and secondly, that the actions available must be commensurate with skills, he proposed an initial theoretical framework in which he identifies a balance of challenge and skill as the determining criterion for inducing a flow experience. The balance between action opportunities and action capabilities is however fragile: if the challenges exceed the skills, the person becomes anxious, as illustrated by position Axy in Figure 2. If the skills exceed the challenges, then the person becomes bored. Experiencing the discomfort of boredom or anxiety is, in Csíkszentmihályi’s view, what impels a person to adjust either their skills or the challenge in order to re-enter the state of flow.

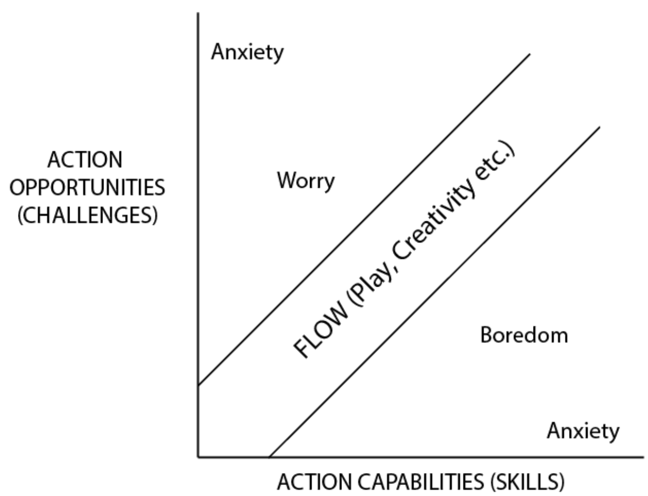


Figure 1. Model of the flow state.
When a person faces challenges greater than skills, the person will experience worry. Conversely, when a person’s skills are greater than the challenge, the person will experience boredom. If the ratio between challenges and skills is overwhelmingly high or low, the person will experience anxiety. (Csíkszentmihályi, 1975a, p. 49)

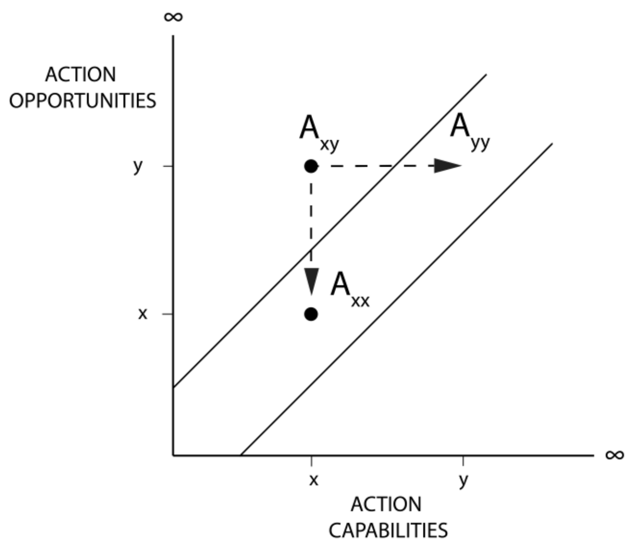


Figure 2. Two ways of experiencing flow.
Person A, with skill level x, has to perform an action which requires a higher skill level y. In order to experience flow, person A has two choices: either decrease the challenge back to x, or increase one's skills to y so that they can match the higher challenge. (Csíkszentmihályi, 1975b, p. 60)

Secondly, Csíkszentmihályi noted that throughout the interviews conducted for his research, the participants reiterated the same characteristics of the experience, which led him to crystallize their observations into the following *components of the flow experience* (1975b):

1. *Merging of action and awareness* – Csíkszentmihályi has written about merging of action and awareness that it is "…perhaps the clearest sign of flow" (1975a, p. 38). When action and awareness are merged, "all the attention is concentrated on the relevant stimuli…and the activity becomes spontaneous, almost automatic" (1990, p. 63). The concentration is exclusively about what one is doing at the moment, uninterrupted by moments of looking at oneself from outside, moments of self-doubt in which one wonders "how am I doing?". This focus is facilitated by the second characteristic of flow:

2. *Centering of attention on a limited stimulus field* – the person is able to keep all intrusive stimuli away from the focus of attention, leaving all outside thoughts "at the door".

3. *Loss of ego* – when thought and action are merged, there is no need for the self to mediate and negotiate between the person and the environment.

4. *Control of action and environment* – due to the previous components of merging of action and awareness and loss of ego, the person feels

no worry about what might come next. There are no unanticipated threats, no uncontrollable situations.

5. *Demands for action and clear feedback* – the flow experience makes clear, consistent demands, and feedback is available throughout, so that the person can fully and accurately evaluate their actions. Were this component to be missing, then the person would be unable to merge action and awareness.

6. *Flow is autotelic*, that is, it requires no external rewards – it is a goal onto itself.

In most respects, flow theory has not needed major revisions, with two exceptions. The first exception pertains to the experience of time while in flow. Although participants in Csíkszentmihályi's original research (1975b) note that "losing track of time" is part of the flow experience, Csíkszentmihályi did not include this in his list of characteristics. In the years that followed, however, the components of the flow experience were enhanced to include "time distortion" as well (Csíkszentmihályi, 1997c; Csíkszentmihályi & Csíkszentmihályi, 1988; Jackson & Marsh, 1996; Nakamura & Csíkszentmihályi, 2005). The other exception will be discussed in section 2.3.1.

This newly created lexicon of flow reflects Csíkszentmihályi's preoccupation with the subjective experience emerging from an intrinsically motivated activity, rather than the behaviours connected with these activities. This is, in Csíkszentmihályi's view, the main difference between flow theory and its precursors (or contemporaries) pertaining to intrinsic motivation. Unlike Buhler's (1922) "funktionslust", White's (1959) "effectance", Berlyne's (1960) "optimal arousal", Maslow's (1968, 1971) "peak experience", DeCharm's (1968) "personal causation", or Deci and Ryan's (1985) "autonomy", flow theory aims to answer questions of a much more subjective nature: how intrinsic rewards feel, whether they feel the same way regardless of the activity, whether everybody feels the same things when engaged in intrinsically motivated activities, and what exactly makes those activities rewarding (Csíkszentmihályi, 1988).

2.2. Transitioning from the theoretical to the practical

Csíkszentmihályi himself recognized that the questions he sought to ask were unobjective, and the phenomena he was trying to describe seemed impossible to quantify. This lack of scientific rigour did not encourage further research on the subject of flow, and the years following the publication, in 1975, of *Beyond boredom and anxiety*, were marked by what Csíkszentmihályi describes as “benign neglect” (foreword to Engeser, 2012, p. vi). Although it was recognized that the experience was common, and naming it was welcomed, flow was seen as “not amenable to scientific investigation” (ibid.). The main difficulty was that collecting data through diaries, interviews and questionnaires was difficult, imprecise, and retrospective, whereas flow is found in the lived experience, in the stream of consciousness. Csíkszentmihályi and his collaborators recognized the fact that a new, more immediate tool was required, a tool which would enable participants to record their feelings and emotions in the moment. Thus, Csíkszentmihályi and his collaborators developed the Experience Sampling Method (ESM), and first used it to conduct research into the daily lives and experiences of adolescents (Csíkszentmihályi, Larson & Prescott, 1977). Participants in an ESM research carry a pager which sends signals at random times of the day. Whenever they are signalled, participants complete a report (called an Experience Sampling Form) both about their objective situation, and their subjective experience. For instance, participants first report on where they are, what they are doing, and whom they are with. Then, they are asked to answer questions regarding their thoughts, emotions, motivations, and perceptions of their objective situation. These questions are formulated as semantic differentials (active-passive; alert-drowsy, etc.) and Likert-type scales, asking participants to rate how well they were concentrating, how much they felt in control, whether they felt that their skills and the challenge were matched, and other questions relevant to assessing subjective states.

In addition to its many advantages, ESM does however present some disadvantages, which researchers have recognized (Scollon, Kim-Prieto & Diener, 2003). Filling out the Experience Sampling Form may be disruptive, leading to attrition of participants, and thus to self-selection bias. Furthermore, ESM may not be suitable when participants are engaged in activities which cannot be interrupted; in some cases, the interruption itself changes the course of the experience, and has the potential to sabotage the very elements it seeks to measure: loss of self-consciousness, concentration, sense of timelessness. In many of these cases, interviews, questionnaires and observational techniques may be more appropriate (Delle Fave, Massimini & Bassi, 2011; 2010). Nevertheless, due to its ability to capture experiences, emotions, and motivations as they occur, ESM revolutionized the research on flow. The importance of ESM in flow research cannot be overstated: suddenly, there was a systematic approach that revealed the inner meanderings of consciousness. Flow theory was ready to *play in the major league*.

2.3. Flow research takes off

ESM opened a window on the quality of subjective experiences; it became possible to explore activities that were not play or leisure, realms traditionally associated with flow theory. Research conducted by LeFevre (1988) and Csíkszentmihályi and LeFevre (1989) suggests that people have flow experiences not only in leisure, but also at work; in fact, substantially more flow experiences at work than in leisure. Thus, far from being a "marginal diversion", flow theory gradually proved itself to be applicable in numerous areas. According to Hektner, Schmidt and Csíkszentmihályi (2007), among the most fruitful topics for research were (1) optimal experience of flow, (2) the experience of adolescents, especially at school, (3) the experience of work, (4) the experience of families, (5) the experience of issues pertaining to mental health, (6) the experience of media, particularly TV, (7) cross-cultural research on quality of daily experience, (8) gender differences and the quality of experience, and (9) the experience of friendship and solitariness. While flow research is all very useful and enlightening, this review will mainly concern itself with categories (1) and (2) in the next sections.

2.3.1. Research on optimal experience and the Milan group

Early ESM research was not without its trials and tribulations: when the data generated by the studies was analyzed, it became apparent that, although the imbalance between challenge and skills had the expected effects – boredom or anxiety, this was not the case for the theoretical prediction regarding the balance of challenge and skills as a basis for an optimal experience. Csíkszentmihályi recognizes that in those early studies, subjects who experienced a balance between challenge and skills were not always in flow, and admits that this aspect remained "for years […] a frustrating puzzle in an otherwise fruitful research program" (1988, p. 260). Efforts to modify the Experience Sampling Form, in an effort to obtain a more detailed picture of inner fluctuations, were unsuccessful, until a conceptual breakthrough occurred under the aegis of Fausto Massimini, from the University of Milan. Massimini and his team posited that flow occurs when the challenge and the skills are in balance, and **above the person's average** level during the testing period (Massimini & Carli 1988).

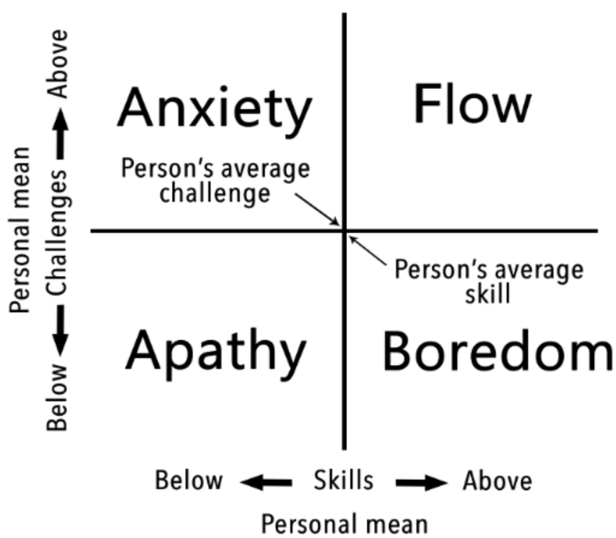


Figure 3. The four-channel model of flow.
The origin represents the subject's average level of challenge and skills during a given period of time. Situations when the challenge and skills are in balance, but are situated below this cut-off point, are experienced as apathy. (Csíkszentmihályi & Csíkszentmihályi, 1988, p. 261).

The new model (see Figure 3) mitigates one of the limitations of the first flow model, clearly diagnosed by Csíkszentmihályi in his original article. When discussing the model, he observed:

> The problem is that whether a person is going to be in flow or not does not depend entirely on the objective nature of the challenges present or on the objective level of skills. In fact, whether one is in flow or not depends entirely on one's perception of what the challenges and skills are. With the same objective level of action opportunities, a person might feel anxious one moment, bored the next, and in a state of flow right afterward. So it is

impossible to say with complete assurance whether a person will be bored or anxious in a given situation. (1975b, pp. 56-57)

In the four-channel model, since the baseline for the individual average is now standardized through ESM, the element of uncertainty that concerned Csíkszentmihályi is very much reduced. Furthermore, through the four-channel model, a new state was revealed: apathy, in which both challenges and skills are low, and which represents “a sphere of stagnation […] the inverse of flow” (Csíkszentmihályi & Nakamura, 2005, p. 95).

Even more refinements to the model emerged from further studies conducted by the Milan group. Following their study on Italian adolescents, Massimini, Csíkszentmihályi and Carli (1987) proposed a model with eight sectors (see Figures 4, 5), each representing a state of the challenge-skill ratio:

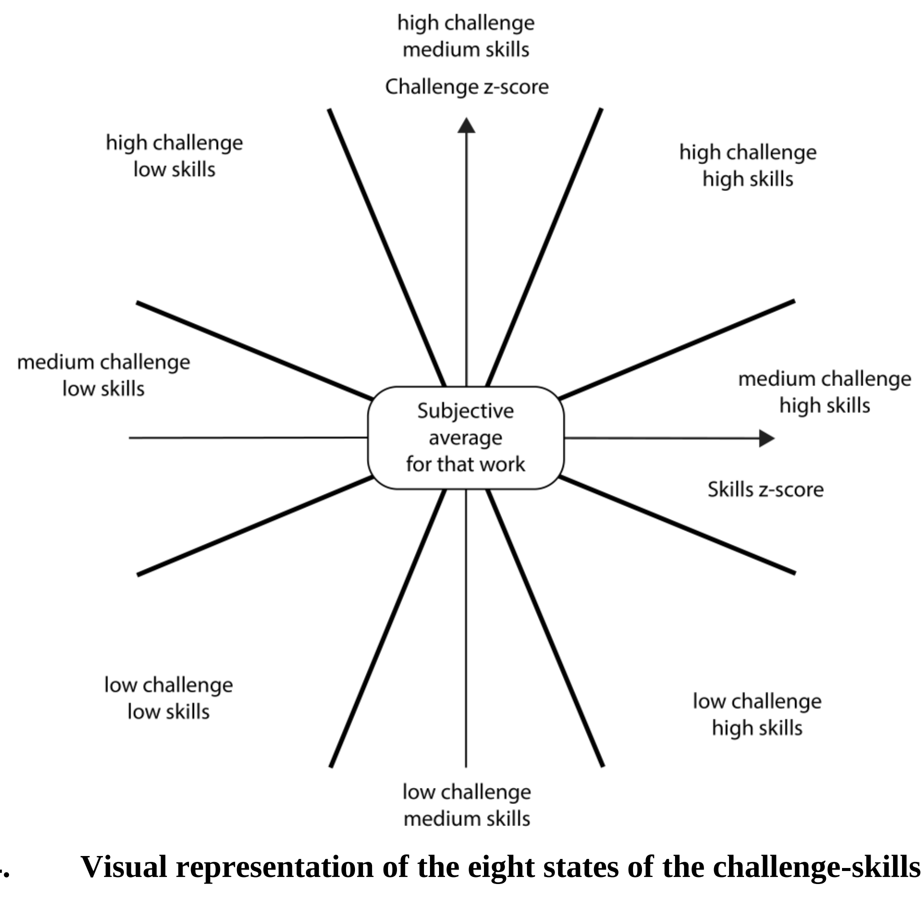


Figure 4. Visual representation of the eight states of the challenge-skills ratio

(ibid p.46)

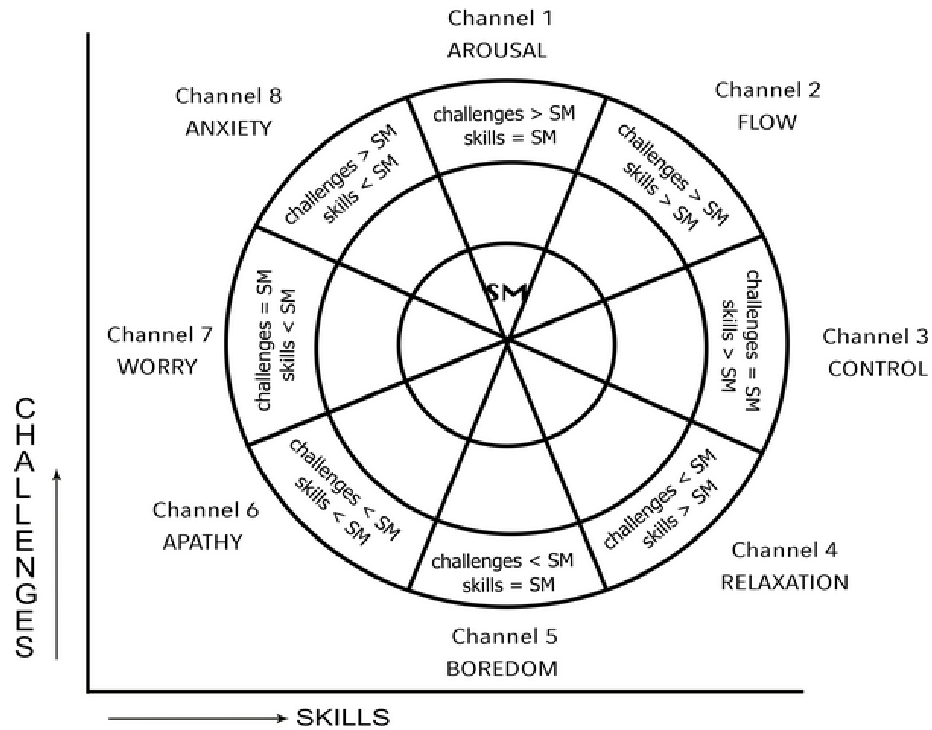


Figure 5. The Experience Fluctuation Model.

The Experience Fluctuation Model, also known as the "octant" model, is modification of the previous model. (Delle Fave et al., 2011; 2010, p. 73). The experience becomes more intense in the rings further away from the average levels of challenge and skills, denoted by SM (subjective mean).

ESM research, conducted at the University of Chicago, and at the University of Milan, on a variety of samples consisting of both adolescents and adults, has confirmed that channel 2 is a state of flow, characterized by concentration, control, clear goals and feedback, intrinsic motivation and enjoyment (Csíkszentmihályi & Csíkszentmihályi, 1988), and that after channel 2, the best experiences are situated in channel 1 – arousal, high cognitive engagement, due to a tolerable discrepancy between above average challenges and average skills, and channel 3 – control, concentration, skills above average and challenge approximately average (Massimini & Carli, 1988). Surprisingly, it is not channel 8 – anxiety, associated with negative affect and lack of control, but high cognitive involvement and high concentration – which presents the worst experience, but channel 6 (apathy), with low values for all components of the experience (Delle Fave et al., 2011; 2010). The effects of having frequent apathy experiences in daily life go well beyond a mere lack of flow, and may lead to pathological outcomes for mental health (Delle Fave & Massimini, 1992, 2005).

While numerous studies which measure flow using ESM have confirmed that the balance between challenges and skills has a positive effect on the quality of the experience (Chen, Wigand & Nilan, 1999; Delle Fave & Massimini, 2005; Guo & Poole, 2009; Keller & Bless, 2008; Moneta & Csíkszentmihályi, 1996; Pearce, Ainley & Howard, 2005; Sherry, 2004), some results showed deviations from theoretical expectations (Hektner, 1996). New research directions gave rise to some uncovered contradictions and an interest to probe even more deeply into the subtle variations of flow experiences, were emerging.

2.3.2. A different conceptualization of flow

All the models presented above are based on Csíkszentmihályi's original insight that flow was relative to the balance of challenge and skills. However, from the very beginning, Csíkszentmihályi insisted that flow is a "holistic sensation that people feel when they act with total involvement" (1975a, p. 36), and subsequent research has established that all components are equally important and, indeed, highly correlated (Engeser & Schiepe-Tiska, 2012). Paying attention only to one component of the flow experience – the balance between challenge and skills is likened by Engeser and Schiepe-Tiska to flying a plane while keeping an eye on a single flight instrument (ibid., p 4). They point out that, firstly, the balance of challenge and skills is not a guarantee that flow will occur, but a mere indication that flow is *more possible.* Secondly, although the components of flow are highly correlated, they could, at times, be dissociated: for instance, "centering the attention on a limited stimulus field" could be a sign of flow, and equally a sign of high anxiety (Eysenck, 1992). Nor is the experience of temporal distortion by itself a sign of flow: temporal contraction can occur not only when the experience is enjoyable, but also in extremely adverse circumstances (Flaherty, 1999).

Thirdly, it has been argued that the value and the stakes of the activity influence the dynamics between flow and the balance challenge-skills: for activities with low stakes and unimportant consequences, a balance between challenge and skills leads to flow; however, for high-stakes activities, with important consequences, a person feels more in control, and is more likely to be in flow, when the skills slightly exceed the difficulties (Engeser & Rheinberg, 2008). Furthermore, even within the same context, the balance between challenge and skills may have a positive effect on some flow components and no effect on others (Moneta & Csíkszentmihályi, 1996). Finally, the balance between challenge and skills also affects people differently: Csíkszentmihályi spoke about the existence of "autotelic personalities" (1975b, 1990), individuals who are "*never bored, seldom anxious, involved with what goes on"* (1990, p. 209). In such circumstances, it may be difficult to assess whether a person is in flow, based solely on measurements taking into account the balance between challenge and skills.

Many researchers have therefore turned to flow questionnaires and standardized scales which assess the componential aspects of flow. These tools were first used in sport research (Andrew & Jackson, 2008; Jackson & Marsh, 1996), and are now becoming more prevalent in other areas as well, for instance studying for exams (Cermakova, Moneta & Spada, 2010; Engeser & Rheinberg, 2008), working (Fullagar & Kelloway, 2009), researching involvement and motivation (Keller & Bless, 2008), or on flow and incentives (Schüler, 2010). As with ESM flow research, a variety of componential models have been devised. Most researchers use either the Marsh and Jackson model (1999), in which the nine components are correlated, and all contribute to the flow experience (Figure 6), or the Quinn model (2005, Figure 7), which starts from the premise (set by Csíkszentmihályi) that "the clearest sign of flow is the merging of action and awareness" (1975a), and separates the rest of the components into antecedents of flow (goal clarity, balance of skill and challenge, concentration and feedback) and consequences of flow (sense of control, autotelic experience, loss of self-consciousness, transformation of time).

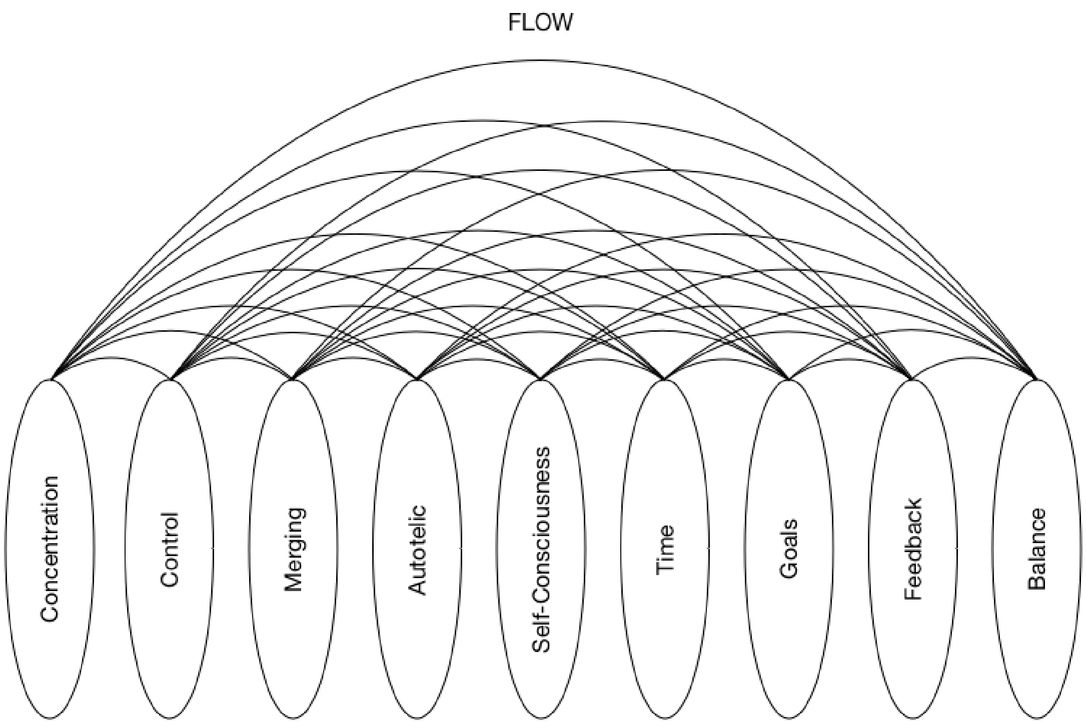


Figure 6. The Marsh-Jackson model

(adapted from Moneta, 2012, p. 42)

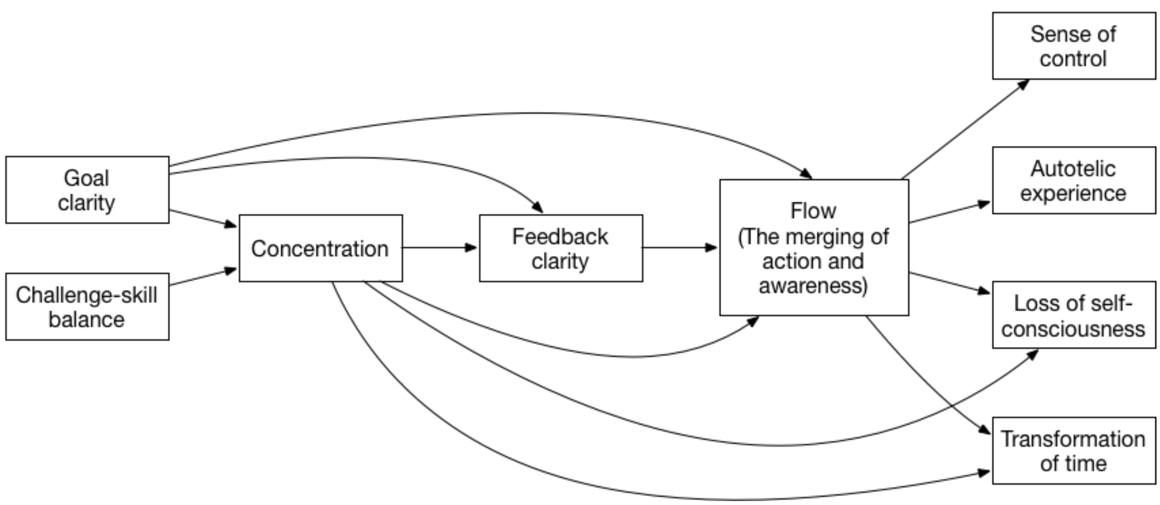


Figure 7. The Quinn model

(adapted from Quinn, 2005, p. 617)

Both models are functional and appropriate. Indeed, Csíkszentmihályi's view is that

> There is no way to decide this except to try. You know you try one way; you try another, and see which one works best because there's no outside measure of authority that can tell you which is it definitively…I think you can work with either one of those two and just see which one seems to be more supported by the data. In my mind, they're not that different. I think all nine components are important. It's true that the first three or four may be conditions. (interview by Beard, 2015, p. 357)

In research, this is exactly what happened: in some cases, data favours the Marsh-Jackson model as a framework – for instance, in research conducted into elementary school teachers' flow (Beard, 2008), while the Quinn model was more suited to researching flow in engineers and technicians engaged in knowledge work (Quinn, 2005).

The process of developing and refining methods for measuring flow is by no means complete. Giovanni Moneta states that:

> […] no existing measurement method for flow and associated model is watertight and […] a gold standard for the modeling and measurement of flow is not at close reach. (2012, p. 46)

Nevertheless, despite divergences and contrasts that appear when fine-grained research methods are employed, it remains clear that a person experiences the highest levels of well-being when their challenges are in line with their skills (Csíkszentmihályi, 1975, 1982b; Csíkszentmihályi & Csíkszentmihályi, 1988; Csíkszentmihályi & Nakamura, 1989; Massimini et al., 1987). These findings have been confirmed by research conducted in other countries and in other cultures (Csíkszentmihályi & Wong, 1991; Delle Fave, 2007; Delle Fave et al., 2011; Delle Fave & Massimini, 1988; Massimini et al., 1987), and support Csíkszentmihályi and his collaborators' view that flow is a universal experience (Csíkszentmihályi & Asakawa, 2016; Massimini, Csíkszentmihályi & Delle Fave, 1988).

2.3.3. New directions in flow research - physiological measures of flow

It has now become possible, through new technological devices, to monitor the dynamics of flow without disrupting it. Csíkszentmihályi, who was initially reluctant to use physiological measures, arguing that "it's hard enough to understand what people say… knowing that something happens in your brain is not going to help you really understand much" (interview by Beard, 2015, p. 359), has somewhat nuanced his position and has declared himself interested in results emerging from physiological research which "illuminate flow" (ibid.), for instance research which recorded changes in vital signs in professional piano players in flow. He is referring to research conducted by Blom and Ullén (2008) and de Manzano, Theorell, Harmat and Ullén (2010) who, using an impressive array of instruments, tracked blood pressure, heart rate variability, heart period, activity of the zygomaticus major muscle, and respiratory depth, and uncovered a particular pattern of biological correlates of the flow experience.

Other studies indicate that flow is associated with low levels of salivary cortisol, which in turn is correlated with lower levels of stress and blood pressure (Adam, 2005; Matias & Freire, 2009). Equally interesting is the work of researchers who are using fMRI of Transcranial Magnetic Stimulation in order to allow them to observe particular regions of the brain when a person is in a state of focused attention (Buzsáki, 2006; Dietrich, 2004; Raz & Buhle, 2006; Weber, Tamborini, Westcott-Baker & Kantor, 2009). However, such research is still in its infancy, and most current flow researchers heed Csíkszentmihályi, who feels that the best way to find out what people experienced is to talk to them – *"because that's the bottom line, the experience"* (interview by Beard, 2015, p. 359).

2.4. Flow in education

In the conclusion of his very first article on flow (1975b), Csíkszentmihályi presciently wonders:

> Is it possible to restructure standard settings for activities (e.g., jobs, schools, neighborhoods, family interactions, and so on) in such a way as to increase the flow experiences they can provide? This question is important for its ecological consequences. As long as we continue to motivate people mainly through extrinsic rewards like money and status, we rely on zero-sum payoffs that result in inequalities as well as the depletion of scarce resources. It is therefore vital to know more about the possible uses and effects of intrinsically rewarding processes. (p. 61)

Firstly, it has to be pointed out that although he placed "jobs" and "schools" in the same context, these two environments are not the same, flow-wise. Csíkszentmihályi established early in his research that people have consistent flow experiences at work (Csíkszentmihályi & LeFevre, 1989), whereas in school, "not so much" (interview by Beard, 2015, p. 356). The work environment, Csíkszentmihályi argues, is structured in a way that is more flow friendly: there are "clear goals, immediate feedback, you can use your skills and if the work is at all reasonable, you can experience flow" (ibid.). In contrast, in schools, even when one has skills and the work is reasonable, even talented students have difficulty experiencing flow (Csíkszentmihályi, Rathunde & Whalen, 1993).

With that in mind, the question remains: why is it important for students to be in flow at school? At the time when Csíkszentmihályi first called for more flow in education, his reasons were based merely on his axiomatic conviction that flow is a good experience to have. However, ensuing studies suggest that Csíkszentmihályi's initial thesis was correct, and the effects of flow go far beyond momentary satisfaction.

2.4.1. The effects of flow

Several research avenues have been explored with regards to the consequences of flow: affective consequences, cognitive consequences, and performance-related consequences.

2.4.1.1. Affective consequences

In Csíkszentmihályi's view, a central part of flow is the autotelic experience, defined as

> The state in which people are so intensely involved in an activity that nothing else seems to matter; the experience itself is so enjoyable that people will do it even at great cost, for the sheer sake of doing it. (1990, p. 4)

Although his original observations about flow were situated in the context of artists and mountain climbers, whose motivation to engage in said activities is purely intrinsic, Csíkszentmihályi also pointed out that the concept of autotelic experience does not imply that the activity cannot have external goals or rewards: "such an assumption is not necessary for flow" (1975a, p. 36). Indeed, flow can be experienced in any activity, as evinced by his later research (2014b): it is possible to enjoy an activity that one is obliged to perform, provided that one is willing to make an "initial expenditure" of skill practice (ibid., p. 167). New conceptualizations of flow argue that in an autotelic experience, the intrinsic motivation stems from the engagement in the activity, regardless of the context of the activity (Schüler & Engeser, 2009). In view of this, it may be more appropriate to adopt Sansone and Smith's definition of intrinsic motivation, which states:

> We consider individuals to be intrinsically motivated when their behavior is motivated by the **actual, anticipated, or sought experience of interest**. (2000, p. 343)

Current flow research distinguishes between, and focuses on, the two ingredients of the autotelic experience: one is motivated to pursue the activity (the motivational component), and one enjoys the activity (the experiential component). Most studies conducted have analyzed the relationship between these components and flow through the lens of the balance between challenge and skills, and suggest that such a fit does, on average, have a positive effect both on motivation (Csíkszentmihályi & LeFevre, 1989; Csíkszentmihályi & Schiefele, 1994; Haworth & Hill, 1992; Keller, Ringelhan & Blomann, 2011; Moller, Meier & Wall, 2010; Moneta & Csíkszentmihályi, 1996) and on enjoyment (Keller & Bless, 2008; Keller & Blomann, 2008). The latter effect is more pronounced when a person has an internal locus of control (Keller & Blomann, 2008) or a high action orientation (Keller & Bless, 2008) – traits which have been associated with autotelic personalities (Baumann, 2012).

Even more provocative is the hypothesis, of which Csíkszentmihályi is a strong proponent, that happiness and flow are inter-connected. He argues that flow is a "dimension of happiness" (1999), and proposed that their relationship is mediated by complexification. He states:

> […] any intentional, intrinsically motivated, and autotelic activity must lead to learning, that is, to changes in the complexity of the organism. Moreover, I shall propose that this kind of learning is the avenue for personal growth that approximates most closely the state of happiness. (2014b, p. 154)

While multiple studies link flow and happiness (Asakawa, 2004; Csíkszentmihályi & Csíkszentmihályi, 1988; Csíkszentmihályi & Hunter, 2003; Csíkszentmihályi & LeFevre, 1989; Csíkszentmihályi & Wong, 1991), other researchers caution about making too close an equivalence between flow and happiness. Engeser and Tiska contend that

> Flow is not happiness, but it is correlated with or related to happiness. A person in flow does not have the conscious experience of being happy. This would even terminate the total immersion in the activity. Flow is not defined by affective means […] However, flow is a rewarding experience, which subsequently leads to happiness and satisfaction. In general, it also provides fulfillment for the person who experiences flow, and lends structure and meaning to life, even to the point of being part of the personal identity. (2012, p. 21)

Though it may seem a trifling point, the fact that flow is generally correlated with happiness rather than identical with happiness is nonetheless important: firstly, because this facile assimilation appears to overlook the investment of effort and attention necessary to reach a state of flow, and the inevitable feelings of frustration due to periodic setbacks which occur during a flow experience; secondly, because happiness derived from enjoyment of an activity is not the same as a global state of happiness; and thirdly, because culturally, we see happiness as a *moral good*, whereas flow is like fire: "you can use to cook a meal or to burn down a house" (Csíkszentmihályi, 1997c, p. 14). Indeed, numerous examples of *maladaptive flow* – flow in service of destructive ends – have been noted and examined, in school crime (Csíkszentmihályi & Larson, 1978), motorcycle gangs (Sato, 1988), stealing (Delle Fave & Massimini, 2005), and drug addictions (Delle Fave & Massimini, 2003).

2.4.1.2. Cognitive consequences of flow

Since concentration is one of the components (or antecedents, depending on what model is adopted) of flow, it is hypothesized that frequent flow experiences may increase a person's attention span and ability to concentrate (Landhäußer & Keller, 2012). Similarly, inasmuch as flow is also characterized by a lack of self-consciousness, which translates into less stress on the individual, flow experiences may also allow for better management of self-regulatory resources, diminishing the risk of ego-depletion (Baumeister, Bratslavsky, Muraven & Tice, 1998). However, at this point, there is not much research to address the topic of cognitive consequences of flow, and the relationships postulated above have not been systematically tested.

2.4.1.3. Performance-related consequences of flow

A positive relationship between flow and performance is highly plausible: firstly, the concentration and sense of control which are defining components of flow are direct mediators of performance (Eklund, 1996). Furthermore, since people experience flow as intrinsically motivating, they are likely to seek to repeat the activities that they enjoyed (Engeser & Rheinberg, 2008). Hence, due to this tendency to replicate an experience that was enjoyable, people will improve their performance in the activity; simultaneously, seeking flow will propel people further up the flow channel, looking for new, progressively more complex challenges, thus leading to mastery and growth (Csíkszentmihályi, 1988; Csíkszentmihályi & Larson, 1978; Csíkszentmihályi, Abuhamdeh & Nakamura, 2005; Nakamura & Csíkszentmihályi, 2009; Shernoff et al., 2003).

It has been argued, however, that since all the studies mentioned are correlational (Engeser & Schiepe-Tiska, 2012), the relationships illuminated above may be reciprocal, and it is yet unclear whether flow causes a good performance, or whether a good performance causes flow. But even taking that into account, it is worthy of note that compelling associations have been found between flow experiences and performance in university courses, as measured by final grades or quizzes (Culbertson, Fullagar, Simmons & Zhu, 2015; Engeser & Rheinberg, 2008; Engeser, Rheinberg, Vollmeyer & Bischoff, 2005) and flow experiences and learning in high school students (Delle Fave & Bassi, 2000; Nakamura, 1988; Shernoff et al., 2003). Correlations have also been found between flow experiences and facilitators of performance, such as a decreased tendency to procrastinate (Lee, 2005), and increase in interest (Culbertson et al., 2015) and curiosity (Kashdan, Rose & Fincham, 2004).

2.4.2. Now that we know flow, what are we doing with it?

For the most part, nothing much: school has been suffering from a (sadly, deserved) image crisis for a very long time. William Blake’s portrayal of the soul-crushing joylessness of a school day, rendered in his poem “The School Boy”, may, alas, still ring true:

> Ah then at times I drooping sit,
> And spend many an anxious hour;
> Nor in my book can I take delight,
> Nor sit in learning's bower,
> Worn through with the dreary shower.

This association between school and lack of enjoyment is present in our daily life: for instance, children celebrate the days off school, with the blessing of many adults in their lives. Equally true ring the observations of anthropologist Jules Henry, who remarked that our very culture is ambivalent about the role of education and scholarship, with schools, just like society at large, divided in the conflict between “hedonistic mindlessness and austere intelligence” (1963, p. 281).

It appears that things have not changed much: contemporary research which attempts to juxtapose “school” and “flow” makes for a sobering, disquieting read. After conducting study after study, Csíkszentmihályi and his collaborators were forced to conclude that students do not experience flow at school, despite the fact that schools may contain some of the elements that would make flow possible. They write:

> Troubling for the field of education is the finding that students rarely experience flow in school. Adolescents spend considerable amounts of time in school, and school is ideally a place where children and adolescents may identify interests and passions that will lead to meaningful and productive careers. The fact that so little flow occurs in school suggests that this potential is not being harnessed in formal educational settings. (Schmidt, Shernoff & Csíkszentmihályi, 2007, p. 555)

This comes in stark contrast with the attitude of children before they go to school, when the natural connection between learning, growth and enjoyment has not yet disappeared and children are still veritable “learning machines” (Csíkszentmihályi, 1990, p. 47). Csíkszentmihályi posits that this lack of flow in schools is due to the fact that learning is no longer an activity freely engaged in, but slowly becomes an obligation, which inhibits the excitement of learning (ibid.); following from this, he deplores what he sees as an underlying assumption that students respond only to rewards and punishments, an assumption which is often built into the fibre of school as an institution (1982a), and which “tends to destroy any enjoyment in learning that may already be there” (Csíkszentmihályi & Larson, 1978).

2.4.3. Schools through a flow lens – the case of Montessori schools

Shernoff (2013) contends that "flow is not free-flowing" in modern schools, and argues that change should take place within whole schools, as the school is "smallest institutional unit in which a culture is built and maintained" (ibid., p. 219). There are significant advantages to having schools where flow experiences are a permanent focus: LeFevre (1988) found that people who had had several flow experiences throughout a week felt happier and more sociable than people who hadn't had these experiences, while Adlai-Gail (1994) uncovered the existence of a "carryover effect" of flow. Shernoff's concern is that episodic flow experiences in school, though enjoyable, will be connected with a specific teacher, course, or activity, and that once the students leave that environment, they re-enter the same unhealthy ecology of external incentives and mismatch between challenge and skills. It comes as no surprise, therefore, that Csíkszentmihályi and other flow researchers hold Montessori schools high in their regard, as examples of institutions whose goals best align with flow theory.

Montessori's educational philosophy is centred on fostering deep concentration and engagement in children, viewing them as trusted, "motivated doers" (Lillard, 2005, p. 28). Montessori charted a detailed curriculum for 3 to 12-year-olds, designed aesthetically pleasing, self-correcting, sequential teaching materials, and established a schedule with long periods of uninterrupted time, all in aid of creating what she called "a prepared environment"; soon, her success in educating children, some with intellectual disabilities, became well-known and internationally admired (Thayer-Bacon, 2012).

Based on her own research with Italian children at the beginning of the 20th century, Montessori elaborated eight principles of learning: 1) physical movement promotes thinking and thus learning; 2) children learn better when have a sense of control over their lives; 3) as a corollary of the previous principle, children should have a say in what they are learning, and should be allowed to choose topics they are interested in; 4) extrinsic incentives, such as grades, gold stars, etc., interfere with children's learning and thus should be avoided; 5) peer collaboration enhances learning; 6) hands-on learning may be deeper and richer than abstract learning; 7) certain forms of adult interaction are associated with better child outcomes; and 8) certain patterns of order in the environment are beneficial to children's learning, supporting, rather than hindering, the freedom of choice that stands at the core of the Montessori's philosophy (Hainstock, 1997). A more detailed discussion of how these principles of learning are situated in the context of current research will take place in subsection 2.4.4.

When Csíkszentmihályi and his collaborators turned their scientific eye on Montessori schools, their research, which compared Montessori students with students from traditional schools, found that students from the Montessori schools reported a vastly superior quality of experience at school, enjoyed what they were doing much more often, and wanted to do academic work more than their traditional school counterparts (Rathunde, 2003). They also reported higher intrinsic motivation and interest, and more concentration and flow experience in academic work than students from traditional middle schools (Rathunde & Csíkszentmihályi, 2005a). Furthermore, Montessori students spent more time doing academic work and chores, more time in collaborative or group work, more time working on individual projects, and spent less time watching media, less time listening to lectures and taking notes, compared to the students in the traditional school (Rathunde & Csíkszentmihályi, 2005b). Finally, an important point of convergence between Montessori's approach and flow theory is the persistent focus on attention and concentration (Lillard, 2005). Montessori called children who could sustain deep concentration "normalized", which was meant to convey the fact that under appropriate (normal) conditions, children are naturally absorbed by the activity they find of interest. Both Csíkszentmihályi and Rathunde recognized that the concept of "normalization" has many similarities to the "autotelic personality", ascribed by Csíkszentmihályi to people with the ability to self-regulate attention (Rathunde & Csíkszentmihályi, 2006; Rathunde, 2014).

Some differences must be acknowledged: unlike Montessori, Csíkszentmihályi makes no prescriptions with regards to how exactly people (and, in particular, children) can reach and maintain a state of flow, other than the general recommendations with regards to antecedents of flow: clear goals, feedback, balance of challenge and skills. Nevertheless, it is not difficult to recognize, firstly, that having more detailed "guidelines", if such guidelines existed, would be helpful; and secondly, that Montessori's educational philosophy intersects flow theory in some essential aspects: the insistence on freedom and choice, the reliance on teaching materials in which both goals and feedback are embedded, the avoidance of extrinsic rewards, the respect for deep concentration and enjoyment, the importance attached to the learning context, and the connections between the Montessori's "prepared environment" and flow theory's "complex environment" (Rathunde, 2001). Some of these possible guidelines are discussed below.

2.4.4. Montessori schools – a possible recipe for background facilitation of flow

Of course, as Rathunde (ibid.) put it, there is no "monopoly" on good teaching, and there may be many schools where students find flow experiences; nevertheless, Montessori education, through its founding principles, appears to cultivate the appropriate characteristics in the school culture for flow to occur predictably and consistently. This section will examine more closely some of the characteristics outlined above and how they impact the environment of the classroom.

2.4.4.1. Physical movement

Juvenal's astute comment that we should wish for "mens sana in corpore sano" has gained new currency with the advent of research into the concept of "embodied mind" (Gallagher, 2005; Foglia & Wilson, 2013; Johnson, 2007; Thompson, 2007). Physical activity offers well-documented physiological benefits: indeed, exercise increases the level of dopamine, serotonin, and norepinephrine, all neurotransmitters that positively affect focus and mood (Willis, 2010). On the contrary, low physical activity is associated with two antagonists of flow, boredom and decreased attention (Ragheb & Merydith, 2001; Smith, 1981; Sommers & Vodanovich, 2000; Thackray, 1981). Csíkszentmihályi laments the passivity in regular classrooms (Rathunde & Csíkszentmihályi, 2005a, 2005b), and offers Montessori schools as an example of environment in which movement is seen as pivotal for learning. Montessori herself stated that:

> Up to the present most educationists have considered movement and muscles as a help to breathing, improving the circulation, etc., or, if movement is indulged in, it is to acquire greater muscular strength. It remains a part of physical education only. What is the individual supposed to do with it? Our new conception stresses the importance of movement as a help to the development of the brain, once it is placed in relation to the centre. Mental development and even spiritual development can and must be helped by movement. Without movement, there is no progress and no health (mentally speaking). This is a fundamental fact which must be taken into consideration. (1949, p. 203)

In Montessori's view, movement is not an optional flourish, but a well thought out strategy to enlist mind and body in the service of learning. Similarly, in an autotelic classroom, physical movement would be wisely incorporated into the very fabric of the classroom, instead of being a cherished exception, or an intermission between two episodes of "real work" (Liljedahl, 2016; Willis, 2006).

2.4.4.2. Choice and control

Two intertwining threads emerge in a discussion about how students can gain control in school. Firstly, there is the matter of the relationship between the person and the institution. Even in contemporary schools, there still are significant structural obstacles which substantially hinder student choice and control. They are not easy to change or remove, due to the persistence of beliefs about human nature being intrinsically corrupt and destructive, beliefs much deplored by Dewey:

> It is sometimes assumed, explicitly or unconsciously, that an individual's tendencies are naturally purely individualistic or egotistic, and thus antisocial. Control then denotes the process by which he is brought to subordinate his natural impulses to public or common ends. Since, by conception, his own nature is quite alien to this process and opposes it rather than helps it, control has in this view a flavor of coercion or compulsion about it. Systems of government and theories of the state have been built upon this notion, and it has seriously affected educational ideas and practices. But there is no ground for any such view. (1990, p. 19)

Just how much control can be taken from the system and (perhaps gradually) given to the students is a matter for experimentation. Given that schools are communal places, meant to serve so many different needs, it is possible that they'll never afford to fully abide by

Aristotle’s wisdom, according to which “life is made enjoyable by the unimpeded activities of our natural state”.

A more fruitful discussion would examine issues of control and choice in the more intimate context of the classroom, where considerably more latitude is available for the students (and for teachers). Here, even small changes can yield great benefits: Csíkszentmihályi assures us that “whatever a teacher can do to transfer the responsibility for the learning process to students ought to bring great dividends in intrinsic motivation” (1982, p. 25), and in that necessary sense of “wanting, choosing and personal endorsement” (Deci, 1992, p. 44). Experience tells us that students do have autotelic experiences at school; therefore, ways exist in which to mitigate a restrictive and restricted environment. It has been suggested that it is possible to achieve an autotelic environment by cultivating a verisimilar “the illusion of control” (Alloy & Abramson, 1979, 1982) and a perception of choice (Perlmuter & Monty, 1979; Perlmuter, Scharff, Karsh & Monty, 1980).

In a classroom, these control-effacing measures may take various forms: flexibility in assessment, a choice and variety of tasks, involving students in taking decisions – all wisely already implemented by many teachers. Further examining the issue of control in a classroom, an approach may be adopted which parallels the Karasek model of an enjoyable workplace (Karasek, 1979; Karasek & Theorell, 1990). The model suggests that high levels of job demands, decision latitude, and social support – in a balanced interaction – will lead to high levels of job satisfaction. The similarity between Csíkszentmihályi’s flow model and Karasek’s model is no coincidence, and stems from the universality of what constitutes a good experience, be it at school, at work, or in leisure. Thus, a situation in which the demands are high, but the control and support are low, is anxiety-provoking. Conversely, a situation in which the demands are low, and decision latitude and support are high, will lead to apathy and boredom. Since a balance between demands and control is just as important as the balance between skills and challenge, it follows that in an autotelic school environment, increases in challenge must come with corresponding increases in decision latitude for students.

Decision latitude and control are however not necessarily the teacher's to give or take. We all – adults, adolescents, children – live in a world where we are not truly free in our choices or our ability to exercise control, a world in which some flo;urish, and others trudge along. What sets apart the former from the latter is the fact that they see new possibilities in unremarkable places. In Csíkszentmihályi's words:

> To most people, a vertical slab of rock does not present opportunities for action; it is something to be glanced at and immediately forgotten. To a rock climber, it might constitute an exquisite sequence of challenges to be savored for hours or days. (2014b, p. 160)

A non-climber cannot see the potential delectations of the rock, or might even fear it, because he or she does not have the skills to recognize the fact that the rock is not a rock, but a game waiting to be played. An autotelic classroom context is able to modify this perception by recognizing the need to increase the skills of the student until the student perceives them as aligned with the challenge. Csíkszentmihályi et al. (2005) posit the existence of an "emergent motivation", stimulated by discoveries made throughout an activity which at first may fail to engage, or may appear opaque and uninteresting. Bandura suggests that a similar phenomenon is occurring for latitude: as the skills of a person improve, and their perception of the activity changes, they begin to see new choices and ways to take control:

> Through agentic action, people devise ways of adapting flexibly to remarkably diverse geographic, climatic and social environments; they figure out ways to circumvent physical and environmental constraints, redesign and construct environments to their liking, create styles of behavior that enable them to realize desired outcomes, and pass on the effective ones to others by social modeling and other experiential modes of influence. […] Growth of knowledge is increasingly enhancing human power to control, transform, and create environments of increasing complexity and consequence. (2001, p. 22)

Thus, in order to create an autotelic environment, the goal is not a complete removal of obstacles, but rather accepting limitations and finding ways to master them.

2.4.4.3. External incentives

The deleterious effect of external incentives on the enjoyment of learning has been thoroughly and abundantly demonstrated (Niemiec, Ryan & Deci, 2009; Vansteenkiste, Lens & Deci, 2006; Vansteenkiste, Timmermans, Lens, Soenens & Van den Broeck, 2008). Owing to the overjustification effect, even bona-fide play can be taken away and transformed into work (Lepper, Greene & Nisbett, 1973). With regards to the tendency to look to projected future rewards as external incentives, researchers take a more nuanced position. Categorically, students should be made aware of the usefulness of what they're learning; indeed, helping students make connections between a classroom activity and their future can assist them in discovering new interests and in developing persistence and mastery (Hidi & Renninger, 2006; Simons, Vansteenkiste, Lens & Lacante, 2004). At the same time, Csíkszentmihályi cautions that that too much insistence on long-term goals takes away from the absorption in the present. He distinguishes between motivation stemming from expectations of future rewards, and motivation based on the rewards of the experience itself (Wong & Csíkszentmihályi, 1991), and notes that, of these, only the latter is fully intrinsic and leads to an enjoyable experience.

2.4.4.4. Collaboration begets optimal experience

Classroom cooperative interactions between students have been extensively researched and shown to yield extensive benefits: not only does cooperative learning significantly increase achievement and retention (Johnson, Johnson & Holubec, 1994; Slavin, 1995) and intrinsic motivation (Shachar & Sharon, 1994), but it also promotes an environment in which the needs for competence, relatedness, and autonomy are satisfied (Deci & Ryan, 1985, 2002). Students who collaborate are more likely to manifest a mastery goal orientation (Nichols & Miller, 1994), more likely to find enjoyment in the task (Quinn, 2006), and less likely to engage in off-task behaviour (Johnson & Johnson, 2008). Csíkszentmihályi is emphatic about the importance of collaboration in class:

> Group work is always top. Students work in a group on a problem, are beeped, and fill out the questionnaire. They don't think "I'm doing the group work." They just fill it out like they have filled out all of the previous ones. But when we analyze the data, we know they were in group work because they write down where they are, and what they are doing. When we

> aggregate all the responses from students, they are so much more in flow, than when they are lectured, which is about the lowest, or anything else, including videos. During video presentations there is almost no flow there. We found that there's more flow taking tests or exams. (interview by Beard, 2015, p. 359)

It is important to note that establishing a collaborative environment is not, in theory, a precondition for experiencing flow. Indeed, it is possible to experience flow in solitary conditions (Logan, 1985). However, only a small proportion of gifted young people are able to find flow while engaged in isolated pursuits (Csíkszentmihályi et al., 1993). Research conducted by Csíkszentmihályi and Larson (1984) suggests that only autotelic students can find flow while working alone, and that most adolescents feel anxious and worried in the same condition. They posit that not only partner or group work alleviate this anxiety, but that it also offers clear goals, timely feedback, and enhanced opportunities to find a balance between the skills of the group and the demands of the task.

2.4.4.5. Patterns of interaction with adults

Rathunde (1988) observes that an authoritarian style of parenting is associated with boring, rigid contexts, and a permissive style with anxiety-inducing contexts. The work of Baumrind (1971) and Wentzel (2002) suggests that optimal experiences can only flourish in an environment which presents a balance between freedom and constraint, and thus an authoritative pedagogy.

Rathunde has also theorized that a teacher who has already had experiences of flow is more likely to nurture, understand, and recognize flow in students (2015). He hypothesizes that such a teacher, having intimate and personal knowledge of the extraordinary power of flow to engender growth and mastery-motivated behaviours, will be more likely to divest from extrinsic motivation tools (exams, grades, etc.), and trust instead that flow is "its own reward".

2.4.4.6. Patterns of order in the environment

In his writings, Csíkszentmihályi uses the concept of negentropy – a state of order and harmony, and the opposite of entropy, which is associated with disorder – to characterize a state of consciousness in which the self experiences no inner conflicts or distractions. He then extends the concept to any social system, and even to the classroom environment. Thus, a negentropic classroom is one in which everybody's attention is fully invested in achieving common goals: *"all of the students and the teacher processing the same information without being distracted by extraneous thoughts and feelings",* while the state of negentropy is brought about by the *"ability to respond to opportunities in the environment that one learns about, or actually discovers"* (2014b, p. 159). In contrast, an entropic classroom is one in which

> […] the teacher's actions will produce information that creates conflict in the students. Instead of paying attention to the lecture or the assignment, the students are conscious of boredom, worry, or anger, or else they withdraw into fantasy. (ibid., p. 157)

Unfortunately, Csíkszentmihályi gives few pointers on how a negentropic classroom environment may be achieved in specific, practical terms, other than to indicate, on other occasions, that striking the right balance of support and stimulation is an important factor in fostering an ethos that leads to growth of complexity (Csíkszentmihályi et al., 1993; Csíkszentmihályi & Csíkszentmihályi, 1993). Further related to the dialectic of support versus stimulation, Csíkszentmihályi alludes to the importance of an environment which is both adult-centred (students accept the challenges posed by the teachers, and try to stretch their skills to master them) and child-centred (students develop the ability to discover new challenges based on current skills). Traditionally, mathematics classrooms are seen by students as more adult-centred, while arts classes are perceived as more student-centred (Csíkszentmihályi & Rathunde, 1997). The benefit of combining the two approaches is, in Csíkszentmihályi's view, the most enriching, as it leads to enjoyment and satisfaction in all activities:

> Distractions must be avoided. Irrelevant stimuli – a lecturer's mannerisms or self-indulgent stories, emphasis on meaningless details or bureaucratic procedures – destroy the concentration that makes involvement in the

> learning process enjoyable. A common source of distraction is an unnecessary threat to the students' ego, such as emphasizing grades or ridiculing performance. Creating self-consciousness is a sure way to distract the learner. Whatever the teachers can do to cut out distractions from their main task will increase the enjoyment of teaching. (1982a, p. 25)

2.4.5. Flow in the mathematics classroom

An individual unconnected with education, but well steeped in the zeitgeist, may be forgiven for thinking that there can be no flow in the mathematics classroom. Public opinion vacillates between concerns about math anxiety (Chinn, 2005), apprehensions about mathematical illiteracy (Paulos, 2001) and distress about the boredom reigning in mathematics classrooms (du Sautoy, 2008). Valid as these aspects may be, it seems that wisely optimistic voices, such as Csíkszentmihályi's, are not often heard. He argues that if humans did not find mathematics enjoyable, they would not have pursued it, and writes:

> If thinkers did not enjoy the sense of order that the use of syllogisms and numbers create in consciousness, it is very unlikely that now we would have the disciplines of mathematics and physics. (1990, p. 126)

Therefore, since humans are meant to delight in playing with mathematical abstraction, Csíkszentmihályi sees no reason why flow cannot happen in a mathematics classroom, as long as the teacher is willing to apply the principles of flow. He acknowledges that there may be differences between mathematics and other subjects, but these differences are not necessarily a disadvantage:

> Science and math, for instance, have the initial disadvantage of presenting too many challenges to students, who start out being anxious and often remain in that state without ever enjoying the learning process. But once skills are matched to challenges, it is probably easier to sustain the flow experience in science and math than in humanities or social sciences because the goals, the rules, and the feedback are much less ambiguous in the former. (1982, pp. 26-27)

One might be tempted to think that all a teacher has to do to in order to keep students in flow is to consummately manoeuvre the axiomatic three: balance, goals and feedback. And yet. Shernoff et al. (2003) also note that most students, although they (correctly) rate mathematics as one of the more challenging and relevant subjects, they also view it as one of the least enjoyable. Equally, although they viewed art as the most enjoyable subject, they considered it to be the least relevant. It may seem unfair that mathematics has such a high bar to clear, whereas all arts have to do is show up and look pretty. It appears however that students experience flow differently, depending, among others, on the stakes of the activity (Engeser & Rheinberg, 2008): high stakes activities lead to a situation where students feel flow in high-skills and medium-challenge conditions. Students' perception of their own abilities and skills, and not only their actual skills and abilities, is also relevant (Seegers & Boekaerts, 1993; Shernoff et al., 2003). However, as students get older and become more sophisticated, their judgment of the school challenges and of their own abilities evolves, and they begin to relish challenges, even if it takes them more time to master them (Hektner & Asakawa, 2000; Schweinle, Turner & Meyer, 2009).

Is the answer then to make mathematics class more like art class? Schmidt, Shernoff and Csíkszentmihályi (2007) offer a qualified "yes":

> Our findings suggest that one hope for facilitating more flow in schools would be for academic classes to restructure activities in a way that allows more room for autonomy and interest. While such a conversion may sound simple, our study suggests that multiple conditions are operating simultaneously when flow is experienced; no doubt application to the classroom is no exception. To more fully understand flow, researchers and practitioners need to focus on the multiple conditions from which the flow experience may emerge. (p. 555)

The need for students to experience flow in mathematics is not a frivolous embellishment. Csíkszentmihályi et al. (1993) contend that students are unlikely to reach high levels of proficiency in mathematics if they do not enjoy learning it; furthermore, their study suggests that students who experienced flow in their mathematics classes were more likely to take advanced mathematics courses at college. Students with high grades, but who did not enjoy mathematics, did not pursue such courses (ibid.).

At this point, it may be of interest to explore what the "multiple conditions from which the flow experience may emerge" are, with a focus on the levers a teacher has to influence mathematical experience. Heine's research (1997) suggests that mathematical tasks have an intrinsic incentive value, higher for some than for others. A task of high intrinsic incentive value is enticing and captivating due to a combination of five aspects: control, diversity, novelty, challenge and meaningfulness. A good task affords both intellective rewards (constructing one's own knowledge, mastering a skill) and non-intellective rewards (a-ha moments, aesthetic delectations). While these experiences are fleeting, Heine argues that when juxtaposed and repeated, they explain students' variations in long-term interest and persistence in mathematics noted by Csíkszentmihályi et al. (1993). Heine does not propose a mechanism for this phenomenon, but his contention is supported by later research conducted by Fredrickson (2001), which put forward the *broaden-and-build* theory of positive emotions. According to Fredrickson, positive emotions have an important adaptive role, expanding a person's repertoire of actions, hence enabling the person to build a set of lasting inner resources which can then be mobilized in order to improve outcomes. It may be hypothesized that broadening-and-building is nature's way to get people become attracted to mathematics:

> […] joy sparks the urge to play, interest sparks the urge to explore, contentment sparks the urge to savour and integrate […] (Fredrickson, 2004, p. 1376)

It is fitting and tempting to return full circle to the idea of "play", which informed Csíkszentmihályi's original writings about flow. However, for mathematics teachers wishing to persuade students of the value of mathematics based on its achievements and its applicability, this emphasis on play and enjoyment to the detriment of usefulness, poses some perplexing problems. Heine scorns such utilitarian arguments and argues they should be irrelevant:

> The primary value of mathematics does not have to depend on what others have done for it or what it has done for others. The intrinsic value is that doing mathematics, in particular, having mathematical insights, is inherently pleasurable. Doing math can result in negentropy, which has its own personal, affective meaning…it should be assumed that math can be enjoyed for its own sake, not just its utility. (p. 150)

Shernoff's recent research (2013, 2015) seems to be somewhat at odds with this view, and closer to the position of the hypothetical, pragmatic teacher. His research, done with high-schools students, indicates that the most influential factor in predicting student engagement was not the affective meaning, the intimated promise of negentropy, but rather the importance the students ascribed to the activity in the context of their future goals. His recommendation is therefore that teachers explain to students how and where the task is situated in this larger picture, so that students can fully recognize its value.

Both Heine's and Shernoff's positions seem to be in some disagreement with Csíkszentmihályi's. In his view, the topic itself is a mere pretext, and all that students should focus on is becoming good at it – whatever "it" is. He states:

> Learning Latin or trigonometry can be enjoyable. However, it is crucial that emphasis not be on the mastery of the subject matter but on the process of mastery itself. The important point is not that students learn trigonometry but that they learn to enjoy the act of learning. (Csíkszentmihályi & Larson, 1978, p. 335)

Throughout his writings, Csíkszentmihályi comes back again and again at this conviction that any learning experience can, and indeed must, become a flow experience (1990, 1993, 1996, 1997b). The question becomes then – why doesn't it?

2.4.6. Research question

After having started life as a novel way to explore play and leisure experiences, flow theory has proved itself a valid framework to investigate experiences beyond this scope, including in school. However, research has also uncovered that flow experiences at school are mostly the preserve of non-academic courses (Schmidt, Shernoff & Csíkszentmihályi, 2007). Nevertheless, flow experiences in mathematics are possible - indeed, without them, there wouldn't be any mathematics at all (Csíkszentmihályi, 1990). In my classroom, too, I had already noticed instances of what appeared to be flow experiences, albeit haphazard and seemingly beyond my control. The students also appreciated these special interludes, during which they were so absorbed in their task that they lost track of time and any concerns for external rewards.

My research question, stemming from my effort to recreate this experience more consistently, is as follows:

“What are the key factors involved in occasioning the flow experience for high school students in the process of learning mathematics?”

As indicated by the literature (Csíkszentmihályi et al., 1993), an exploration of these key factors must have a double focus: the student and the environment. Therefore, I am going to examine both salient aspects of students’ experience, taking into consideration all nine components of the experience, and the patterns of a flow-friendly mathematics classroom, in an effort to find a coherent image of what helps and what hinders flow.

Chapter 3. Methodology

> *The only time that is unforgettable is that time during which one forgets that times exists. Only that time is fertile which remains chaste and unsullied by the touch of consciousness…*
>
> Arthur Koestler, 1937 - *Dialogue with Death*

This chapter introduces the context, the participants, and the logistics of the research. I describe relevant background features of my classroom, and outline the moves I undertook in order to establish an environment in which flow was more likely to occur. Lastly, I give a rationale for and a description of the processes of data collection and analysis.

3.1. Setting: the students, the classroom, the course

This research takes place in three French Immersion grade 9 mathematics classes, in a public secondary school in the Lower Mainland. The classes participating in the study were each composed of approximately twenty-five grade nine students. There is no selection for students in French Immersion, so the classes contain a typical mix of students: high ability, average, anxious, disengaged, conscientious but uninterested. The students come from three feeder elementary schools. For this reason, the students don't have the same experience of mathematics education, and the differences between individuals are compounded by the differences in approach taken by their elementary school teachers. At our school, there is only one other teacher who teaches grade 8 French immersion mathematics. Thus, approximately two-thirds of the students in grade 9, who are the participants in the research, have already been my students in the previous year, while approximately one third are new to my class.

As usual in my teaching, the provincially mandated curriculum was being followed, and a provincially suggested textbook ("Math Makes Sense") was loosely and infrequently used. The duration of a class is eighty minutes. The instruction during class took the form a problem-solving approach, using either vertical whiteboard surfaces or small individual whiteboards. The problems that the students tackled every day varied, from practice problems for current topics, to tasks meant to encourage reasoning and problem solving. Therefore, the instructions given to the students varied too, in accordance with the goal of each particular lesson. As a rule, however, students were given all information on matters pertaining to mathematical conventions, and were encouraged to establish by themselves all concepts that are generally discoverable by a process of reasoning. For instance, I, the teacher gave instruction on how to write powers and exponents, but expected the students to find the exponent laws by themselves.

While the students were thus engaged, the teacher walked around, observing and giving feedback to groups or individual students. Practice problems were provided when appropriate, together with complete solutions. As usual, the problems and their solutions were also posted on-line, so that students were able consult them at their convenience if necessary. Students' grades came from a blend of communication/problem solving marks and chapter tests. While students wrote frequent "quizzes", these were never part of the mark. Collaboration between students during a quiz was expected, and appropriate feedback was available – from peers, from the teacher, other materials – whenever students requested it. Students were more than eager to take advantage of the low-stakes quizzes, and appreciated the benefits of getting feedback without risking the discomfiture engendered by unsatisfactory marks. In the same vein, the students were made aware that no marks would be given for their work on problem solving. I felt that attaching grades to their effort would be inimical to experiencing flow, and it would have been self-defeating: my purpose was precisely to propose activities the students would find worthwhile in and of themselves.

Since this is a French Immersion environment, the students are, in theory, supposed to communicate in French in written and oral forms. Practice is, however, very different: many students find combining mathematics with French onerous, and thus, for the purposes of this research, no attempt was made to mandate a language for peer-to-peer communication. I also used language in a flexible manner, depending on the circumstances. This has been done to facilitate the transmission of mathematics ideas, taking into account research about language use in a French Immersion mathematics classroom (Tang, 2008) which found that in order for complex discussions to develop, students may need to lower their cognitive load by using English to express themselves. Students were also aware that there would not be graded for any of the tasks used during the research.

On the other hand, students did not generally have a choice with regards to seating. At the beginning of the year, each student was assigned a number. Afterward, when they came to class, they had to find their number on a randomly filled grid, and took their place accordingly. The groups changed every class. Long before this research had started, and the students first tried the system, there were some complaints, as a few insisted on sitting with their friends. However, they soon saw the advantages – making new friends, collaborating with, and learning from, someone else – and began to enjoy the underlying idea of change. Indeed, when, due to the randomness of the group formation, they ended up either with the same partner or in the same physical location, they let me know so that next class I would ensure there would be no further repetition.

It has to be noted that the issue of the seating is a puzzling one, especially in the context of optimal experience, in which "control and choice" are central concepts. ESM research conducted in mathematics and science classes in US high-schools by Uekawa et al. (2007) found that, paradoxically, when students are given a choice about where and with whom to sit, their engagement level is substantially lower. This is but another illustration of how, when charting a course intended to maximize the likelihood for the students to experience flow, the limiting blocks that one establishes are just as important as the limiting blocks that one takes away.

3.2. Flow – laying the groundwork

The strategy for influencing the experience of the students had two components: first, the mathematical tasks, and secondly, my interventions.

3.2.1. The tasks

For the purposes of this study, the students were presented both with advance-curricular and non-curricular tasks. All the tasks were selected with four of Heine's criteria in mind:

control, diversity, novelty, and challenge (Heine, 1997). I cannot judge the *meaningfulness* of each task for the students, other than to say that the goal was to provide personal meaningfulness derived from the desired flow experience engendered by the task.

1. There is only one advance-curricular task discussed in this study: factorization of trinomials, which is a grade 10 topic. In the study, this task was the context for the case of James and Marie.

The rest of the tasks were non-curricular, as follow

2. The staircase problem (adapted from Youcubed).

   "In my house, there is a staircase with 13 steps. Being a person who gets easily bored by repetition, I like to climb my staircase in a different way each day. However, I do not climb more than two steps at a time, lest I hurt myself. Are there enough ways to climb the staircase so that I never have to repeat myself for a whole year?"

   In this study, this task was the context for Karl and Brandon's case.

3. The four digits problem (credit to John Grant McLoughlin).

   The students are asked to propose random digits, and I select four or them from their proposals. My selection is not wholly random, as I try to include digits with "possibilities", such as 4 and 9, together with at least one prime number. In this study, the digits selected were 1, 4, 7 and 9. The task consists of writing all the numbers from 1 to 100 using only the four digits permitted, and mathematically correct operations.

   In the study, this task was the context for the case of Chloe and Tamara.

4. The four 4s problem (Ruth Carver)

   Similar to the problem above, the students have to write all the numbers 1 to 100 using only four digits of 4 and mathematically correct operations.

   In the study, this task was the context for the case of April and Lucy, and Glen and Linus.

5. The Mickey Mouse fractal (Peter Liljedahl)

   A fractal image is created by attaching two smaller circles ("ears") as tangent to a larger circle of radius 1 ("head"). The radius of the "ears" is always half of the radius of the "head", and the process continues ad-infinitum. What are the values of the perimeter and the area of the resulting image?

   In the study, this task was the context for the case of Lonnie and Nikolaus.

6. The cryptography problem (Centre for Innovation in Mathematics Teaching)

The students were asked to decipher the message below, encrypted with a substitution code. The secret message was in English, and that, unlike the Enigma code, the "value" of a letter did not change throughout.

AUHC MVKFK V BYZUGC ISMC CJ GUMBZYAZD UKUVM. VC HZZGZB CJ GZ V HCJJB PD CFZ VYJM KUCZ AZUBVMK CJ CFZ VYJM KUCZ AZUBVMK CJ CFZ BYVWZ UMB OJY U IFVAZ V TJNAB MJC ZMCZY OJY U IFVAZ V TJNAB MJC ZMCZY OJY CFZ IUD PUYYZB CJ GZ.

In the study, this task was the context for the case of Bianca and Nadyia.

As can be observed, the tasks are neither trivial mathematically, nor impossibly hard to reach for students in grade 9. With the exception of the first (factorization of trinomials) and the last (cryptography), they can be given orally; the mathematical notations emerge from the reasoning process. Some require moderate amounts of prerequisite knowledge: for instance, knowledge of the Fibonacci sequence, although not essential, had the potential to increase the interestingness-factor of the problem. Similarly, the cryptography problem, an example of a substitution code, was presented in the discussion about encryption during WW2, with an excursion through codes in history. The Mickey Mouse fractal problem was presented in the context of a general exploration of fractals in nature. Finally, the tasks are purposefully not tethered to the curriculum (which, for students, is the textbook), as they were intended to infuse a sense of freedom and playfulness into the problem-solving process. A more detailed discussion on this topic can be found in sections 6.5 and 6.6.

3.2.2. Teacher interventions

The other prong of my endeavour to occasion and maintain flow was deciding, first, whether I should intervene, and, if yes, when and how to intervene. My goal was to be "less helpful" (Meyer, 2009), and only mediate when it was absolutely necessary. The threshold of *absolutely necessary* was evaluated through the "disciplined noticing"

(Mason 2002; 2005) of a list of behaviours which acted as a barometer of the interest and energy level of the students.

Table 1. Behavioural cues.

Verbal cues	**Non-verbal cues**
Exclamations	Eye contact
Laughter	Nodding, shaking of the head
Sighs	Posture (leaning in vs. leaning out, expanding vs. shrunken)
Tone of voice	The movement of the pen (energetic and bold vs. sluggish and timid), combined with the frequency of writing and erasing
Words or phrases denoting acquiescence or disagreement	Various other eccentric hand gestures, such as hands over the head, arms folded, high fives, etc.

(Adapted from Williams, 2000, 2005)

When the group reached a prolonged impasse, the next step was to decide on the appropriate intervention: teacher feedback or peer feedback. Before offering my own feedback, I always asked the students whether they wanted it. It was also the students' decision whether they just wanted to find out whether they were right or wrong about some particular hunch or an aspect of their solution (which I will call "informatory feedback"), or whether they wanted something more substantial, intended to move their skills in harmony with the challenge, or the challenge in line with the skills, and which I will call "upformatory feedback". Both kinds of feedback had one purpose though, which was to provide the students with a stream of data about their understanding and give them the opportunity to access the flow channel.

Generally, however, I tried to utilize peer resources as much as possible, especially since very rarely did a group experience a difficulty that nobody else had encountered before them. Students were also free to ask questions and communicate as they saw fit. The only rules were, first and foremost, that "one doesn't tell the solution", and that "one shouldn't push people too far ahead" – which is to say, the ideal feedback would move the student adjacently upwards from the current position, and no further. The metaphor I gave the students – and which I followed myself, too – was that of "the stepping stone in the river": if the stepping stone is too far, then one still cannot cross, and the problem remains too difficult. Conversely, if the stepping stone is too near, then it's no longer "fun" to cross, and the problem becomes too easy.

Giving extensions to problems ensued naturally from the same model of feedback. I gave extensions whenever I noticed students teetering on the edge of boredom, and, at times, students themselves noticed the possibilities arising from their reasoning and asked themselves questions which were themselves stepping stones to new discoveries. Due to the time-intensive nature of the tasks, giving extensions did not occur frequently enough so as to be captured in this study. It has to be pointed out that despite my best efforts, my feedback did not always achieve the perfect timing, nor the Goldilocks positioning: neither too far, nor too close. A more detailed discussion of the highs and lows of flow intervention is forthcoming in sections 5.1 and 5.2.

3.3. The data

Before launching into the discussion on this topic, it may be worthwhile to reiterate the nine components of the flow experience, in the Quinn (2005) frame of reference that distinguishes between antecedents and consequences:

1. Antecedents of flow: concentration; goal clarity; balance of skill and challenge; feedback.
2. Consequences of flow: sense of control; autotelic experience; loss of self-consciousness; merging of action and awareness; transformation of time.

It is obvious that not all components need the same instruments in order to be measured or examined. Thus, the "concentration" component was considered included in the proxies for engagement (see below). The remainder of the components was evaluated via questionnaire, interview, or both.

3.3.1. Data source 1: Proxies for Engagement

The possibility that students may be in flow was inferred via the Proxies for Engagement (PFE) (adapted from Liljedahl, 2016). For all groups, I recorded indicators A, B and C, and used my experience and professional judgment for subjective indicators D, E, F, G. All indicators received a mark from 0 to 3, with 3 being "excellent" and 0 being "unsatisfactory".

The Proxies for Engagement are as follows:

A. Time to task – how long did the students take to begin the task
B. Time to first mathematical notation
C. Time on task
D. Eagerness to start
E. Discussion within the group (participation)
F. Persistence
G. Distractibility

The PFEs were intended to be the "early warning system", employed to select the groups and students who merited further attention. As indicated above, the PFEs give a measure of concentration on the task, too (particularly indicators D, E, F and G); therefore, *concentration* as a flow component was not formally assessed, as I considered it evident from the students' behaviour whether or not they concentrated.

Very often, so many groups and students had high scores when this initial tool was employed, that limitations to my attention, my time, and my students' time, were significant considerations when deciding who would participate in the next steps of the flow evaluation process. In other words, not everybody who had high scores on the PFEs was interviewed or was able to fill out a questionnaire. Out of the pool of students I interviewed and who answered the questionnaire, I selected as subject of this research a sample of pairs who presented, in my view, varying and interesting facets of the flow experience. I stopped interviewing and distributing questionnaires once I found that new data failed to bring anything that had not been uncovered already.

3.3.2. Data source 2: Questionnaires

More details about the experience were gleaned from a Likert-style questionnaire, completed by students who demonstrated high levels of engagement when assessed with the PFEs. The questionnaire consisted of more detailed questions intended to assess specific components of the flow experience, ascertaining where students stood with respect to three components of flow: sense of control (questions 1 and 2), merging of action and awareness (questions 3 and 4) and loss of self-consciousness (questions 5 and 6). These components were, in my opinion, hard to get at during an interview, particularly the component of *merging of action and awareness.* The feelings and states they engender are mostly inexpressible, unless one has both the sensitivity to notice them, and the eloquence to decorticate them – which is why I deemed it necessary to have a clear, consistent, unambiguous formulation. In the event, no students requested my help in deciphering what the questions asked them to reflect on.

Flow questionnaire

(adapted for ease of comprehension from Quinn, 2005, and Engeser & Rheinberg, 2008)

a. When you first started out, how optimistic were you about your ability to do the task?				
very optimistic	somewhat optimistic	Neutral	not very optimistic	not at all optimistic
b. How good did you feel about what you were doing, while you were doing it?				
very good	somewhat good	Neutral	not very good	not at all good
c. How much does the following statement reflect your experience? ***"During the activity, I felt I knew what to do and how to be successful at it."***				
all the time	very frequently	Occasionally	very rarely	never
d. How much does the following statement reflect your experience? ***"During the activity, my mind was thinking of other things"***				
Never	very rarely	Occasionally	very frequently	all the time
e. How often did you judge yourself negatively during the activity (for instance, "I'm no good at this")				
Never	very rarely	Occasionally	very frequently	all the time
f. How often did you worry about others judging you during the activity? ("others" = your partner(s), other classmates, the teacher)				
Never	very rarely	Occasionally	very frequently	all the time

3.3.3. Data source 3: Interviews

At the end of the class, the same students were interviewed in order to obtain a more fine-grained image of their experience in class. The timing was imposed on me by the demands of teaching, and usually took about 10 minutes of break time, which both the students and I had to forfeit. In one instance, the interview took place at the end of the day. However, my goal was to capture the experience as quickly as possible after it had occurred, in its rawest form, in order to avoid contaminating it with the influence of subsequent events and moods, and in an effort to stay as close as possible to the spirit of the ESM, which, as indicated in the section 2.2, samples experiences *in the moment.*

The interview consisted of series of questions in a semi-structured form. I aimed to keep it akin to a guided conversation, rather than a scripted interaction. The first question is non-specific, as it is meant to prompt the students' own reflection on how they perceived their experience. Questions 2 and 3 were not always necessary, as some of the students had already volunteered information on getting stuck and how they resolved their impasse. The aim of questions 4 to 8 was to prompt the students to discuss their perception of the other components of flow (autotelic experience, balance of challenge and skills, feedback, clear goals, and time distortion). All the questions formed a skeleton which allowed a richer conversation to take place, and were not always asked in the order written here. Often, students felt the need to elaborate on their experiences and trajectory, sometimes in great detail. Thus, the interviews became a venue in which students felt impelled to share details about their experiences not only during the mathematics class, but about their school life in general, and sometimes even their hobbies.

Basic interview questions:

1) How did the problem work out for you?
2) Tell me if you got stuck at any point, and how did you feel about it?
3) Did you get help from anybody and when?
4) Did you enjoy the task? Please give reasons why.
5) Did you feel anxious while you were doing the task? If yes, when? Please tell me more about it.

6) Did you get bored? If yes, when? Please tell me more about it.
7) How hard did you think the activity was?
8) Did you look at the clock during the activity, and if yes, how many times?

All the interviews were recorded and transcribed.

3.3.4. Data source 4: Field notes

As the students were working on problems, I observed the groups and made notes about any elements of significance occurring in their behaviour, their attitude, and their interactions with me and with each other. At the end of each class, I also made notes about my own experience. Since I already know how it feels to be in flow, I considered it sufficient to examine my teaching experiences through the lens of other previous flow experiences and decide whether or not they were similar.

3.4. Flow as a framework for analysing experiences of learning as shared play

Csíkszentmihályi and other researchers have traditionally investigated flow from an individual perspective, focusing on its components, as discussed in chapter 2. All these are necessary for understanding why and when an individual student may be in flow. However, for my research, I found that this view neglected the social context in which flow occurred: in a school, in a classroom, as part of a pair or a group amongst other groups. The communal nature of teaching and learning, the fact that enjoyment of mathematics was evidently so tightly intertwined with enjoyment of the interaction with others, the effervescence engendered by the cross-pollination of ideas, all speak to the relevance and influence of social setting in my exploration of flow.

In giving due attention to this aspect, I do not consider that I have deviated from flow theory. On the contrary, such a shift in perspective is a welcome return to Csíkszentmihályi’s initial observations about flow, which were spurred on by his interest in play (1975b). Thus, since for children – and students are still children – play is shared, so is the flow experience. To bring this feature of flow back from obscurity, I use a modified version of Glăveanu and Lubart's framework of analysis (2014), which, although originally employed to research sociocultural aspects of creativity, has demonstrated its utility in the context of flow research (Gaggioli, Milani, Mazzoni & Riva, 2015) and in research on enjoyment (Gajadhar, de Kort & IJsselsteijn, 2008). In my research, I will zoom in on the core of the pairs' collaboration experience as contained in a space defined by four dimensions: regulatory, informational, motivational, and ludic. Their role is to highlight different nuances of the interaction between students: the regulatory dimension pertains to the way the students set up the rules of their game, how they establish their guidelines; the informational dimension refers to how the students check each other's work, and act as "a mirror" or "another pair of eyes"; the motivational dimension will consider how the students offer each other encouragement, collaboration and competition; and, finally, the ludic dimension looks at whether the students see each other as a play partner. Illustrating and disentangling these conduits of peer feedback will take place within a part of the analysis that I call the Shared Resource Space.

Chapter 4. Results: Student experiences

. . .Time past and time future
Allow but a little consciousness.
To be conscious is not to be in time
But only in time can the moment on the rose-garden,
The moment in the arbour where the rain beat,
The moment in the draughty church at smokefall
Be remembered; involved with past and future.
Only through time time is conquered.

T.S. Eliot, 1943 - *Four Quartets: Burnt Norton, II*

As indicated in chapter 3, the students who became participants in the study were selected based on their proxies for engagement scores, **and** on their availability. I purposefully did not select any of the students who, based on my knowledge of them, enjoyed mathematics and were always focused in my class, without any special effort on my part. The students I interviewed had manifested, throughout their time in my class, medium or low levels of engagement. Put simply, they were students who were generally either unconscientious, or joyless, or both. I begin each vignette with a presentation of the students from my point of view, a narrative of their episode, as reported by me, together with other pertinent observations related to their experience and interactions, followed by excerpts from their interviews, and conclude with their scores on the PFE tool, their questionnaire scores, and an analysis of their interaction in the Shared Resource Space. In order to preserve student anonymity, all names are pseudonyms.

4.1. The experience of Karl and Brandon

At the time of the interview, Karl and Brandon had been my students for almost two years. They were both quiet and pleasant, and while they didn’t go out of their way to look for challenges, they did not refuse them either when they were put to them. They both had easy-going personalities, and worked very well with whoever was in their group. Neither was very talkative, and I suspect they agreed with that whatever was decided by the group without much protest. Karl often relied on Brandon for catch-up, as he had a chronic condition which frequently kept him away from school. However, he always did his best, and didn't use his ill-health as an excuse for slacking off at school. Another activity that Karl and Brandon had in common was table tennis, and, as we had our class right before lunch, they always rushed out of the class to find a table to play. From all I could see, their passions lay somewhere other than the mathematics class.

The problem of the staircase was presented to the class as a personal story: in my house, I have a staircase with 13 steps, and being a novelty-seeking sort of person, I wanted to know whether there are enough ways for me to climb the stairs at home so that I never use the same way twice in a whole year. While many students were amused by the context, in a "we're worried for your sanity" kind of way, Karl and Brandon were willing to suspend their disbelief and engaged with the mathematics of the problem, despite its implausible context. As Brandon was still unsure about how to begin, he asked Karl and me some clarifying questions:

> Brandon: "But can she skip three? [to me]: Can you skip three, Madame?"
>
> Karl: "No, she said she only skips a maximum of two! That means one or two, not three!"
>
> Brandon: "But why would she …."
>
> Karl: "That's the math teacher way!" [Both laugh]

Karl and Brandon's first move was to draw a staircase with thirteen steps on the board, and they made a brave attempt to count the ways in which they could be climbed. I talked to them at this point, when they were slowly realizing that it was difficult to count all the possibilities. The following discussion ensued:

> Me: "So how is your solution going?"
>
> K: "We're trying to count all the ways. But we don't know … Does the first one count?"

He placed his marker on the bottom step of the staircase they had drawn.

> Me: "Show me how you're counting."

Karl started making small arched arrows over steps, of length one step (for a skip of one), except the last one, when his arrow covered two steps. He then started again, and followed the same pattern, except for the last four steps, above which he drew two arrows.

Me: "So what have you drawn here?"

K: "The first two possibilities. That's is a two-step jump…one…or two…. Up to 6."

Brandon appeared to want to say something.

Me: "What do you think, Brandon?"

B: "Ummm…. Rien." [Pause] "And we'll count the twos that are here." [with his hand, he indicated the middle region of the staircase]

Karl and Brandon tilted their heads and looked at their drawing. They appeared a bit bemused. Brandon shared with us his – correct – intuition that, due to the way the problem was asked, and the mention of "a year", the solution would undoubtedly uncover around three hundred ways to climb the stairs, otherwise, in his view, the problem "made no sense". At this point, I reinforced to the students the idea that, since they were expecting a big number, they would be well-advised to find another way to count, something that would be less confusing than drawing arrows.

After a few minutes, I looked at Karl and Brandon again. They were writing their possibilities with strings of 1s and 2s, rather than arrows, and appeared to be very content with the new way of recording information. However, they reached a point when I saw them writing and erasing in quick succession. At some point, Karl erased everything, seemingly in frustration. Brandon protested loudly, and begun to re-write what had been erased. Soon, they turned away from the board, ceased writing, and adopted a passive posture. It became obvious to me that Karl and Brandon were no longer in flow, and that they were, in fact, losing momentum.

In order to lower their frustration to a tolerable level, I intervened and offered them upformatory feedback. My suggestion was to try to see what happens for smaller numbers of steps. Again, I left them, but observed them while they counted the ways to climb the stairs for 1, 2, 3, 4, 5, and 6 stairs. Suddenly, they were feverishly writing on the board then erasing, in quick succession, and finishing each other's sentences. The moment when they reached 6, they suddenly saw the pattern of the Fibonacci numbers, and they both stopped, and remained completely still for a few seconds.

Karl: "Wow."

> Brandon: “Wow.”
>
> Karl: “Is this Fibonacci? Wow!”

Their silence continued for a few more seconds, while they looked up and down their table. Then they surprised me by performing a complicated fist bump and lifting one another up on their backs.

I began their interview by asking them about the specific moment when they saw the Fibonacci sequence, since it was such a marked moment of surprise. Both Karl and Brandon viewed it as a startling revelation:

> Brandon: “I remembered the sequence from last year. But I didn’t expect to see it in this problem”
>
> Karl: “Yes, I was like - what’s Fibonacci doing here?”

Each student had a different reason for being surprised. For his part, Karl thought that Fibonacci numbers appeared only in flowers, based on a video watched in grade 8 about plants and the Fibonacci sequence. Brandon was surprised because he expected to see linear growth, an expectation which he imposed – incorrectly – on his solution. More specifically, when they created their table, Brandon was lulled into a false sense of linearity by the first entries: for one step there was one possibility, for two steps stairs there were two, for three steps there were three possibilities. The pattern broke down when they reached four stairs, for which Brandon wrote without hesitation and without reflection as having four possibilities, whereas the correct answer was five. The mistake became obvious for both students when they reached five steps, which they realized could not have only five possibilities. This led them to backtrack, correct their mistake, and build an understanding towards a reasoned solution, which emerged when they noticed that the table entries 5 and 8 added up to the next value, which was 13.

Karl and Brandon’s answers to the questions pertaining to the quality of the experience indicated that there was a high probability they had been in flow while solving the problem. Brandon had not looked at the clock at all, while Karl added:

> “I couldn’t believe that there were only 2 minutes [of the class] left.”

Karl also admitted to not having noticed that another classmate had tried to get his attention to talk about a matter related to another course. They both reported satisfaction with their level of concentration, enjoyment, and a complete lack of worry and anxiety about their ability to find a solution. Furthermore, while they appreciated getting "hints", they recognized that "too many hints make it [the problem] too easy".

> Me: Were you worried you might not do it?
>
> K and B: No.
>
> Me: Why not?
>
> B: You gave us a hint!
>
> K: Yeah…it's nice when you get hints. Like in a game.
>
> B: But not too many.
>
> Me: What's wrong with that?
>
> B: Then it's too easy!

Both students clearly made a connection between the level of difficulty of a problem and the level of enjoyment they would derive from it. In such circumstances, thinking about a mathematics problem is akin to being in a game (presumably, a computer game), in which one gets artful hints which propel one forward, without marring the enjoyment of the game by making it too easy.

<u>How I saw Karl and Brandon through the Proxies for Engagement (PFE)</u>:

A. Time to task – 2 min
B. Time to first mathematical notation – 2 min
C. Time on task – the remainder of the class
D. Eagerness to start – 3
E. Discussion within the group (participation) – 3
F. Persistence – 3
G. Distractibility – 0

<u>How Karl and Brandon saw themselves in the questionnaire</u>:

a) When you first started out, how optimistic were you about your ability to do the task?

Karl – neutral	Brandon – not very optimistic

b) How good did you feel about what you were doing, while you were doing it?

Karl – very good	Brandon – very good

c) Statement: “During the activity, I felt I knew what to do and how to be successful at it.”

Karl – very frequently	Brandon – very frequently

d) Statement: “During the activity, my mind was thinking of other things.”

Karl – never	Brandon – never

e) How often did you judge yourself negatively during the activity?

Karl – never	Brandon – very rarely

f) How often did you worry about others judging you during the activity?

Karl – never	Brandon – never

<u>Karl and Brandon’s Shared Resource Space</u>:

Karl and Brandon

Regulatory:	*Informational:*
Brandon's commenting that the number will be big enough to justify the choice of timeline. Correcting their mistake in the table of values through arguing with each other.	Developing their own way of counting. Filling out the table of values together.
Motivational:	*Ludic:*
Brandon's protests and his attempt to re-create what had been erased. In the final stages of the solution, the students were "of one mind", finishing each other's sentences.	Celebrating the completion of the problem with fist bumps and back lifts. Comparing their experience with that within a game.

4.2. The experience of Glen and Linus

Glen and Linus were two very different personalities, who nevertheless formed a very successful team. Glen was frequently late to class, and when he came he tried to delay the moment when he would have to begin work by greeting his friends with ostentatious warmth, shaking hands, bumping fists, and generally making a small spectacle of his late entrance. When he sat down, he reclined in the manner of a sunbather on a lounge chair, and appeared to have difficulty in keeping his eyes open. It was hard to predict how his behaviour would be every class, because his energy waxed and waned, depending on circumstances unknown to me. Some days he yawned all the time, and other days he showed glimmers of concentration and enthusiasm. According to his parents, Glen had been a very talented young mathematician in elementary school, but all that was now in the past. His marks were low, and, more worryingly, his experience in the class was clearly not a fulfilling one. His parents were often in contact, and appeared to be very anxious about Glen and his disengagement from school in general.

Linus' story couldn't have been more different. His parents were frequently away, and he had to manage many aspects of his life with very little help from them. Linus was a problem solver par excellence, highly energetic, polite, albeit with a streak of eccentricity. For instance, he only sat down in class when he was writing a test, and even then, he often got up to move. He once explained to me that he "thinks better" when he's standing. His interests were very eclectic, from music to cooking, in addition to sports and game apps. He also had a great sense of humour, and was able to laugh at his own mistakes. When he would catch himself making errors, he liked to display exaggerated and theatrical frustration, which was in direct contrast with Glen, who, despite my exhortations, still erased the board the second he as much suspected a mistake. Linus was also not shy to show pride when he completed a task, no matter how small, whereas Glen's reactions to success were much more muted, even when his accomplishments were significant.

Glen and Linus worked on the problem of the four 4s: create all the numbers from 1 to 100, using exactly four digits of 4. I was surprised to see that Glen and Linus worked until the end of the class, albeit with some pauses. The conversation between them was very heated, but almost always focused on mathematics. I heard arguments, questions, and laughter in harmony many times throughout the class. I did intervene several times with goal-clarifying questions, such as “what operation are you planning to use?”, or “have you chosen a target number?” when I noticed them going off track. I did not allow them more than a few minutes of disengaged behaviour, and they were perfectly willing to go along with this.

Glen and Linus were able to complete several numbers by themselves, and after each success they were getting more and more enthusiastic. However, after about 15 minutes, I saw them going off-track and talking about something completely unrelated to the task. I went to see them and they told me that they ran out of ideas. I then gave them a new operation: summation, and the sigma operator. With it, they were able to do a few more numbers, again with great energy and impassionate discussions. The cycle repeated again: finding solutions with eagerness and speed, then, once the ideas were not coming with the same intensity, the initiation of a conversation about other matters. My subsequent interventions occurred to remind the students about some other available operations (.4 and the gamma function), and twice to console them when their target number had been “taken”. I encouraged Glen and Linus to find another solution to some numbers that had already been done once, but they did not proceed on this avenue.

While they were working on the task, Glen and Linus sounded like a well-oiled machine, with their efforts working in perfect tandem. The following is an excerpt from their animated conversation:

> G: “And what if we do that?”
>
> L: “Wow! It works!”
>
> G: “No it doesn’t. Put a minus.”
>
> L: “Aha! Success!”
>
> G: “Is 89 on the board?”
>
> L: “Not yet! Come on, before somebody takes it!”

Whenever I was showing them a new operation, they were both very eager to use it, and even rejected solutions which didn't contain it. The frenetic rhythm was contagious, and I was constantly surprised to see Glen work with the same enthusiasm and concentration as Linus, which was unusual for him.

I began the interview with an invitation for the students to give me a general overview of how they thought the day went. Both Linus and Glen were unequivocal in their expressions of enjoyment. Linus stated that "it was fun to have someone to share the joy with", and added that he liked the new symbols and the new operations he had learned:

> "…it's fun when it's new math. If it's just things you already know…it can be fun, but it's a lot more interesting when you learn something new. I'd never heard of these symbols and I never knew existed."

He also discerned his own "moments of realization", noting how "enjoyable" they felt, and expressed his surprise about the problem:

> "I never knew that you could make all these numbers with just 4s. It was … very satisfying. Especially at the end, when we filled out the last ones!"

Glen also reported that "it was fun to work together and discover something", and that he liked the fact that the problem was "nice" and "different". He expounded:

> "[…] when it's not new, it's sometimes… repetitive. I don't like repetitive. I don't have the same feeling."

Asked to define what he means by a "nice problem", he stated:

> "Something where it's not too hard, but still presents a challenge."

Both students reported that they were not bored, and did not look at the time. On the contrary, they stated that they felt excited to work on the problem. When I reminded them about the intervals during which they were not working on the problem, they explained that "they were waiting for the ideas to come". Considering how quickly they got back to work after such episodes, it may well have been true.

<u>How I saw Glen and Linus through the Proxies for Engagement (PFE)</u>:

A. Time to task – 3 min
B. Time to first mathematical notation – 2 min
C. Time on task – the remainder of the class

D. Eagerness to start – 2 for Glen, 3 for Linus

E. Discussion within the group (participation) – 3

F. Persistence – 3

G. Distractibility – 1

<u>How Glen and Linus saw themselves in the questionnaire</u>:

a) When you first started out, how optimistic were you about your ability to do the task?

Glen – neutral	Linus – somewhat optimistic

b) How good did you feel about what you were doing, while you were doing it?

Glen – somewhat good	Linus – very good

c) Statement: "During the activity, I felt I knew what to do and how to be successful at it."

Glen – very frequently	Linus – all the time

d) Statement: "During the activity, my mind was thinking of other things."

Glen – very rarely (it depends)	Linus – very rarely (it depends)

e) How often did you judge yourself negatively during the activity?

Glen – very rarely	Linus – never

f) How often did you worry about others judging you during the activity?

Glen – never	Linus – never

Glen and Linus’ Shared Resource Space:

Regulatory:	*Informational:*
Periods of contagion: discussion on other topics.	Negotiating the best way to achieve certain target numbers.
Glen and Linus	
Motivational:	*Ludic:*
Glen felt inspired by Linus and worked with previously unseen enthusiasm; sharing the joy. The students urged each other to run to the board and place their solution.	Joking, laughter and good spirits.

4.3. The experience of Chloe and Tamara

Chloe and Tamara were conscientious students, never disruptive, always smiling. Both of them were my students for the second year, and during that time I had seen in Chloe significant positive change in terms of her self-confidence. Tamara was less self-assured, and at times seemed unwilling to contribute her ideas, especially when she was in a group with boys. Her small voice – both literally and figuratively – had difficulty in being heard. I was hoping that working with Chloe would give her more opportunities to contribute.

Chloe and Tamara’s task was to make all the numbers from 1 to 100 by using the randomly chosen digits 1, 4, 7 and 9. They started working on the problem with some diffidence, and their undertakings lacked complexity. For the first minutes of work, they ran on the same ideas: addition, subtraction, concatenation. Many of the numbers they were finding had already done by others, but they were still unwilling to branch out and try new operations. However, what was also fascinating was the extraordinary perseverance with which they covered all the (limited) possibilities available to them within the confines that they themselves had established. Everybody in the class knew about factorial, so I suggested they use the factorial and try a few combinations of numbers, which they did, and finally obtained a number that had not been done before. After that, their demeanour underwent a subtle change, from dutiful students to engaged creators. I checked in with Chloe and Tamara three more times, and they didn’t need me for anything else other than praise. Their solutions showed more and more creativity, for instance they combined a .1 and a .9 to make a 1, used a summation and a 9 to the power of root 4, which then they combined in a different manner to make 4 to the power of root 9.

This burst of creative energy did not come uninvited, though: I heard Chloe and Tamara reject three solutions because they were not "interesting enough". A few minutes later, when I checked in with them again, they told me that they were looking for something with "a sum and a root", then something "with a repeating decimal", then they spent a lot of time playing with the gamma function. I use the word "play" advisedly, because they were really like children playing with new toys. In a very short time, they went from timid and restrained to unstoppable, and their standards for what they could and would achieve were increasing. Knowing how shy and reserved they usually were, I also noted how talkative they had become. Both were smiling and laughing, and appeared not to get tired at all for the duration of the class.

The first thing that Chloe and Tamara said during the interview was to remark on how they had "lots of fun". When prompted for details, Chloe reported that she was intrigued by the fact that the problem was so flexible, the numbers so random, and yet the goals were so attainable:

> "I liked how I could make all these numbers. When you first told us what we had to do, I thought it could not be done. I really thought it was impossible."

For her part, Tamara liked the fact that there "wasn't just one answer", and that "there were no rules", which stimulated her and Chloe to "combine things in a new way every time", in order to find "something they really liked". The students quoted this flexibility as one of the reasons for not feeling stuck at any time.

As their quest for the most interesting combination of numbers and operations was rather rare, I asked them to explain what was behind it:

> Me: Usually students just take the first right answer they can get…or am I wrong?
>
> Chloe: I think this one I took it as a challenge, not as a lesson. If it's a challenge, I want to complete it […] I wanted to feel accomplished. And if you choose to do something difficult, you have a chance, but if you don't even try…"

Both students stated that they found the problem to be "neither too easy, nor too hard", to which Chloe added that when something is more difficult, she feels emboldened to try harder, whereas when she perceives a task as easy, she feels less motivated to make effort.

When I asked them about looking at the time, they both stated that they hadn't. Nor did they feel bored during this class, although they mentioned that when they work "from the book, […] reviewing the same thing", they may at times feel bored. Both Tamara and Chloe indicated they did not feel worry, although Tamara mentioned that she had a test during the following class, which caused her some anxiety.

<u>How I saw Chloe and Tamara through the Proxies for Engagement (PFE)</u>:

A. Time to task – less than 1 min
B. Time to first mathematical notation – less than 1 min
C. Time on task – the duration of the class
D. Eagerness to start – 2
E. Discussion within the group (participation) – 2, increasing to 3.
F. Persistence – 3
G. Distractibility – 0

<u>How Chloe and Tamara saw themselves in the questionnaire</u>:

a) When you first started out, how optimistic were you about your ability to do the task?

Chloe – not very optimistic	Tamara – not very optimistic

b) How good did you feel about what you were doing, while you were doing it?

Chloe – very good	Tamara – very good

c) Statement: "During the activity, I felt I knew what to do and how to be successful at it."

Chloe – all the time	Tamara – very frequently

d) Statement: "During the activity, my mind was thinking of other things."

Chloe – never

Tamara – occasionally [Tamara elaborated that she worried about an upcoming test]

e) How often did you judge yourself negatively during the activity?

Chloe – never

Tamara – very rarely [Tamara explained that at times she thought herself less quick than Chloe]

f) How often did you worry about others judging you during the activity?

Chloe – never

Tamara – occasionally

Chloe and Tamara's Shared Resource Space:

Regulatory: Not recorded.	*Informational:* Negotiating the best way to achieve certain target numbers.
Chloe and Tamara	
Motivational: Both students found their voice, and together relished the challenge of the problem.	*Ludic:* Playing with operations and numbers in order to achieve self-imposed standards of novelty and interestingness. Smiling and laughing.

4.4. The experience of Lonnie and Nikolaus

At the time of the interview, Nikolaus had been my student for two years, while Lonnie was a new student in my class that year. I knew that Nikolaus usually preferred working on his own, because I had had to coax him numerous times into working with a partner. He seemed an eccentric introvert, with a notable interest in construction toys. His marks were generally good, but I had the feeling he wasn't really that involved in the class, satisfied to do his "best minimum". He occasionally came to life when the class discussion turned to more esoteric subjects, such as the first ever locomotive, or the French Revolution. His interests were very eclectic, and he always came to class with a book, which he read from in the beginning of the class until he couldn't politely continue to do so, and to which he returned at every possible occasion.

Lonnie was more of an unknown quantity to me. He was also very quiet, but he also seemed alight with curiosity for everything and anything. He was always very grateful for any feedback I gave him and always took it very seriously. Once I finished talking, he appeared to replay my sentences in his head, then nod gravely and gesticulate with his index finger, as a sign of understanding. His penchant for using body language as a way to communicate with me was a bit unsettling in the beginning of our acquaintance. For a long time, I felt that I did not know Lonnie as well as I would have liked. His grades were decent, and he worked in class with considerably more interest than Nikolaus.

On the occasion of the task of the Mickey Mouse fractal, both Lonnie and Nikolaus were initially assigned to other groups. However, soon after starting work, they drifted apart from their own group and each followed their own idea, separately at first, and then together. What united them was that they had the same (wrong) idea, which they pursued vigorously for about fifteen minutes. It was for their dedication to their chosen, unfruitful path, that I found their interactions so interesting.

My interactions with them were as follows:

1. About 5 minutes in, I talked to them and they indicated that based on their intuition, the value of the sum 1+1/2+…has to be infinite. I did not contradict them, and merely encouraged them to continue whatever they were doing.

2. After 15 more minutes I went to how they were progressing. They were mildly frustrated, owing to the fact that they chose to pursue their exploration using decimals, rather than fractions. I suggested they switch to fractions, to make the calculations and the noticing easier. This, they did not do immediately. They continued with decimals for one more fractal level, until they erased everything they had and started anew.

I left them to their own devices for a while after that. I could see their work on the board, and their discussion indicated to me that they were making progress.

N: So, it's not infinite then?

L: Look!

3. I asked how they were doing, and Lonnie explained

“every time we add more fractions, the part we’re missing [to get to 1] gets smaller and smaller. So I think the sum is going to be almost 1.”

After a few more minutes I checked in with then again, because their body language seemed to indicate to me that they were feeling frustrated, and asked them if they wanted a hint. They looked at each other and Lonnie replied “not yet”. Again, I left, and when I came back again, they were staring at the board in silence. I took the opportunity to ask what they were doing:

Lonnie: “We sit and ponder.”

Me: “What are you pondering?”

Lonnie: “The sum is exactly 1!”

[Nikolaus put his hands on his head in a gesture of amazement.]

Both Lonnie and Nikolaus continued work on other geometric series, and were able to found the sums of 1/3 and 1/4, and then a general rule about *1/k*, based on the pattern they noticed.

In this instance, too, I begun the interview with a question about their moment of surprise, as it seemed to me that it made a great impression on both students. Indeed, Lonnie was effervescent:

Lonnie: “It [the problem] exploded my brain.” (and then demonstrated with his hands and sound effects how it explosion went). “But explosions are good.”

Me: “Why did it explode your brain?”

Lonnie: “It just didn’t make sense in my brain. What made sense to me at the time for it was to be infinite”

Nikolaus: “Yeah, exactly. I didn’t believe you when you said it wasn’t infinite.”

Lonnie: “I still can’t believe it.”

Nikolaus reflected that they got stuck because “they kept working with decimals”, which were “too complicated”, observed that the problem ran counter to his expectation, which had been “to add and get big numbers”, and remarked that he found this to be "interesting". Both he and Lonnie felt that the problem, although different from others, had not caused them anxiety or worry:

> Nikolaus: We didn't have time to worry [laughs].
>
> Lonnie: It wasn't that kind of problem.
>
> Me: What do you mean?
>
> Lonnie: [hesitates] I mean…I don't know, it was different.
>
> Me: Are you worried when you solve other kinds problems on the board?
>
> Nikolaus: For me…[pause]… yeah, it was different too. I just felt relaxed.
>
> Lonnie: Yeah… It wasn't like a test. It was not from the book. It felt …I don't know, like a game?
>
> Me: In what way?
>
> Lonnie: [pause] Because we didn't have to do it, there was no test…we did it because we wanted to do it.

I was also interested to know why they refused "hints", even in moments in which it was clear they were floundering. Lonnie stated that he "wanted to discover himself". Nikolaus concurred:

> Nikolaus: Yeah, exactly! It's more fun! It's not fun when it's too easy. I get bored if it's too easy.
>
> Me: So would you say this problem was too easy, too hard, or just right?
>
> Both Lonnie and Nikolaus: It was a bit hard in the beginning … but no, it wasn't too hard.
>
> Lonnie: In the end, it all came together.

Both students stated that they did not get bored, on the contrary, they enjoyed themselves, and that they did not check the time or look at the clock at all during class.

<u>How I saw Lonnie and Nikolaus through the Proxies for Engagement (PFE)</u>:

A. Time to task – less than 1 min
B. Time to first mathematical notation – less than 1 min
C. Time on task – the duration of the class
D. Eagerness to start – 3
E. Discussion within the group (participation) – 3

F. Persistence – 3
G. Distractibility – 0

<u>How Lonnie and Nikolaus saw themselves in the questionnaire</u>:

a) When you first started out, how optimistic were you about your ability to do the task?

Lonnie – somewhat optimistic Nikolaus – neutral

b) How good did you feel about what you were doing, while you were doing it?

Lonnie – very good Nikolaus – very good

c) Statement: "During the activity, I felt I knew what to do and how to be successful at it."

Lonnie – all the time Nikolaus – all the time

d) Statement: "During the activity, my mind was thinking of other things."

Lonnie – never Nikolaus – never

e) How often did you judge yourself negatively during the activity?

Lonnie – never Nikolaus – never

f) How often did you worry about others judging you during the activity?

Lonnie – never Nikolaus – never

Lonnie and Nikolaus' Shared Resource Space:

Regulatory: Drifting away from their original groups and following their own idea about solving the problem.	*Informational:* Checking and critiquing each other's work.
Lonnie and Nikolaus	
Motivational: Persisting with their idea despite the obstacles, despite doing something different from everybody else.	*Ludic:* The use of language, shared sound effects and gestures to denote surprise. The common awareness that it was all "like a game".

4.5. The experience of James and Marie

Both James and Marie were part of the minority of students who had not been my students in grade 8. When they came to my class in grade 9, I had concerns about how well they would feel with the change in style in teaching. Fortunately, I needn't have worried, because they adjusted very well to not taking written notes. They were always in a good mood during class, especially when we worked on individual whiteboards, and very keen to try new things. I had noticed already that both enjoyed helping and assisting classmates, but two new traits came to light one day when they were working on polynomials. I like to introduce grade 9 students to factoring polynomials, mostly because I noticed that grade 9 students feel really excited when I tell them that what they're doing is "grade 10 stuff", whereas the grade 10 students are not at all stirred by the same information.

In order to make the context clear, I feel I need to sketch the pedagogical approach I have chosen:

Step 1: I remind students how to multiply two numbers by using the rectangle method, which was done in grade 8.

Step 2: I ask them to multiply two binomials, using the same method.

Step 3: I then give a trinomial and ask them to factor by "undoing" all the steps they had done before.

Working together, both James and Marie were very successful in accomplishing steps 1-3. In my experience, the transition from step 1 to step 2 may be sometimes awkward for some students, so I asked Marie to lift her whiteboard and show her rectangle to her classmates. When step 3 was introduced, James entered a frenzy of writing and re-writing, while all the time keeping a close eye on Marie's whiteboard. When he finished, he looked at her triumphantly, and she demonstratively put her pen down mere seconds after him.

A not-so-subtle competition started, and for the rest of the class James and Marie were always trying to out-do each other. I also noted, however, that all this was done in a friendly spirit, as James assisted Marie with her placement of the signs, and Marie returned the favour with a hint on the appropriate pair of factors. Anytime one of them was correct, they would whoop, make the victory sign, or give each other high-fives.

Soon, however, both Marie and James were looking for new challenges. Since everybody in the class had also mastered factorization of trinomials without a leading coefficient, nobody was in need of help anymore. I asked the students if they want another example, and James shouted:

> "Can we have something saucier, please?"

I wrote on the board the word "sossy", and under it I wrote a trinomial with a leading coefficient. James and Marie worked on it together, and after a short while, when I looked at them, they had not only factorized the trinomial, but they had also drawn a "Sossometer" with the indicator needle on level 2 (easy), which they proudly showed to the class (see section 6.4). Thus, the concept of "sossiness" was born. Subsequently, whenever the students were presented with an exercise, they requested to know its level on the "sossometer", and asked for progressively higher levels of "sossiness".
[Note: a more aesthetically pleasing sossometer was manufactured soon after and is in use to this day.]

James and Marie were interviewed when they were still in high spirits, and peppered their sentences with exuberant exclamations. They focused their attention on their friendly competition as being the engine of their enjoyment. Marie was careful though to point out that, in her view, not all competition is created equal:

> Marie: […] we wanted to compete. I was…I mean today I felt I understood everything. It seemed really easy for me. James was like "ok can you beat me to this" and then I thought "ok I gotta go gotta beat James".

For James, "everything worked out really well", and the level of the work, which I had labeled as grade 10, was "just right". Although both he and Marie encountered things they didn't know how to do, neither saw it as "getting stuck", and thus did not have any reason to feel anxious. On the contrary, they "had fun" working out which were the right pairs of numbers that would fit in. James took the opportunity to delve into the concept of "sossometer", explaining that it was his way to avoid repetition and boredom, and ensure that he gets examples that are harder and harder, which is how "he learns best". Both he and Marie expressed how much they like the so-called "monsters", examples of a high level of difficulty, "because there's more thinking". Marie noted that part of her enjoyment of "monsters" came from the fact that we worked on whiteboards, and thus she felt she had much more support and feedback than in the past.

Both James and Marie reported that they did not look at the time for the duration of the class, and did not feel bored or distracted at any time.

<u>How I saw James and Marie through the Proxies for Engagement (PFE)</u>:

A. Time to task – instantly
B. Time to first mathematical notation – less than 1 min
C. Time on task – the duration of the class
D. Eagerness to start – 3
F. Persistence – 3
G. Distractibility – 0

<u>How James and Marie saw themselves in the questionnaire</u>:

a) When you first started out, how optimistic were you about your ability to do the task?

James – somewhat optimistic — Marie – neutral

b) How good did you feel about what you were doing, while you were doing it?

James – very good — Marie – very good

c) Statement: “During the activity, I felt I knew what to do and how to be successful at it.”

James – all the time — Marie – all the time

d) Statement: “During the activity, my mind was thinking of other things.”

James – never — Marie – never

e) How often did you judge yourself negatively during the activity?

James – never — Marie – very rarely [Marie explained that she felt at times she was slower than James]

f) How often did you worry about others judging you during the activity?

James – never — Marie – very rarely

<u>James and Marie’s Shared Resource Space</u>:

Regulatory:	*Informational:*
Willingly engaging in a competition.	Teaching each other. Checking each other's work.
James and Marie	
Motivational:	*Ludic:*
Climbing a spiral of increased complexity that stemmed from their interaction as much as from the task itself.	Whooping, high-fiving, inventing a new way to measure difficulty.

4.6. The experience of Nadyia and Bianca

Nadyia and Bianca were not only new students in my class, but also very apprehensive about mathematics. I often noticed gestures indicative of anxious uncertainty (for instance, frequent and indiscriminate erasing of the whiteboard), uncertainty which led them to ask numerous reassurance questions that generally started with “is it true that …”, followed by a mathematical statement. Both Nadyia and Bianca were also struggling with the challenge of the language, in addition to the mathematics. They always tried their best, however, and were slowly, cautiously, allowing themselves to take more risks as they got used to the new environment. In my view, Bianca was slightly more tenacious than Nadyia, but not by much. Their grades were in the C range, and they seemed resigned to this.

The task they worked on was not a mathematics problem per se, but a cryptography problem. The topic of cryptography arose from my collaboration with the Social Science teacher, who was, at the time, leading her students through the study of World War II. In mathematics, the students watched Numberphile videos about the Enigma code, and read a book about life at Bletchley Park during the war. However, the first text that the students had to decode was not encrypted with anything more difficult than a simple substitution code. The encrypted text was the famous beginning of the novel “Rebecca”, by Daphne du Maurier:

> Last night I dreamt I went to Manderley again. It seemed to me I stood by the iron gate leading to the drive, and for a while I could not enter, for the way was barred to me.

The passage in code can be found in subsection 3.2.1.

Just as I had mischievously anticipated, some students, including Bianca and Nadiya, started from the premise that the passage was about mathematics, and assumed that one of the four letter words was MATH, and played with various whole-language strategies, trying to guess the sentence, rather than make the effort to reason and decipher letter by letter, following the logic of the text. Thus, Bianca posited that AUFC was MATH, which she wrote above the encrypted string. After that, they concluded – correctly – that U was

A, but also, incorrectly, that A was M, etc. They filled out the letters they had up to then, looked around very satisfied, and pronounced in a loud voice: “The first word is MATH, right?”. Many voices replied vehemently that no, it wasn’t, and they seemed so crestfallen that I intervened and suggested they look at the text and the length of words and think what the short words could be.

My feedback helped them refocus, and they erased everything they had (including the correct U->A), and started again from the observation that there were two one-letter words, and one had to be A, and the other I. They selected V to be A (incorrectly), but also made some better guesses about the two-letter words, of which they made an incomplete list. For instance, BY was not on their list, but TO and IT were, which helped them advance on other disparate letters and words, including the three-letter string CFZ, for which they considered only two possibilities - either AND, or THE. Once the T was found, they correctly deciphered CFZ as THE. This discovery filled out some more gaps, and, through experimentation, they were finally able to correct the initial assumption that V was A.

When I went to talk to them again, they did not remember how exactly they found their mistake, but they said that “something didn’t work”. At that point, since they were also able to observe the text with more knowledge about the letters, they were finally able to use their original language based strategy, and reject some constructions as “not making sense in English”, for instance the fact that a verb cannot follow the article “a”. Interestingly, the letter D->Y was the most difficult to guess, probably because it appeared in “Manderley” which was to them a new word, and also did not roll naturally off the tongue when they were repeating “the wa…the www…”, trying to guess the last letter of “way”. An interesting aspect of this group was that once they finished, and high-fived each other, they asked me for another example of the same difficulty. I observed their work and I noticed that not only had they incorporated all the successful strategies they had learned, but also begun testing a new one, which was to write down the alphabet and cross out the letters they had found.

I began my interview by congratulating them for having worked so hard. Nadyia's first reaction was to express her surprise that she could have "so much fun" on a Monday morning. Her explanation for that unusual circumstance was that the problem was not from the textbook, "which really helped". As I probed more, she referred to her perception of monotony when she has to work from the textbook:

> "I know I have to practice, but it's so repetitive… page whatever… number whatever…"

Unlike Nadyia, Bianca stated that she had no complaints about textbooks, and that her enjoyment of the class was a result of working with someone she felt she "matched", on a problem that connected her favourite topics:

> "I like everything that has to do with Socials and English… and I liked working with Nadyia… I like it when I have somebody beside me who is at the same level… yeah… sometimes I feel bad when I work with other people and we don't really match that well."

Both students assessed the problem, somewhat inconsistently, as "a bit hard", but "fun". They reflected on their "many wrong guesses" with amusement and confidence:

> B: Everything worked very well. […] In the end, it was fun. I liked how it felt when we filled more and more of the letters. It was like a puzzle.
>
> N: I think for me…well, I liked that it had to do with codes. Codes are cool!
>
> B: Yeah, I had never seen a code before…it was interesting. I didn't know it could be so easy, once you thought about it. When I first looked at that paper, I thought no way we can guess what this is, but in the end it felt so good, to go through and find more and more words.

When they got stuck, they "argued" or "kept going back". Nadiya viewed the latter as her main accomplishment:

> N: I liked that I was not afraid to start again. I mean, after a wrong guess.
>
> Me: You're sometimes afraid to start again?
>
> N: Yeah…maybe not afraid, but sometimes not motivated enough. If I know where the mistake is, sometimes I don't go back. But now I really wanted to complete the whole thing."

Both students reported that they did not look at the time for the duration of the class, which, they admitted, was so uncharacteristic, that they were themselves surprised. And although they identified instances of anxiety, such as when they discovered a letter was incorrect, overall, they viewed their experience as satisfying and even happiness-inducing.

How I saw Nadyia and Bianca through the Proxies for Engagement (PFE):

A. Time to task - less than 1 min

B. Time to first notation - 2 min

C. Time on task - the duration of the class

D. Eagerness to start - 2

E. Discussion within the group (participation) - 3.

F. Persistence - 3

G. Distractibility - 0

How Nadyia and Bianca saw themselves in the questionnaire:

a) When you first started out, how optimistic were you about your ability to do the task?

Nadyia - not at all optimistic	Bianca - not very optimistic

b) How good did you feel about what you were doing, while you were doing it?

Nadyia - somewhat good	Bianca - very good

c) Statement: "During the activity, I felt I knew what to do and how to be successful at it."

Nadyia - very frequently	Bianca - very frequently

d) Statement: "During the activity, my mind was thinking of other things."

Nadyia - never	Bianca - very rarely [Bianca explained that she had moments during which she thought about an upcoming test]

e) How often did you judge yourself negatively during the activity?

Nadyia - very rarely	Bianca - very rarely

f) How often did you worry about others judging you during the activity?

Nadyia - never Bianca – never

<u>Nadyia and Bianca's Shared Resource Space</u>:

Regulatory: Venturing some early assumptions. Designing a comfortable ambience in which they could make mistakes.	*Informational:* Debating which words were possible, eliminating the impossible through trial and error.
Nadyia and Bianca	
Motivational: Finding the energy to go back and fix mistakes. Feeling that "they were matched". Wanting to complete "the whole thing".	*Ludic:* Laughing, smiling, joking. Appreciating the puzzle and the "coolness" of codes.

4.7. The experience of April and Lucy

April and Lucy were two grade 9 students who were in my class for the second year in a row. April was a student with an Individual Education Plan, but who, through sheer determination and with a lot of help from her teachers (including learning support specialists) was able to achieve much better grades in grade 9 than she had in grade 8. In fact, while she was in grade 8, there had been some discussion about pulling her out of French Immersion, as she was suffering from anxiety at school. To everybody's delight, in grade 9 she had made excellent progress, both in her managing her schoolwork and in controlling her anxiety. She continued however to be very pessimistic in her outlook, and appeared to have very little confidence in herself. She never took the initiative to ask for assistance in class, but if I asked her whether she needed any feedback or help, her first words were "I don't understand anything", and she would display body language signalling great anxiety. Her gloomy self-assessment was seldom warranted, because most of the time, when asked to give more details, she understood much more than she gave herself credit for. Lucy, on the other hand, was the kind of student whose report card is likely filled with glowing praise. She was conscientious without ostentation, always had a genuine smile on her face, followed instructions with great alacrity, and was well-liked by her peers. In short, Lucy was the perfect student. Possibly because of this perfection, it was hard to get to know her very well.

I followed April and Lucy while they were solving the problem of the four 4s. They began their work without delay, Lucy with her customary diligence, and April making a visible effort to apply herself to the task. They had some success in the beginning, but avoided using any of the new operations. At some point, about 10 minutes in, I noticed them losing focus. I went to talk to them and the following conversation took place:

> Me: "So, how are you doing here?"
>
> L: [hesitation]. "All the numbers we find are already done."
>
> Me: "Ah, let's look. There are still many big numbers missing… you're getting smaller numbers. Have you tried aiming for something over 70?"
>
> A: "It's hard… my numbers are too big…"
>
> Me: "Can you show me what you're trying to do?"

April had however erased everything and was unable to show her ideas, and Lucy had not even written it, as she was doing everything with a calculator in hand.

> Me: "you know, if you write your expressions, even if they're wrong, you can still tinker with them a bit and then maybe you get something useful out of it."
>
> L: "Ok…"

April and Lucy applied themselves to writing on their whiteboard, and I went to talk to another group. I came back and I saw they were still not very productive, so I sent them to look at another group's work and find some inspiration. After that, I did not have any more interactions with April and Lucy, but from a distance they appeared to work and collaborate very well. My misconception became clear during the interview, when their body language signalled sentiments far from ease and satisfaction.

Although initially unwilling to say anything negative, after some persuasion they used the politest way to indicate that, indeed, they had not enjoyed themselves, and had not liked the problem. On the contrary, Lucy found it "hard to grasp". She indicated that in the numerous moments when they were stuck, they hadn't known what to do. They felt they did not benefit from looking at other groups, and found themselves unable to build on their ideas. April was open about the fact that she had felt bored, and, when asked, stated that she had checked the time very often – upon which Lucy nodded. Both students expressed frustration: Lucy, with the fact that she "didn't know what the point [of the problem] was", while April stated that she was "a visual person…and [likes] problems better if they're on a piece of paper". Although I asked, she was unable to pinpoint why she found working on whiteboards so radically different from paper. Both Lucy and April seemed ill at ease. It was quite obvious that, far from experiencing flow, they had had an unpleasant experience, and were reluctant to talk about it in detail.

<u>How I saw April and Lucy through the Proxies for Engagement (PFE)</u>:

A. Time to task – 1 min
B. Time to first mathematical notation – 1 min
C. Time on task – approximatively 20 minutes
D. Eagerness to start – 3
E. Discussion within the group (participation) – 2
F. Persistence – 3
G. Distractibility – 1

<u>How April and Lucy saw themselves in the questionnaire</u>:

a) When you first started out, how optimistic were you about your ability to do the task?

April – neutral	Lucy – neutral

b) How good did you feel about what you were doing, while you were doing it?

April – not at all good	Lucy – not very good

c) Statement: "During the activity, I felt I knew what to do and how to be successful at it."

April – never	Lucy – very rarely

d) Statement: "During the activity, my mind was thinking of other things."

April – very frequently	Lucy – all the time

e) How often did you judge yourself negatively during the activity?

April – very frequently | Lucy – occasionally

f) How often did you worry about others judging you during the activity?

April – very frequently | Lucy – occasionally

<u>April and Lucy’s Shared Resource Space</u>:

April and Lucy

Regulatory: Accepting the limitations of the problem, but feeling frustrated about them.	*Informational:* Observing what numbers were still not completed, sharing of calculator work.
Motivational: Not reported. More likely, "de-motivational", in their case.	*Ludic:* Not reported, likely non-existent.

Chapter 5. Analysis: the umbra of flow

The river flows, the seasons turn,
The sparrow and starling have no time to waste.
If men do not build
How shall they live?

T. S. Eliot, 1934 - *Chorus from The Rock*

While in chapter 4 I presented mere descriptions of students' experiences, in this chapter I will look at these experiences through the lens of the components of flow[1]: goal clarity, balance of skill and challenge, feedback, transformation of time, sense of control, merging of action and awareness, loss of self-consciousness, and autotelic experience, and thus set forth the reasons for my assertion that certain students were in flow.

5.1. Clear goals – the rules of the game

While some students are able to self-guide in the direction of flow right away, others are unable to enter flow without some intervention from the teacher. The first of these interventions is goal clarification, which serves to covertly indicate to the students where to channel their energy and prioritize their attention. Confusingly, there is no fixed prescription on how to clarify goals. Sometimes, students merely need to hear the problem restated. For instance, Brandon asked me a question regarding the maximum allowable jump in the problem of the staircase, a question that was answered by Karl. While this may at first appear to be a lack of confidence, it is obvious that Brandon knew what the parameters of the problem were, and he was rather looking for a way to open a conversation between me, him, and Karl. In confirming that yes, the most I could skip were two steps, I did not tell Karl and Brandon anything they didn't know. The unstated purpose of the exchange was to establish that all parties are on the right path: Karl and Brandon will be looking for a solution, while I (the teacher) will be there to illuminate the way out in case they got lost.

[1] As discussed in section 3.3.1, the concentration component was assessed through the PFEs in the initial stage of data collection.

The experience of Chloe and Tamara was similar. As they began working on the problem of the four digits, their body language suggested some hesitation. They, like all students in the class, knew about the new operations like factorial or summation, but they needed to hear the rules of the game repeated, for instance, that they could not use more than one digit of 1, and that 2 as an exponent was not permissible. In contrast, Glen and Linus did not ask for any clarification, but since I noticed them initiating a conversation unrelated to their problem (the problem of the four digits of 4), I took the initiative of suggesting that they find a solution using the factorial symbol. James and Marie did not need any clarification of goals from me, because the goals were intrinsic to the task: factor this polynomial. Bianca and Nadyia's goal was equally clearly delineated: decipher this encrypted message. Nikolaus and Lonnie, who were working on the problem of the Mickey Mouse fractal, were quick to begin work on their problem, and were able to clarify the goals of the problem for themselves.

As a teacher, I waver between worrying that I don't provide sufficient clarification, and vexation that I give too much away. Much calibration and recalibration goes into getting clarification right, and I wasn't always successful in curating a flow experience for the students. For instance, in the case of April and Lucy (also working on the problem of the four 4s), although I tried to help set clear goals for them, for instance in suggesting target numbers, they found the problem "hard to grasp". As I perfected my technique for providing clear goals for the students, I found that my task was very much to re-state, elucidate, and de-mystify "the rules of the game", and that once that was done to the students' satisfaction, they were more than willing to start work with confidence and even with enthusiasm.

5.2. Feedback – the game is afoot

Feedback is the other instrument in the toolbox of the flow-seeking teacher. As indicated in section 3.2.2, I distinguish between two kinds of feedback:

a) informatory feedback, which is merely a yes or a no, without any other information.

b) upformatory feedback, which consists of more elaborate assistance, meant to guide the student through the flow channel

Both kinds of feedback have one purpose though, which is to provide the student with a stream of data about his or her understanding and to give the student the opportunity to access the flow channel.

Students' need for feedback varied greatly. For instance, Lonnie and Nikolaus [Mickey Mouse fractal] distinguished themselves through their perseverance. When I first noticed their mounting frustrated, I asked them if they wanted a hint, which – to my surprise – they refused. After 15 more minutes, during which they circled the solution, still wavering, I suggested they multiply the sum by 2 in order to get a definitive answer. Tenaciously clinging to the mistaken hypothesis that the series was divergent, they accepted a hint only when they started having serious doubts about their intuition, because nothing else made sense.

Lonnie and Nikolaus put me in a dilemma in terms of what, when, and how much upformatory feedback I should give them. They were in flow while pursuing the wrong solution, and I became concerned that once they discovered their error, it would take away from the enjoyment that they felt during their circuitous journey, and would leave a bitter taste about their experience. However, I was happy to be proven wrong. The students did not seem in any way disaffected, and reported in very visceral language that they had a very enlightening experience. Much time in the mathematics classroom is dedicated to the search for correct answers, but there seems to be no connection between flow and correctness, other than the minor jolt of satisfaction when one is right.

Karl and Brandon [13 steps staircase] were first stuck on the issue of counting. Although I clarified that matter for them, they were still unsure about how to reason in order to get a "lead" on the problem. I could see them getting frustrated, so I intervened with upformatory feedback suggesting that they write a table of values for smaller numbers of stairs and look for a pattern in the numbers. This was sufficient to propel Karl and Brandon back into the flow channel. Once they saw the Fibonacci sequence, and taking into consideration their increased skill level, my second intervention was to ask them to explain the apparition of the Fibonacci sequence in what was, to them, an unexpected place. This led to another few minutes of flow until a satisfactory explanation was found. When interviewed, the students reported that they did not become too anxious while they were solving the problem, because I “gave them a hint”. They also noted that they didn’t want “too many hints”, as that would make things “too easy”, and that the right amount of hints made the experience “like in a game”. I was intrigued to hear that they mentioned games in their interview, as many computer games are purposefully designed to offer ingredients of flow (Blythe, 2003).

In my classroom, I found that a good way to duplicate the experience of playing a computer game is by using the individual whiteboards, and thus give personalized feedback to each student. In their interview, both James and Marie [factorizing polynomials] clearly indicated that, for them, using the whiteboard contributes to increased enjoyment of the class. While they do not use the word “feedback”, Marie does state that in elementary school “the teacher didn’t know what we didn’t know”, whereas since I “explain what’s wrong right away, it makes things a lot easier”. The rapid feedback cycle afforded by the individual whiteboards is stunningly effective in occasioning a state of flow, even for the most mundane of tasks. The only limitation is my speed of reading and thinking. However, students soon learn to seek feedback from one another: just like most students, James and Marie constantly checked (and checked-out) each other’s work.

When discussing feedback, one cannot overlook the importance of the feedback that the students give the teacher, which the teacher may not always judge accurately. Seen through the lenses of this feedback loop, Lucy and April’s experience is very understandable. I misread their quiet toil as engagement, and I did not offer enough upformative feedback. While they did not directly express dissatisfaction with the feedback they had received, when interviewed, they gave voice to a slew of negative emotions, ranging from indifference and purposelessness to boredom and discomfort.

5.3. Skills in balance with challenge, challenge in balance with skills

Csíkszentmihályi proposes the balance of skills and challenge as a state in which a person feels “there is something for them to do, and they are capable of doing it” (Csíkszentmihályi & Csíkszentmihályi, 1998, p. 30). That there exists a correlation between the balance of skills and challenge and flow has been thoroughly documented in experimental studies by Eisenberger et al. (2005), Engeser and Rheinberg (2008), Keller and Bless (2008), Keller and Blomann (2008), and Schüler (2010). These findings have been further refined to establish that, in addition to the existence of a balance between skills and challenge, it is also necessary for the skills and the challenge to be higher than the average for a person (Csíkszentmihályi & Csíkszentmihályi, 1988; Csíkszentmihályi & Rathunde, 1993; Massimini & Carli, 1988). The balance between skills and challenge is therefore dynamic, and an impulse for continuous growth. Once a skill has been mastered, in order to remain in flow, a person has to continuously increase the challenge, or risk loss of enjoyment or boredom. This “spiralling complexity”, which both fuels and engenders ever more flow experiences, is a force for creativity, discovery (Csíkszentmihályi & Csíkszentmihályi, 1988, p. 30) and eudaimonic happiness (Waterman, Schwartz & Conti, 2008).

All the students who were interviewed in my research – again, with the exception of April and Lucy – expressed, in various ways, the sentiment that the task achieved a balance: “just right” (James), “not too hard, but still presented a challenge” (Glen and Linus), “a bit hard in the beginning, but not too hard” (Lonnie and Nikolaus), “a bit hard… but in the end fun” (Bianca and Nadyia), “neither too easy, nor too hard” (Tamara), “doable…like a game” (Karl and Brandon). Karl and Brandon’s comparison between their process of solving the problem of the 13 steps and the process of playing a game is in line with the observed ludic nature of flow (Csíkszentmihályi & Bennett 1971). Similar to flow, games offer a framework where players are invited into a structured universe where they must find their own “sweet spot”, a place of perfect fit between capacities and demands. Good games persuade the player to come back because they offer meaningful challenges, and entice him or her to maximize their opportunities for action, while being limited by clear rules.

As mathematics teachers, we are very fortunate that a large part of mathematics is just that: play within some rules. Moreover, the game of mathematics can be simplified or complexified at will, both by the teacher, and – even more satisfyingly – by the students. Consider, for instance, the experience of Glen and Linus, who were creating numbers with four digits of 4. Discussing their experience, they remarked:

> It helped that you gave us the new operations all the time. […] it's a lot more interesting when you learn something new. I'd never heard of these symbols and I never knew they existed.

Assisted by my feedback, they showed great enthusiasm and creativity while problem-solving: anytime they learned a new operation, they insisted on using it as much as possible, to exploit its advantages in new and sometimes unorthodox ways, mastering and pushing the rules of the game at the same time. As predicted by previous studies, we see that the equilibrium between skills and challenge is fragile, and that effort must be made constantly in order for the balance to be re-adjusted, in keeping with the new skills or the new challenge.

To return to April and Lucy, who were also working on the problem of the four digits of 4, it is clear that they were unable to achieve a fit between skills and challenge. They found the task "hard to grasp", got stuck "a few times" without feedback, and "didn't know what was the point [of the task]". The game was meaningless, failed to excite their interest despite their best intentions, and not being able to play caused them embarrassment and frustration.

5.4. The passage of time

A close look at how students perceive time is particularly pertinent in the context of a discussion about flow, since Csíkszentmihályi considers "transformation of time" as one of the core characteristics of flow: when one is in flow, time passes very fast, and hours feel like minutes (1990). This inbuilt awareness of time is a "sensitive index of the basic function of emotion" (Droit-Volet & Meck, 2007), and can be used as a proxy for identifying various states of being. For instance, when students attend to the passage of time, it is a clear indication that they are bored (Danckert & Allman, 2005; Eastwood, Eastwood, Cavaliere & Fahlman, 2007), unable to pay attention (Eastwood, Frischen, Fenske & Smilek, 2012), or even sad (Gil & Droit-Volet, 2009).

In my experience, during a regular class, students don't often lose track of time, and frequently cast quick glances at the wall clock or at their phones. However, all the students I interviewed, with the exception of April and Lucy, responded that during the activity they did not look at the clock at all (Brandon, Nikolaus, Lonnie, Chloe, Tamara, Marie) or only once, close to the end of the class (Karl, Glen, Linus, James). Many also remarked on this as an anomaly. For instance, in the case of Nadyia and Bianca, who worked on the cryptography problem, one of my questions was whether they were aware of how long it took them to decrypt the message. Nadiya expressed surprise:

> I couldn't believe that we worked on that almost the whole class!

She indicated that she did not look at the clock at all, while Bianca stated that she looked only once, when she had finished the first task − in contrast with her usual behaviour during class, when she looks at the clock approximately every ten minutes. Interestingly, at this point in the interview, Bianca added:

> My mother says that when you're not looking at the clock, it means you're having fun.

When asked whether she herself had fun, she enthusiastically confirmed it. It's gratifying that young adolescents (and their parents) make the seamless connection between losing track of time and enjoyment, and it should come as no surprise. In his original research about peak experience, Csíkszentmihályi hypothesized that most humans have had, at some point, the experience of total immersion in an activity, to the complete loss of awareness of time, and that this experience serves as a point of reference for other situations (1975a, 1975b).

If April and Lucy experienced a sense of distortion of time, it can only be the feeling that time passed excruciatingly slowly. April gamely admitted that she looked at the clock "more than Lucy", while Lucy stated she checked the time "often enough to notice [they] were not making progress". The impression we get is one of crushing discouragement, rather than satisfaction. Based on this measure, it is very unlikely that April and Lucy were in flow. However, the loss of time awareness is not, taken in isolation, an indication of the presence of flow, but rather one of the predictors on which a determination of flow can be made. We will therefore reserve judgment both about April and Lucy, and about the other students.

5.5. A sense of control

From the point of view of flow theory, engendering a sense of control is essential for occasioning flow (Csíkszentmihályi & Csíkszentmihályi, 1988). Even if flow were considered too lofty a goal, a host of other behaviours that we want to encourage in students, such as curiosity, interest, the pursuit of mastery, eagerness for challenges, intrinsic motivation, better performance, are associated with students feeling a sense of control and autonomy (Adie, Duda & Ntoumanis, 2008; Csíkszentmihályi et al., 2005; Reinboth, Duda & Ntoumanis, 2004; Sheldon & Krieger, 2007). Conversely, a lack of "decision latitude" is a cause of mental strain and dissatisfaction (Karasek, 1979), or boredom and frustration (Allison & Carlisle Duncan, 1988).

For the purposes of my research, I asked students to assess their subjective feelings of control after the task, using Likert-type scales.

a) when you first started out, how optimistic were you about your ability to do the task? (students choose from **very optimistic 5; somewhat optimistic 4; neutral 3; not very optimistic 2; not at all optimistic 1**
b) how good did you feel about what you were doing, while you were doing it? (students choose from **very good 5; somewhat good 4; neutral 3; not very good 2; not at all good 1**
The results were telling:

Student	Rating for question a)	Rating for question b)	Change
Glen	3	4	+1
Linus	4	5	+1
Karl	3	5	+2
Brandon	2	5	+3
Nikolaus	3	5	+2
Lonnie	4	5	+1
Chloe	2	5	+3
Tamara	2	5	+3
Nadyia	1	4	+3
Bianca	2	5	+3
James	4	5	+1
Marie	3	5	+2
April	3	1	-2
Lucy	3	2	-1

Table 2. Students' self-assessment of their sense of control during the task.

With the exception of April and Lucy, all the students' scores increased, sometimes quite dramatically (Nadyia and Bianca, for instance). The students who reported an increased control score also reported that they felt no anxiety or worry, with Nikolaus adding that they felt "relaxed", and that they "didn't have time to worry". This is an intriguing observation, which seems to connect the carefree self-confidence that comes from having a sense of control with the other characteristic of flow: the distorted perception of the passage of time. Further, Lonnie remarked that the activity felt different, because "it was not from the book" (see section 6.6 for more discussion on that topic). He also stated that the task felt "like a game… there was no test", and "[they] did it because [they] wanted to do it" – which indicates not only that he felt a sense of control, but also that he made the conscious choice to engage with the task. We can also observe again that April and Lucy's rating of their sense of control decreased during the activity. They began in the neutral stance, which could have been promising enough, however, at the end of the activity, their rating plummeted to the lowest or second lowest. It’s becoming more and more unlikely that April and Lucy experienced flow during the class when they were interviewed.

5.6. Merging of action and awareness and loss of self-consciousness

Both merging of action and awareness (MAA) and loss of self-consciousness (LSC) are measures of how absorbed a person is in an activity. Action and awareness become one, and the “oneness does not require effort in flow” (Jackson & Csíkszentmihályi, 1999, p. 19). This apparent lack of effort does not imply that the activity is easy, but is rather a direct consequence of the exquisite balance between skills and challenge mentioned in section 5.3. A person that has mastered the necessary skills to be successful will allow their consciousness to surrender to flow, trusting that all the right things will happen without the need to worry or think too much (ibid., p. 27). The lack of worry speaks to the loss of self-consciousness as well: the oneness that characterizes MAA means that the state of psychic entropy, the inner turmoil of thoughts in which humans often find themselves, is replaced by negentropy, the state in which the self is in harmony, free to focus on growth and enjoyment, rather than chaos and dissatisfaction.

Perhaps a quick disclosure is necessary at this point: my first tries in eliciting and capturing flow were not successful, because I only looked at balance, goals, feedback, and temporal distortion. Gradually however, through listening to the students, I realized that these are not sufficient to make a determination about flow. The students reported feeling the four characteristics mentioned above when writing a test, activity during which they did not enjoy themselves, after which they did not feel good, and which was not, by any measure, an autotelic experience. This was an eye-opener, and made me recognize how MAA and LSC get at the heart of the subjective experience of flow, **and** how delicate the task of extracting information about MAA and LSC can be. In the interviews, I was able to ascertain that many students did not feel worry at all (Chloe, Karl, Brandon, Glen, Lonnie and Nikolaus), and some worried “a bit” (Nadyia, Bianca). Furthermore, some students even evoked sentiments that seemed to indicate that they indeed experienced MAA and/or LSC. I'm referring, for instance, to Lonnie's remark, "*in the end, it all came together",* to Maria's observation, *"…today I felt I understood everything. It seemed really easy for me",* and to Tamara's commentary *"We just made them [the ideas] up as we went along… There weren't any rules really",* all of whom offer a glimpse into the effortlessness and spontaneity characteristic of the MAA and LSC components of flow.

However, it seemed to me that it was hard to get students to talk about what had just happened: we had all seen or felt the "oneness" of flow, and yet, in their interviews, the students seemed unable to articulate what had made the experience so rewarding, and mostly kept repeating that it was "fun". I was reminded of the unsatisfactory interviews of great athletes, who, after having performed incredible feats, are asked by eager reporters on live TV "and how did it feel to…?" and can barely muster a "it was great, really awesome." Seen through the lens of the flow model, such reticence of language is understandable, and reflects the difficulty of pouring the inexpressible into conventional forms of communication. As David Foster Wallace wrote in his essay "How Tracy Austin broke my heart":

> The real, many-veiled answer to the question of just what goes through a great player's mind as he stands at the center of hostile crowd-noise and lines up the free-throw that will decide the game might well be: nothing at all. (Wallace, 2005, p. 154)

In the case of my research, answering "nothing at all" would not be sufficient to assess whether my students experienced MMA and LSC. Hence: the questionnaire (see section 3.3.2), intended to bring more clarity and nuance to their experience.

I remind that questions a) and b) assessed merging of action and awareness:

a) How much do the following statements reflect your experience?

a) "During the activity, I felt I knew what to do and how to be successful at it."

Students selected an answer from the range **all the time; very frequently; occasionally; very rarely; never.**

b) "During the activity, my mind was thinking of other things"

Students selected an answer from the range **never; very rarely; occasionally; very frequently; all the time.**

Their ratings were as follows:

Student	Rating for question a)	Rating for question b)
Glen	4	"it depends"
Linus	5	"it depends"
Karl	4	5
Brandon	4	5
Nikolaus	5	5
Lonnie	5	5
Chloe	5	5
Tamara	4	3 [a test]
Nadyia	4	5
Bianca	4	4 [a test]
James	5	5
Marie	5	5
April	1	2
Lucy	2	1

Table 3 **Students' self-assessment of their merging of action and awareness during the task.**

To assess whether the student experienced loss of self-consciousness I asked the following questions:

c) ***How often did you judge y'ourself negatively during the activity (for instance, "I'm no good at this", etc.?)***

Students selected an answer from the range **never; very rarely; occasionally; very frequently; all the time.**

d) ***How often did you worry about others judging you during the activity? ("others" = your partner(s), other classmates, the teacher)***

Students selected an answer from the range **never; very rarely; occasionally; very frequently; all the time.**

Their ratings were as follows:

Student	**Rating for question c)**	**Rating for question d)**
Glen	4	5
Linus	5	5
Karl	5	5
Brandon	4	4
Nikolaus	5	5
Lonnie	5	5
Chloe	5	5
Tamara	4 [not fast enough]	3
Nadyia	4	5
Bianca	4	5
James	5	5
Marie	4 [slower than james]	4
April	2	2
Lucy	3	3

Table 4. Students' self-assessment of loss of self consciousness during the task.

In analyzing the scores the students gave to the four questions, several things become apparent:

1. Most students who communicated in their interviews that they had had a positive experience gave high scores to questions a-d. An exception may be Tamara, who stated in the interview that she "had lots of fun", but in her scores for questions b) and d) indicated

that her mind was occasionally preoccupied about matters unrelated to the activity, and that she occasionally worried about being judged by others. The case of Nikolaus and Lonnie, who rated a 5, despite being the groups who encountered the most difficulties, is also compelling. Their individual experiences will be discussed in more depth in the section 6.1.

2. Glen and Linus, who were partners during the activity with the four digits of 4, crossed out the suggested scale for question b) and wrote "it depends" over it. When asked to clarify, they explained that they took breaks during the activity, during which they talked about other topics of interest to them (and, they claimed, *waited for ideas to come)*. In their view, however, it did not count as "during the activity", but as "during a break", so they considered themselves a 5, as they were focused solely on the activity *while working on the activity*. I was inclined to accept their argument, as it is difficult to sustain long intervals of flow without interruptions. Csíkszentmihályi observes that episodes of flow, when one is "in ludus", are disturbed by instances of loss of concentration, and temporary rifts between awareness and action, which he calls "inter ludes" (1975a, p. 38). In light of this, it is not unlikely that all the students who gave high scores on questions a-d experienced this kind of ebb and flood, but were able to re-enter flow without great delay. This re-entry was facilitated by the "limited stimulus field" characteristic of flow-inducing activities (ibid., p. 48). When the goals are clear, the feedback is appropriate, and the challenges are in balance with the skills, one feels able to reject distractions, force order on psychic entropy, and concentrate on the coherent demands of the task at hand.

3. Not surprisingly, Lucy and April did not give high scores to any of the questions a-d. The scores confirm what they already stated in their interview: they could not figure out what to do during the activity, they judged themselves and feared being judged by others – and though they did not say it outright, based on their hedging and hesitations during the interview, I presume it was my judgment they were concerned about. I was intrigued by the discrepancy between reality and appearance: were an outsider to glance in that day, he or she would have seen Lucy and April "working hard": eyes on the board, pens in hand – and yet, they were categorically not in flow. As Csíkszentmihályi insists time

and time again, flow is a subjective experience (1975a; 1990), and thus not always easy for an outsider to diagnose.

5.7. Autotelic experience

An autotelic experience (from the Greek auto – *self* and *telos* – goal) is the kind of experience people seek not for gain and reward, but for the pleasure it gives in and of itself (Csíkszentmihályi & Nakamura, 1989), and rests on the two complementary pillars of enjoyment and involvement (Landhäußer & Keller, 2012, p. 72). Of the 14 students I interviewed, 12 categorically stated that they "had fun" or had an "enjoyable" experience. They singled out various characteristics of the activity as engendering this pleasurable feeling: "learning new math… something new" and having "moments of realization" (Linus), "working together and discovering something" (Glen), "the problem was not too easy" (Nikolaus), the fact that an "explosion" took place in his brain (Lonnie), the fact that "there wasn't just one answer" (Tamara), the power that came from yielding mathematical operations to make all the numbers from 1 to 100 with only four digits and the surprise that it was possible (Chloe), the pleasure and satisfaction of a puzzle (Bianca, Marie), the interestingness of codes (Nadyia), the progressive difficulty of the task (James), and the thrill of recognizing the Fibonacci sequence in an unexpected place (Karl and Brandon). As predicted by previous research (Csíkszentmihályi, 1975a, p. 30), students clearly identified discovery, exploration, problem solving, surprise, novelty, harmonious collaboration and appropriate challenge as essential ingredients of their autotelic experience.

Some students were also sensitive to the fact that the activity had no practical goal, to be tasted for its own sake, rather than part of the school diet. For Lonnie and Nikolaus, it was an opportunity to embrace the freedom of the game:

> Nikolaus: For me […] yeah, it was different too. I just felt relaxed.
>
> Lonnie: Yeah… It wasn't like a test. It was not from the book. It felt … I don't know, like a game? […] because we didn't have to do it, there was no test… we did it because we wanted to do it.

On the other hand, April and Lucy remarked on the fact that they didn't see "a point" to the task as a cause for dissatisfaction and lack of engagement. However, even if they had "seen the point", and pursued the problem for that reason only, they still would not have been in a state of flow unless they had also been able to ignore the very "point" they felt was missing. Their case limpidly illustrates the difference between doing something because one has to, because it's useful, and doing something because one wants to, not only regardless of its utility, but also willfully blind to its inherent coercion (it is obvious that Lonnie and Nikolaus perceived freedom of action is a mirage: they had to wake up in the morning, they had to come to school, they had to do work during class, etc.). Benefits of an external nature may accrue in both situations, but only the virtuous illusion of the latter can occasion an autotelic experience.

5.8. Conclusion

Based on the data described in sections 5.1 to 5.7, I can confidently make the determination that, with the exception of April and Lucy, all the students who took part in the research were individually in flow at the moments described, based on the characteristics of flow proposed by Csíkszentmihályi. They felt that their skills and the demands were in balance, reported distortion of temporal experience, engaged with the task of their own free will, without expectations of external rewards, felt in control, totally absorbed, and one with the actions they were undertaking. This was, however, only part of my research. In the following chapter I will highlight other themes which emerged from the data.

Chapter 6. Discussion: themes and motifs

"Human beings make life so interesting. Do you know,[…] in a universe so full of wonders, they have managed to invent boredom."

Terry Pratchett, 1996 - *Hogfather*

In this chapter, I propose to look at themes that emerge out of the episodes and experiences described in chapters 4 and 5. I will describe and examine various types of flow experiences observed throughout the research, and discuss how certain features of the classroom environment may enhance the flow experience or derail it into boredom or anxiety. I will also consider what types of task are more likely to induce a state of flow, take a look at unusual manifestations of flow, and explore the enduring conundrum about the connection between teachers' flow and the students' flow. Through these considerations, I will shed further light on the characteristics of the flow experience in the classroom.

6.1. Individual differences in experiencing flow

As the research progressed, it became obvious that not all students were provoked into flow by the same experiences, and that not all students who experienced flow felt it with the same intensity. Some liked problems that went beyond the letter of the textbook, and required them to stretch their imagination and their skills; others found flow in the more prosaic classroom practice. Some students were able to remain in the flow channel for the entire duration of the class; others only lasted for minutes, and then sought stimulation in other pursuits. These differences in how and when flow is experienced are not surprising, and depend on a variety of individual traits and contextual factors. Csíkszentmihályi has remarked that "flow exists on a continuum, from extremely low to extremely high complexity" (1975a, p. 141).

To wit, I noted with great interest the case of Glen and Linus as an example of oscillation between these two extremes. The students went periodically through cycles of **deep flow ——> extraneous banter ——> deep flow**. As the new goals (finding a number, mastering an operation) became manifest to them, they went through a period of great exuberance during which they exhibited all the signs of flow, and seemed hungry for the intellectual stimulation of the problem. After a while, their appetite was sated and they would begin a period of carefree badinage. As soon as I noticed them, and presented them with a new tool, they re-entered deep flow immediately.

Far from being resentful of me for interrupting their conversation, they were always very eager to start again in the new context. It was as if they had settled in the flow playground, and were trying on rides of various levels of risk. As discussed in section 5.6, they gave themselves a 5 for merging of action and awareness, while admitting that they had moments of shallower concentration. However, it did not seem to them that much had changed between the "in ludus" and the "inter ludus". Their conversation may have seemed inconsequential chitchat, but it was obviously neither anxiety, nor boredom.

It is my contention that these interludes were instances of *microflow,* more specifically *social microflow*. Csíkszentmihályi described microflow as occurring during "trivial activities…at a lower level of complexity…which may be as intrinsically rewarding as deep flow activities." (ibid.). Through his research, he established that at a large part of microflow activities occur in the social sphere, and involve "unnecessary talking and joking with other people" (1975a, p. 146) – which is exactly what Glen and Linus were doing. Further research conducted by Davis (2010) situated microflow as a way to "wait well", and hypothesized that microflow may be a way to keep the brain in a state of arousal and positive affect until an opportunity for flow arises. It is therefore no accident that Linus, when asked to explain their moments of "going off on a tangent", explained that they "were waiting for the ideas to come". It may well be that had I not intervened, ideas might have been a while longer in coming. However, since microflow is perceived as enjoyable, and a good use of time (Davis, in the research cited above, calls it a *pseudopeak experience),* it is nonetheless a desirable, albeit low-complexity state, which may serve as a safe harbour from the boredom of waiting, as well as a springboard for deeper optimal experiences.

The case of Nikolaus and Lonnie presents interesting characteristics pertaining to the way students respond to challenges during a flow experience. While working on the problem of the Mickey Mouse fractal, Lonnie and Nikolaus encountered difficulties in determining the value of the geometric series 1+1/2 … etc. As mentioned in section 4.4, their union was born out of the desire to pursue the same ineffective avenue, i.e. working with decimals instead of fractions. This meant that while the other groups were making solid progress, Lonnie and Nikolaus were mired in decimals, valiantly persevering despite the challenge. As they continued their endeavour, it seemed to me at some point that they had had enough fruitless experimentation, so I intervened with feedback – which, to my great surprise, they completely ignored for a while. This left me perplexed, for it seemed to me that Nikolaus and Lonnie could not have been in flow at that time: neither were their skills in balance with the challenge, nor did they find any use for my "right" feedback, preferring instead the "negative" feedback originating in their "wrong" solution. In order to shed light on their experience, it is worth considering some aspects going beyond feedback and the balance between challenge and skill. I'm referring here to their goals, their motivation for the activity, their personality, and their unique form of *communitas*, all of which affected the way they responded to the problem.

Firstly, what emerges from their own words is that they saw themselves as completely intrinsically motivated when engaging with the problem. Lonnie stated that "it felt… like a game… we did it because we wanted to", Nikolaus felt "relaxed… we didn't have time to worry [about not being be able to solve it]". Secondly, the use of the pronoun "we" denotes the value they attached to their stance: working together, but as outsiders, as rebels going on their own path. Thirdly, unlike many students, whose goals seemed to be focused on outcome, Lonnie and Nikolaus appeared to be more focused on the process, as a form of play. Because they viewed the activity as a game, they were freed from the fear of failure and from the stress of social comparison, factors that usually impair the onset of flow (Abuhamdeh, 2008). Removing all expectations about performance tipped the balance in favour of a higher challenge and higher risk, as illustrated by their unique strategies and their refusal to accept my help. Thus, it appears that Lonnie and Nikolaus' experience reflects what Liljedahl (2017, p.8) calls "a moment of imbalance", and they found themselves situated not in the flow channel, but in its outskirts, in the boundary region Liljedahl calls *perseverance* (see Figure 8). Students who find themselves in the *perseverance* channel (or in its *tolerance* counterpart) “use the buffer created by perseverance … to avoid frustration as they [seek] to correct the imbalance between skills and challenge” (p. 18).

Figure 8. The tolerance and perseverance channels.
(Liljedahl, 2017)

However, playing in the fringes is a high-risk activity – if the gambit does not succeed, frustration or boredom await. An athlete interviewed by Csíkszentmihályi and Jackson (1999) describes a similarly unstable experience: “it’s riding the razor… you can fall one way or the other” (p. 129). For an observer, it may seem that “riding the razor” is dangerous and counter-indicated. Indeed, my reaction to Lonnie and Nikolaus’ circuitous approach was that of a concerned parent watching her child climbing a precarious cliff. Upon further reflection, I have come to believe that Lonnie and Nikolaus enjoyed their adventure precisely because of the element of suspense and uncertainty contained therein. Abuhamdeh, Csíkszentmihályi, and Jalal (2014) explored the very experience of students who enjoy “outcome uncertainty and suspense” to such a degree that considerations about the balance between skill and challenge become secondary[2]. It is not an accident that in

[2] It is important, however, to note that these findings hold only in circumstances in which concerns about one’s performance are absent; only then will a person seek and enjoy activities that may lead to “defeat” (ibid.)

their interview, Lonnie and Nikolaus hint abundantly at the enjoyable soupçon of peril in the problem, mentioning the thrill of "discovering for oneself" and "good explosions". I consider that here lies the key to their experience, and that what distinguished Lonnie and Nikolaus from their colleagues was not better skills, but a subversive taste for exploration and mathematical risks; they are simply better at *the work of play*.

The experience of James and Marie contains elements of the tolerance referenced above in Liljedahl's model. As mentioned in section 4.5, James and Marie were working on learning how to factor trinomials. The work of adjusting the balance between skills and challenge was done by me, and, inevitably, this meant that at some point the task was not sufficiently challenging for them – a fact easy to ascertain by the lowering frequency of their exchanges and collaboration. As they mastered more and more of the skills required in order to solve the challenge, they didn't need each other's help for solving the examples and they started working more independently. They kept on working like this for a few more minutes, although they had started exiting the flow channel and descending the lower border of *tolerance*. As their skills increased, the challenge that a few minutes previously had elicited so much enthusiasm was becoming, in Liljedahl's model, *mundane*.

However, as described in section 4.5, their tolerance was not long-lived, and soon they requested an increase in the challenge via the sossometer (see section 6.4). I found the experience of James and Marie very interesting for two reasons: firstly, they seemed to have journeyed through perseverance, flow and tolerance for the mundane, all in the space of no more than 20 minutes, in a striking illustration of the fluid nature of flow and its contiguous states. Secondly, in their case there appears to be a qualitative difference between the tolerance and perseverance borders: the time spent in the perseverance border was considerably longer than the time spent in the tolerance border (it could be said that James and Marie had little toleration for *tolerance*). This observation supports the forthcoming discussion in section 6.7 about how students perceive boredom as more unpleasant than anxiety. We can expect these differences to be even more marked in the borderlands of flow, when the memory of the enjoyment of flow is still fresh in a person's mind, and can motivate and guide back to the optimal experience.

The dyads Chloe and Tamara and Bianca and Nadyia also experienced moments of frustration while solving their respective problems (four random digits for Chloe and Tamara, encrypted message for Bianca and Nadyia), and seemed at times in danger of *falling off the razor.* Their *perseverance buffer* kept them trying, and they eventually re-entered the flow channel. However, there are some other notable aspects that are worth exploring. Firstly, Tamara, although she stated in her interview that “she had lots of fun”, did indicate in her questionnaire that her mind was occasionally preoccupied about matters unrelated to the activity (a test, in her case), and reported occasional worries about being judged for not being “fast enough”. In a follow-up discussion, Tamara was surprised that I saw a contradiction between having fun and being occasionally worried. It seemed to me that her natural state was one of mild disquiet. She stated that she did not “wish to have been doing something else” – a question that Csíkszentmihályi frequently asks when measuring flow (Csíkszentmihályi & Csíkszentmihályi, 1988, p. 256); rather, she was unable to stop completely all intrusive thoughts about her upcoming test. On the contrary, Chloe, her partner, who was also due to write the same test, expressed relief that working on the problem helped her not to think about the test at all.

Tamara also reported that she judged herself for being “too slow”. She admitted that no one imputed her for being slow – she made this judgment herself and allowed the thought to occasionally intrude on her consciousness. Bianca also reported being very rarely worried about a test, while her partner, Nadyia, reported no such thoughts. In fact, Nadyia not only surprised me, but she also surprised herself by concentrating so well “on a Monday morning”, which to her presaged discomfort rather than engagement. Marie, who was partnered with James, also demerited herself to a 4 for the infrequent thought that she was “slower than James”. When asked whether she compared herself to others when playing volleyball, for instance (I knew that Marie was on the school team), she admitted to doing it “a lot”. I can only exclaim in frustration: *what is up with girls?*

As a matter of fact, many things are up with girls. Although Csíkszentmihályi insists throughout all his writings that gender is irrelevant to the flow experience (1990), and that girls experience at least as much flow as boys (Schmidt et al., 2007), a significant body of research seems to suggest that girls tend to worry more than boys (Balding, 2006), and that they are more likely to perceive the world through anxious glasses. Coleman and Hagell (2005) observe that

> […] girls are more affected by stress than boys, […] see setbacks and adversities as more threatening than boys, and are more likely to expect the worst in stressful situations. (p. 168)

Some of the female students I interviewed certainly appeared more anxious than any of the male students. The male students seemed to care less not only about what others thought of them, but also, as seen above, even about their success in the activity. Among the female students, we can also distinguish some differences: some occasionally concerned about a future unpleasant event, giving rise to the hypothesis that some girls are more sensitive to variations in the meanderings of a problem, and in those moments of uncertainty, the self-consciousness barrier created by the positive experience of flow deteriorates, allowing outside worries to infiltrate. Support for this hypothesis comes from research conducted by Schmidt et al. (2007), in which they argue that positive feedback and perceptions of competence and success affect adolescent girls more than boys. They advise therefore that "young females [be provided] with opportunities to engage in activities with clear goals and criteria for success, where they would be most likely to earn positive feedback" (p. 563). Other female students reported comparing themselves to their partners, which again, is not unexpected: girls are also more likely to engage in social comparison, and to judge themselves with diffidence (Guimond, Chatard, Martinot, Crisp & Redersorff, 2006).

Nevertheless, I note that according to the students' own reports, these negative thoughts were but fleeting, and that not all female students experienced such mental intrusions. With that in mind, I reiterate that the flow model calls for the balance of *perceived* skills and *perceived* challenge. The fact that there is a substantial subjective element to the experience has both positive and negative consequences: some students persevere for long periods of time, sustained by an ounce of knowledge and a ton of optimism. Others, equally competent, need much more feedback and encouragement to keep their belief in their abilities. If these students fall off the flow channel, it may not be because their skills are not high enough, but because of their perception of those skills is inaccurate and errs on the side of modesty. While self-criticism is a valuable way to gain feedback, it is also a dangerous weapon to yield: carry it to excess, and the loss of self-consciousness that is a mark of flow dissipates like a sand castle swept away by waves. Fortunately, although some of the female students in my research were more vulnerable to intrusions of extraneous thoughts, this did not diminish of their enjoyment of the task. Flow seems to be a resilient, dynamic state, and there may be a certain *tolerance* for unplanned, but not totally unexpected, interferences into the mental absorption that is characteristic of flow, similar to the tolerance posited by Liljedahl with regards to the interplay of skill and challenge.

Although more sensitive to inner adverse fluctuations, girls were, on the other hand, more likely to articulate explicitly how they were carried along by undercurrents of aesthetic desires. I commented on this in my interview with Chloe and Tamara, after watching them select carefully, from possible solutions, one which “they liked”. This, as any teacher knows, is a unique situation, in which the students, instead of catching the fastest solution that presents itself, prefer to take the “scenic route”. At the time, I jokingly compared their strategy with “choosing a dress” – a comparison which elicited exclamations of approval. However, behind this seemingly trivial exchange about articles of clothing, I discern a thread that runs through the experiences of other students as well, namely the connection between flow and aesthetic experiences.

The relationship is closer than may be suspected[3]: indeed, a side-by-side comparison reveals compelling similarities:

[3] It is no coincidence that Csíkszentmihályi’s first reflections on flow were inspired by watching artists - people whose entire life revolves around aesthetic experiences.

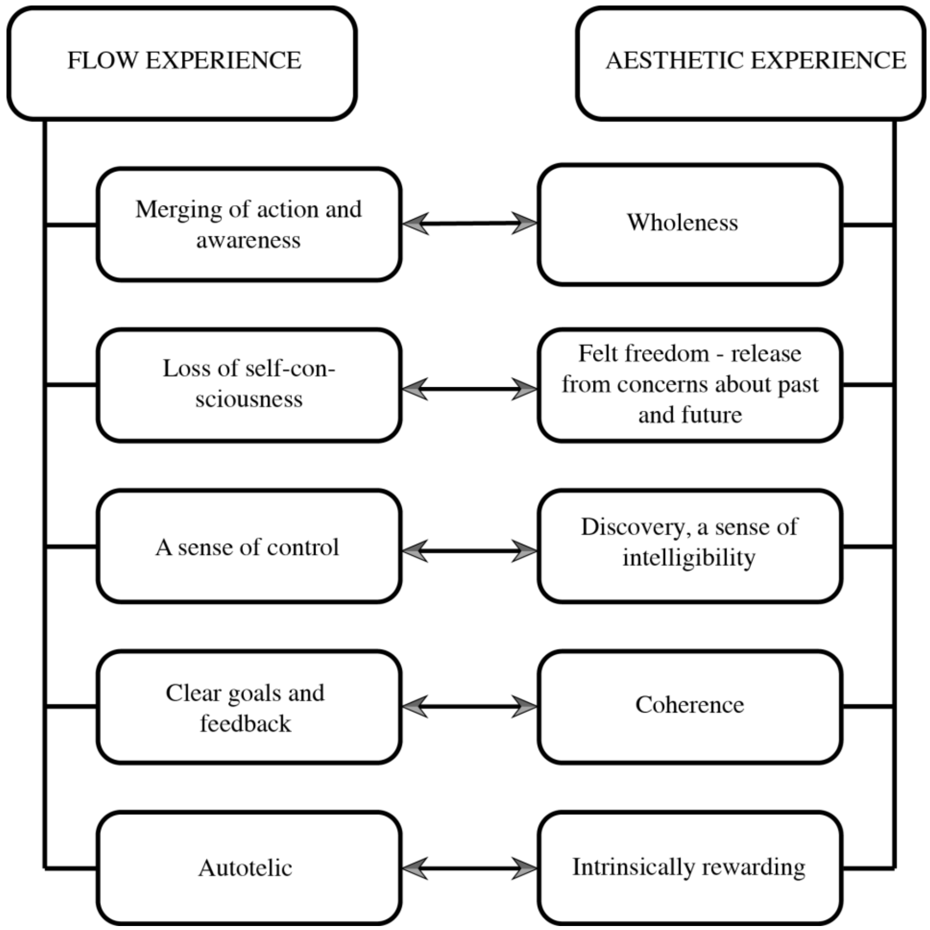


Figure 9. Comparison between the flow experience and the aesthetic experience.

(Csíkszentmihályi, 1975b; Beardsley, Callen & Wreen, 1982, Csíkszentmihályi & Robinson, 1990)

The aesthetic experience is further similar to flow insofar as it punctuates reality with moments pleasurable of intensity. In the words of Maxine Greene, aesthetic experiences have the ability to "place every day in parenthesis, and ward off chaos, without denying it" (as cited in Csíkszentmihályi, 1997a) – thus pointing us to another zone of tangency: Csíkszentmihályi's concept of "negentropy", the state of order and harmony, and opposite of chaos and entropy. Contemporary philosopher Roger Scruton comments on this sense of harmony and integrity thus:

> Aesthetic choices correlate with a particular kind of satisfaction: the satisfaction that comes when things look, sound, or feel just right. (Scruton, 2011, p.312)

And “just right” is precisely the phrase used by many students in their interviews. This exquisite dovetailing may not be confined to the realm of their experience with the problem, but may also be seen as referring to the sense of contentment that comes from perceiving themselves as a perfect match for the world and its challenges. True, the students made fewer explicit reflections of aesthetic matters in their interviews. This apparent paucity may however be due to the fact that they may have never learned the habit of contemplating their own aesthetic experiences in the mathematics classroom. As Sinclair (2011) puts it, there is a tendency to

> treat the aesthetic as epiphenomenal, overly vague, or even frivolous […] which allows the aesthetic to operate in somewhat covert ways within the mathematical community […]. (p. 3)

Nevertheless, the fruitful, enjoyable “aesthetic choices” made by Chloe and Tamara, or Lonnie and Nikolaus, and the sense of satisfaction expressed by Glen and Linus, together with their uncharacteristic observation that the problem was “nice”, are evidence that these students felt an aesthetic thrill at least at some point throughout the meanderings of their experience.

6.2. Collaboration and flow

The style of collaboration described in this research was not scripted, and the students were not assigned roles. In keeping with the flow component of control, the students had no constraints in how to collaborate, as long as their interactions remained productive. As a result, all the students I interviewed expressed a marked preference for group or partner work. Evidently, each dyad had their own way of inhabiting the Shared Resource Space: some mentioned the exchange of ideas and cooperation (Chloe, Brandon, Glen, Karl), others, working with someone with a similar learning profile (Bianca, Nadyia), the satisfaction of sharing the fun (Linus, Glen, Marie, Tamara), or the gratifications of friendly competition (Marie, James). Seen through the prism of the flow model, the student’s experience in collaboration clearly aligns with components of flow: feedback and clarity of goals, an autotelic experience, intense concentration, the ability to center the attention on a limited stimulus field, and balance between skills and challenge. Also worth noting is the fact that the regulatory and the informational dimensions included in the Shared Resource Space were alluded to only by a small number of students. Most of them pointed to the advantages of collaboration in terms of affective benefits, situated in the motivational and ludic dimensions. Although as an outsider I was able to observe all four dimensions, what was salient for the students was mostly their emotional engagement with *the other*.

Student-speak	**Flow-speak**
"Sharing the joy", "fun"	Autotelic experience, concentration
"Learn from somebody else", "exchange ideas"	Feedback and goal clarification, centering attention
"Working with someone at the same level"	Balance between challenge and skills
"Competition"	Centering attention, concentration

Table 5. Viewing student's collaboration experience through the lens of flow theory.

In effect, the effects of collaborations make themselves seen in all the antecedents of flow, thus raising the possibility that, at least for students, the Quinn model of flow presented in chapter 2 could be modified to reflect the findings that this study has brought to light:

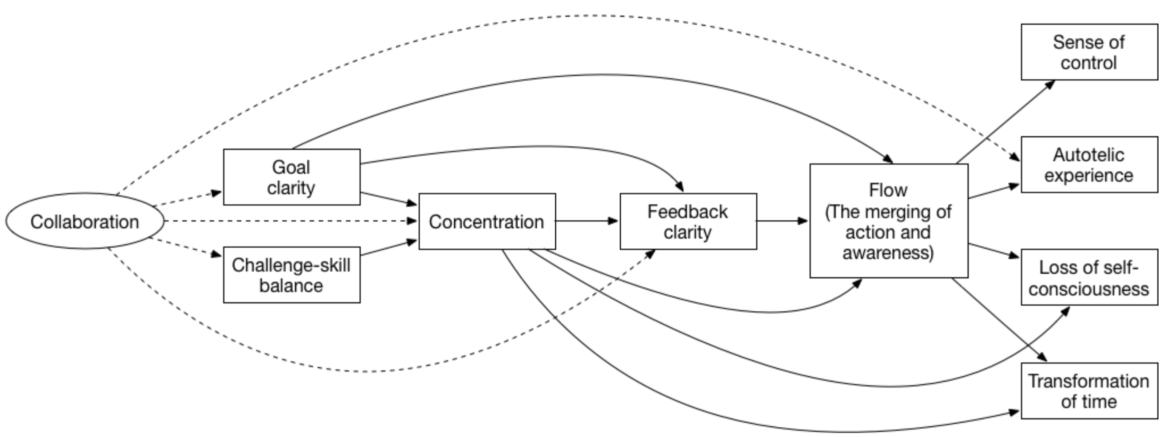


Figure 10. The Quinn model, modified: the effects of collaboration on flow.

It may be instructive to examine the students' experiences via the findings on the social facilitation effect. Indeed, working with others has been shown to increase the arousal level (Zajonc, 1965), however, when the task is new or more complicated, performance suffers (Strauss, 2002). This seems to be the case for the pairing of April and Lucy, whose experience was definitely not flow – possibly because of a combination of two factors. Firstly, while both students were generally persistent and industrious, Lucy, being more methodical and confident than April, may have been perceived as having a "higher academic status" (Cohen, 1994). Secondly, they also had to contend with a challenging task (the problem of the four 4s): both students tried to articulate that there was, in their view, a problem with it, April mistakenly believing that it was not "visual" enough, and Lucy stating that it was "hard to grasp". Thus, despite my expectations, the differences between April and Lucy were greater than their similarities, and Lucy did not take the lead in the collaboration. As Cohen remarks:

> If the task is very challenging and ambiguous and has an ill-structured solution, and if a heterogeneous pair is left alone to agree on an answer, then the confidence of the more developmentally advanced can be shaken. (ibid., p. 12)

While most groups formed in the class were randomly decided, this was not the case for Bianca and Nadyia. They were both late for class that day, and they chose to sit together, rather than join their allotted groups. While initially I had some misgivings about that, they had such a good working experience together that it made me reconsider my original inflexible position about upholding random grouping at all times. This was not the only non-conforming aspect of Bianca and Nadyia's experience, as they are also the students who preferred to repeat the same task level, rather than increase the difficulty. We may hypothesize that since their confidence in their knowledge and skills was low, only a minimal risk environment – both in terms of work partner and task – would be deemed as safe to experiment in.

The case of James and Marie is also interesting, insofar as it sheds some light on a self-declared instance of competition. Although cooperative environments have consistently been shown to be more effective than competitive ones in building academic and social competence (Roseth, Johnson & Johnson, 2008), in some instances, competition can galvanize the flow experience. Csíkszentmihályi (1990) states that competition is a reliable way to increase complexity, however, with a caveat:

> The challenges of competition can be stimulating and enjoyable. But when beating the opponent takes precedence in the mind over performing as well as possible, enjoyment tends to disappear. Competition is only enjoyable when it is a means to perfect one's skills; when it becomes an end to itself, it ceases to be fun. (p. 60)

If James and Marie competed, judging by their enjoyment, their competition was clearly a means to an end. Not only did they assist each other several times, but out of their interaction was born the "sossometer", which indicates that they saw each other as partners in solving the challenge, rather than adversaries. Furthermore, James and Marie were in complete control of what to do and whether or not to engage in competition. Ultimately, it may be that it is not the competition itself that is detrimental, but rather its obligatory nature.

All the successful groups interviewed were characterized by a high degree of positive interdependence. Positive interdependence implies that the members of the group:

> […] perceive that they are linked with groupmates in a way that makes it impossible for anyone to succeed unless the entire group succeeds (and vice versa) and that they must coordinate their efforts with their groupmates to complete a task. (Johnson, Johnson & Holubec, 1994, p. 27)

Furthermore, no member of the pairs in flow separated to pursue his or her own ideas, nor were there instances of "freeloading" or "showing off". Worth noting are also the instances of convivial conflict, such as the exchanges between Glen and Linus (what operations would be most advantageous), or Karl and Brandon (disagreements about how to model the problem). Johnson and Johnson (1979) have shown that controversy in a cooperative environment stimulates higher-level reasoning, curiosity, and creativity, all of which fuel the growth of complexity associated with flow.

It follows that giving students plentiful opportunities to work productively in togetherness and camaraderie will increase the likelihood that each one will be experiencing flow. One puzzling aspect remains yet unexplored: is the flow of the group merely the sum of the individual flow experiences? After researching the flow of groups of jazz musicians and improvisation actors, Sawyer (2006) argues that the flow of the group is a different entity than the added flow of the group members. In his view, the added element emerges from the dynamic of the members, the unfolding dialogue which he termed “interactional synchrony” (p. 157), and which refers to the fact that performers must constantly attend to each other so that they can offer a timely and appropriate response. Berliner (1994) quotes musician Franklin Gordon who described the experience of group flow thus:

> Every jazz musician wants to be locked in that groove where you can’t escape the tempo. You’re locked in so comfortably that there’s no way you can break outside of it, and everyone is locked in there together. It doesn’t happen to groups every single night…these are the magical moments. (p. 388)

Admittedly, a mathematics class does not resemble playing on a stage; nevertheless, there is an element of uncertainty and improvisation present in mathematics which makes a comparison quite apt. In all the groups that were in flow, the collaboration went beyond the realm of the intellect, as the students appeared to share a state of emotional arousal as well. Perhaps a fitting term would be that of “collective effervescence”, introduced by Durkheim (1965) to denote the ineffable energy that seems to course through a community undergoing an intense, electrifying experience. In my research, the students appeared to be closely attuned to their partner, finishing each other’s sentences (Glen and Linus, Chloe and Tamara), completing each other’s ideas (Bianca and Nadyia, Karl and Brandon), writing over each other’s solutions (Karl and Brandon, Glen and Linus, James and Marie), always responding in a way that built on and enhanced the work of the other. The collective effervescence observed is, in my opinion, the ingredient that distinguishes group flow from the flow of the individual. Unfortunately, Sawyer rightly cautions us that “group flow”, just like individual flow, cannot be predicted, due to “intangible factors” (p. 158) pertaining not only to the *chemistry* between individuals, but also, in the case of teenagers, on their levels of energy, their mood, their perception of their school experience at a particular time, and even the day of the week. As many things that have to do with flow, any prescription is difficult to make, due to the inherent fluctuations in the life of an adolescent.

There is much to be said for the benefits of collaboration in the classroom, and much of that is acknowledged by Csíkszentmihályi himself. Why then does flow theory itself not include any mention of its importance, and why does it position itself at times in a kind of aloof, icy social austerity? My experience during this research suggests that it would impossible to achieve flow experiences without relying not only on the foundational role of peer feedback, but also, and, in my view, more importantly, on the motivational and ludic ingredients of that support. Csíkszentmihályi would probably disagree with this view – he relegates "small talk" and "joking around" in the domain of pseudopeak experience (1975a). Donning the cloak of speculation, I offer a few reasons for Csíkszentmihályi's apparent omission: it is possible that he felt that since there exists a small minority of people who can find flow solely through their own powers of concentration and imagination, flow can be explained without the need for introducing the social aspect into the theory. Equally, he may have felt that "small talk" takes away from the intense absorption in the task that he saw in his first subjects. Regardless, a possible way to square this perplexing circle would be for flow researchers to re-consider the flow benefits of social interactions, even when they do not lead to immediate "growth and complexity". What for Csíkszentmihályi is *pseudopeak*, for others is healthy *social snacking* (Gardner, Pickett & Knowles, 2005) – which is, it must be admitted, a much more appealing choice from the menu of life.

6.3. Idiosyncratic manifestations of flow

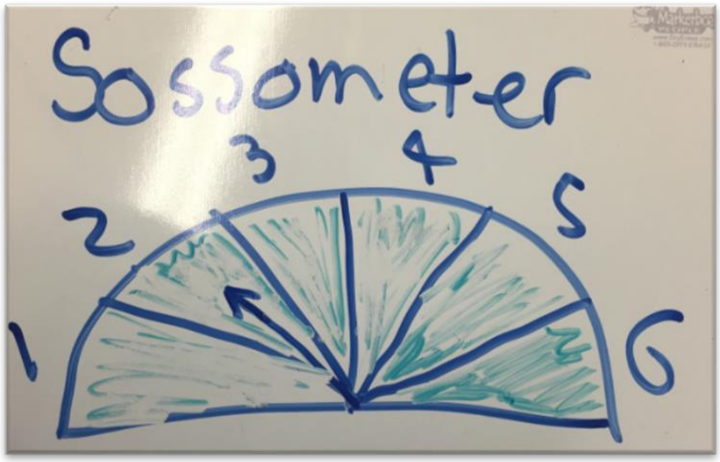
An interesting and unusual aspect I remarked in all the instances in which students reported experiencing the characteristics of flow was the array of physical actions that accompanied their mathematical explorations. This included not only standing, walking, jumping and high-fiving, but other gestures of a physical nature which I found arresting: Lonnie and Nikolaus' large, expansive gestures and sound effects miming of their amazement and head "explosions", Chloe and Glen running towards the board to place their solution lest others get ahead of them, Linus grabbing the pen from Glen and pushing him to fix a mistake at the board, and Karl and Brandon's moments of absolute stillness while observing the Fibonacci sequence, followed by their linking arms back to back and lifting one another upon completion of their problem. Equally, James' sossometer, which I analysed above as a method of communication with me, in order to solicit increased difficulty, may also be viewed as a way to bring outwards an inner reality, and thus, a manifestation of embodied flow just as much as “head explosions”.

I hypothesize that these manifestations would not have happened so spontaneously, had the environment of the classroom been different. The fact the students had freedom of movement, freedom to occupy, unconstrained, the space of the classroom (and were not, as in Blake’s poem cited in subsection 2.4.2., *drooping sat*), encouraged these playful exchanges. Under the influence of flow, particularly the loss of self-consciousness component, and inhabiting an action-friendly space, the students were *taken at the flood*, their emotions channeled into a synesthetic surge during which body expression became free, spontaneous, and, I often felt, as eloquent as a well-written essay.

6.4. Self-induced flow – a puzzle of one’s own

In the course of the research, an interesting phenomenon was observed: students deliberately seeking or creating for themselves a state of flow. I had this revelation when, one day, during the course of regular classroom practice, James requested more challenging examples. The word he used, however, was “saucy”, which, getting into the spirit of things, I wrote on the board as “sossy”. Then, he and Marie drew a “sossometer” (see Figure 10), which looked like a semicircular dial gauge divided into six sectors, and then directed me to push the difficulty of the examples higher and higher on the “sossometer scale”.

Figure 11. The original "sossometer"



At first sight, the behaviour of these students may appear paradoxical. Students do not usually request more difficulty; indeed, it is more common to hear them complain that "math is too hard". However, seen through the lens of optimal experience, these students are in fact acting in accordance with their desire to experience happiness and growth, quest which:

> requires that one be able to find increasingly complex opportunities for action and that one be able to improve one's appropriate skills. (Csíkszentmihályi, 2014b, p. 163).

A student who enjoys learning something new and feels interest knows from past experience of learning that the only way to sustain those pleasant feelings is to proceed to learn something harder, otherwise they will quickly lapse into boredom. There is only one way forward, and that way is up the flow channel. This constant ascent is necessary because, as Csíkszentmihályi puts it, "*the positions at the lower end of the flow diagonal are inherently unstable"* (ibid.), precisely because of this propensity of the mind to feel bored once the challenge disappears.

The fact that the students' pursuit of "sossiness" was not an arbitrary, one-time event is demonstrated by the popularity of the "sossometer" thereafter. James and Marie's idea to visually represent the difficulty level of a problem has remained in use ever since, and it is not the only way in which students have requested an increase in difficulty to match their improved skills. Students have also drawn jars of Ragu Sauce on their whiteboards, sketched monsters (where the monster was a metaphor for a more challenging problem or exercise), or simply shouted “Monstre, s’il vous plaît!” (“Monster, if you please”).

In these examples, the students, although willingly looking for an adjustment of the challenge level, still relied on me to make that adjustment. In other cases, however, they were the ones controlling “the levers” and determining the appropriate intensity of the challenge. To wit, Chloe and Tamara, working on the problem of the four digits, reached a point where they were so confident about their abilities in wielding the new tools, that they afforded themselves the pleasure of mathematical detours, such as using only certain operations or certain combinations of operations. Chloe explicitly stated that she “likes harder things”, because they “make her try harder”, and that she took it all “as a challenge to complete”.

Similarly, Glen and Linus expressed their interest in “discovering” and “learning something new”. They too put themselves into a state of flow, not only by increasing the level of the challenge, but also by paying attention to their classmates’ solutions, both for the purpose of acquiring feedback and to add a layer of competition to the challenge. In the case of Nikolaus and Lonnie, they were comfortably in flow without my feedback, and did not lose their confidence even when their solution was somewhat faltering. They refused hints because, in their words, “it’s not fun if it’s too easy”. They too welcomed the difficulty of the problem with gusto and willingly embraced the opportunity to grow.

Even reluctant mathematicians like Nadyia and Bianca found flow in decrypting the secret message. They were able to discard many of their incorrect assumptions by paying close attention to the text and its mechanics, and enjoyed the task so much that they asked for another one, similar to the first. They enjoyed themselves although, Nadyia’s words, “it [the experience] shouldn’t have worked, because it was Monday morning”. I found it extremely interesting that they asked for another one “like this one”. They were aware that a more challenging offer was available; they glanced at it, noticed that “it looked harder” and preferred to remain at the level they were at before. This, in contrast with other groups in their class, who grabbed the new encrypted text from my hands without a moment's hesitation.

In the past, I might have insisted and encouraged Nadyia and Bianca to push further, but in light of flow theory, I found this move to be not only unnecessary, but downright counter-productive. Had they been coaxed into trying the more challenging task, they would have immediately landed into the territory of anxiety and stress. Explorations of student engagement have uncovered the fact that not all students need ever higher challenges to be in flow (Salmela-Aro, Moeller, Schneider, Spicer & Lavonen, 2016). In fact, some need "protection against overly challenging demands" (ibid., p. 68). The contrast between Nadyia and Bianca and the other, more adventurous groups, brings into focus the importance of trusting students' decisions when determining the accurate point of balance between skills and challenge. Teachers trying to steer the students away from the dangers of boredom and repetition may inadvertently lead them on the doorstep of fear, frustration, and burnout.

The experiences of my students bring into focus the capacity for flow as a "meta-skill" (Csíkszentmihályi, 2014a). Csíkszentmihályi posits that since humans have an impulse to seek to repeat pleasant experiences, and flow is one such an experience, humans will be motivated to learn how to recreate it through a process of "teleonomy of the self" – that is, the tendency to seek goals through which a person feels fulfilled (Csíkszentmihályi & Csíkszentmihályi, p. 24). I have also observed this phenomenon in my research, which leads me to propose the existence of a *"flow mindset"*, which I define as the mindset of students who, once they have already had several successful flow experiences, become adept at extracting information from their own states of boredom or anxiety, and are eager to use that information to re-adjust the balance between skills and challenge, in order to tune in to the frequency of their own flow.

For many students in my classes, the first flow experience is similar to that of James and Marie: practicing exercises of increasing difficulty on the individual whiteboard. For this reason, I consider James and Marie exponents of what I will call "early flow". "Early flow" is the state of flow observed in a student who makes his or her first efforts to seek greater complexity. Those efforts are supported by the teacher, who mediates and meticulously regulates the intensity of the challenge. In the other cases discussed, I discern a slightly different experience, one which I will call “mature flow” – the state of flow observed in students who are more experienced in effecting a journey upwards the flow channel and are able to act in order to set goals, give feedback, and increase the challenge by themselves, for themselves. Due to the level of sophistication required to perform these elaborate and refined tasks, I posit that a student must have solid experience with “early flow” before being able to recreate the experience for themselves and enter a state of “mature flow”. Thus, the transition from “early” to “mature” flow is possible due to the emergence of the *flow mindset.*

6.5. What makes a good flow problem – knotting and unknotting

Csíkszentmihályi observes that “any subject can be taught enjoyably if the teacher understands the principles of flow” (1982a, p. 25), and, indeed, students can be led into a state of flow by goal clarification, feedback, skill development, and challenge, appropriately modulated. The experience of James and Marie, who were working on factorization of trinomials, is an example of such a situation. There is no doubt that from a mathematical point of view, James and Marie were engaged in what might be called a *trivial pursuit*, as are many in our lives. This is not to diminish the importance of the études, scales, and arpeggios of mathematics – if structured with the goal of flow in mind, they can, and do, present opportunities for optimal experiences. Based on my experience after having experimented with various ways of engendering flow in students, I put forward the following observations regarding the use of skill-based questions:

a. They are relatively easy to produce for a teacher;

b. Flow-wise, they are easy to present in an ascending order of difficulty, easy to give feedback on, and the need for goal clarification is minimal;

c. The risk to students is manageable: small, but not zero; it could be said they are having an adventure “encased in safety” (Liljedahl, personal communication). Csíkszentmihályi also remarked on this ambiguous relationship with risk when he interviewed surgeons, and distinguished between “very routine” operations, which left surgeons feeling bored, and merely “routine” cases, which surgeons saw as “relaxing and satisfying” (1975a, p. 127). He made the observation that there is an “enjoyment of the craft” (ibid., p. 128), which makes surgeons perceive these interventions as pleasant, rather than mundane, because they can do other flow-engendering activities, such as teaching, while they operate. This is evident in James and Marie as well: their increasing skills led to confidence and *enjoyment of the craft* of factorization, and enabled them to enjoy a range of diverse activities unrelated to factorization, such as looking for further challenges and competing;

d. The stable, predictable progression of skill development can entice more reluctant students to engage with mathematics. Thus, it is to be hoped that they too can learn to re-engineer their inner state in order to experience the merging of action and awareness which is the nucleus of the flow state, and use this new ability in order to enjoy riskier mathematical adventures (which was the case of reluctant mathematicians like Bianca and Nadyia);

e. On the other hand, pleasant puzzling over a skill can quickly degenerate into meaningless repetition. The teacher has to be acutely aware of the state of the students at all times. Moreover, students master skills at various rates, and while some students may be still in flow, others may be already in toleration for the mundane (see section 6.1), or even boredom;

f. In my view, limiting flow experiences to skill-based questions carries a certain whiff of waste: the flow experience can lead to so much more. Mastering a mathematical skill, while perfectly necessary and useful, is but an exiguous approximation of the pleasures of authentic mathematical problem solving, with its exciting twists and turns.

Csíkszentmihályi recognized that activities which engender autotelic experiences must have certain characteristics in order to “…give participants a sense of discovery, exploration, problem solution—in other words, a feeling of novelty and challenge” (1975a, p. 30). His contention is that these ingredients are relevant to all flow experiences, but it is particularly in school mathematics that I find them poignant. It is a sad reality that students often view of mathematics as a body of knowledge which follows a rigid path, from a clear set of departure parameters to an inexorable solution, which is pre-determined by the chapter studied at the time in class. It follows that short-circuiting this expectation will perplex students, which will engender dissonance, and, at best, will generate wonder and a call to self-transcendence. The dimensions of discovery and exploration are particularly relevant to me, as they underpin not only the growth, complexification and evolution that are at the core of the flow experience (Csíkszentmihályi 1994), but also the very development of the science (art?) of mathematics.

In line with Csíkszentmihályi's remark, that "*no activity can sustain [flow] for long unless both the challenges and skills become more complex"* (1988, p. 30), Williams (2000) developed the concept of "discovered complexity", defined as:

> […] a complexity that becomes apparent during task completion and requires each member of the collaborative group to work with unfamiliar mathematical ideas to understand the complexity discovered…optimal learning conditions would exist each time a complexity was discovered. (p. 55)

Problems that contain discovered complexities are solved through a process of unravelling: every twist and turn of the solution begets feedback, and feedback begets more solutions, often unexpected. Most students in my study experienced discovered complexities, and, exactly as predicted by Williams, they were the engine that propelled them forward and up the flow channel.

Table 6. Discovered complexities in the experience of each group.

Students	Discovered complexities
Linus and Glen	The new operations (summation, gamma function, double factorial); old operations with new possibilities: decimal point, factorial, percent.
Lonnie and Nikolaus	The necessity to abandon decimals and switch to fractions; the representations of the sum.
Karl and Brandon	The new representation of the jumps on the staircase.
Chloe and Tamara	New operations and old operations with new possibilities (repeating decimals), combining old and new (summation to a square root).
Nadyia and Bianca	Making a series of incorrect guesses, followed by realizations that they lead to absurd text; backtracking and offering correct guesses.

It is also worth noting that the demands of the tasks placed the students in situations where they experienced an unexpected rupture in the fabric of their knowledge, a gap between their expectations and the mathematical evidence. Forman and Pufall (1998) call this process *epistemic conflict,* and contend that it forms the cornerstone of constructivist learning: it is when one's old ideas are challenged by reality, that new knowledge has to be constructed. From a flow perspective, the epistemic conflict is often what generates the wonder and exploration that Csíkszentmihályi talks about[4].

[4] Interestingly, for Dewey, the interplay between the new and the old is more about pleasure rather than conflict:

> Neither the absolutely customary, nor the entirely novel, attracts the mind; it is the old amid the new, the novel in the wonted that appeal. (1967, p.113)

Table 7. Epistemic conflict (surprise) in the experience of each group.

Students	Epistemic conflict (surprise)
Linus and Glen	It was possible to make all the numbers with just 4s, operations existed which they didn't know
Lonnie and Nikolaus	The sum was not infinite
Karl and Brandon	There was a Fibonacci sequence hidden in the problem with the staircase, and the growth was not linear
Chloe and Tamara	It was possible to make all the numbers using just the digits 1, 4, 7, 9
Nadyia and Bianca	It was possible to enjoy a Monday morning of mathematics; the pleasure of guessing correctly is worth the effort of going back to check incorrect assumptions

In comparison, James and Marie did not appear to have experienced any conflict. Their flow experience was more predictable, which is to be expected, considering that I set the level of the difficulty, and I purposefully engineered their climb to be smoother and more linear. Flow-wise, both experiences take the students through the flow channel. From a mathematical point of view, however, as mentioned above, there is something frustratingly incomplete about James and Marie's experience: they are missing out on the pleasures of the intellectual struggle. Seen through this perspective, their spontaneous competition may be a reflection of this need for conflict: there was no *wrestling with the problem*, hence the need to wrestle with each other.

However, there is another possible interpretation of their lack of epistemic conflict: since James and Marie were just beginning to learn about the topic, there were no pre-existing assumptions to be disproved. Naturally, in order for epistemic conflict to occur, some *episteme* has to be in place already. For instance, Karl and Brandon could not have recognized the Fibonacci sequence, nor could they have contrasted it with a linear growth, had they not known about these two concepts from other problems. The instances epistemic conflict can therefore be seen as moments of creativity, in which students leave old ideas behind and adopt new ones. As Robert Sternberg (1990) remarks:

> The wise person seeks to understand the meaning and limitations of this [existing] knowledge. The intelligent person seeks to make optimal use of this knowledge. The creative person, though, wishes to be freed from this knowledge. (p. 153)

Together with the discoverable complexities and the opportunities for epistemic conflict, and closely connected with them, an essential ingredient that kept the students in a state that Hobbes calls "perseverance of delight" was curiosity. Kashdan et al. (2004) see curiosity as "proactive, intentional behaviors in response to stimuli and activity with the following properties: novelty, complexity, uncertainty, and conflict" (p. 291). The successful tasks in this study gave rise to *a lust of the mind* (again, in Hobbes' words): in the first place, it propelled the students to enter the rabbit hole, and afterward it compelled them to stay the course until a satisfactory resolution was found. For many of the students in the research, what made them curious was difficult to express. Nadyia and Bianca articulated most clearly that they were curious to know "what the [encrypted] message was all about". For the other students however, the course of curiosity was more a succession of peaks and valleys, in which new interesting questions were continually emerging as points of interest, mirroring the complexities discovered.

Alas, the underlying question: what mathematics makes people curious? is more difficult to answer. Some students are curious about everything and anything, others only about specific subjects, and a small minority have had curiosity almost completely beaten out of them. The common advice given to teachers: find what the students are interested in, and enhance that, is worth following. One may however object that many students may not be curious about numbers and sums and other mathematical abstractions. While true, it is however an obstacle that can be overcome: Susan Engel, in her book "The Hungry Mind" compares curiosity with "a delicate plant…in order to flourish, it needs to be cultivated" (p. 193). Interestingly, Engel's list of strategies and circumstances that cultivate curiosity has numerous commonalities with the strategies discussed for inducing flow: a teacher who values curiosity, a classroom dynamic where learners have choice and autonomy, an ethos which, rather than being content with curiosity emerging serendipitously – and infrequently – keeps a deliberate and enduring focus on curiosity.

Alongside with the need for discoverable complexities and the need for conflict, in my view, good problems for occasioning flow have a flexible architecture with multiple entrance points, multiple solutions, multiple approaches. Thus, most students find a way to enter flow wherever their skills were at the moment; once in flow, due to the discoverable complexities, they will find opportunities to increase the challenge as required, alternating between provocation and satisfaction, to create what Glen calls "an enjoyable experience". All the non-curricular problems used in this research had the spongy structure which entices students to wander and explore. In this respect, too, curriculum-centered tasks, of the kind that James and Marie worked on, fall short. Often, curricular-based questions, even when well-planned, of an appropriate challenge, and engineered to enhance flow, offer less flexibility: one solution, not much diversity of approach, and not a lot of suspense, unless it emerges from social interactions.

Thus, due to its importance in the emergence of the flow experience in the classroom, the task could be incorporated in a second, and final, modification of the Quinn model, thus:

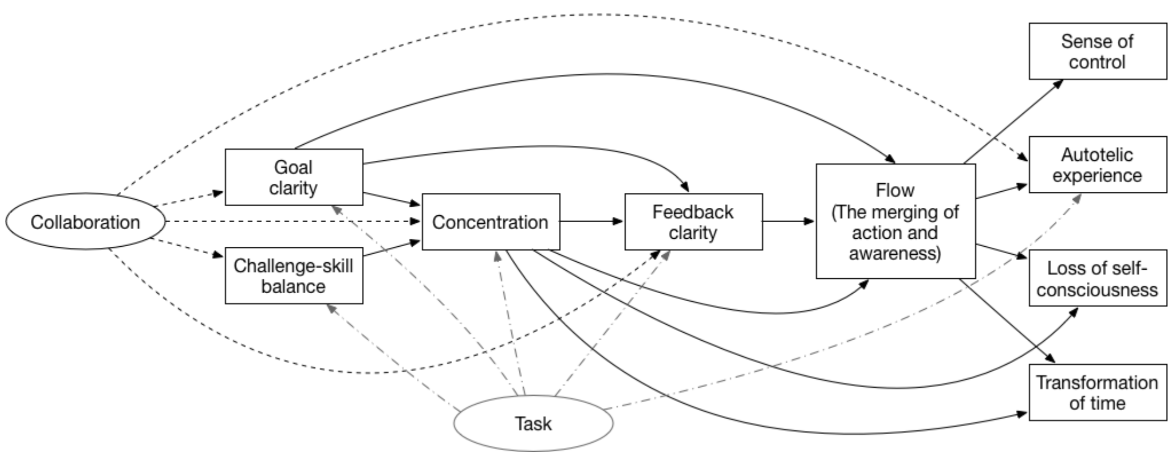


Figure 12. The Quinn model, modified: the importance of the task.

Finally, while I did not proceed with the declared goal of looking for aesthetically pleasing problems, most of them did satisfy *my own criteria* of – dare I say it – mathematical beauty. True, when deciding whether to use a problem, a matter of first importance on my mind was its "flowability". However, beyond my conscious awareness, *behind my mind*, were primordial judgements about aesthetic value, which are now so ingrained so as to be invisible to me. Out of the tasks used in this study, the only one I personally consider utterly lacking in aesthetic value was factoring trinomials. I chose it solely for the challenge that it posed to me – can I make something out of a topic which I see as the very embodiment of mathematical ugliness? (It turns out that I could[5], and the implications of the power that flow could confer to a consummate practitioner will be discussed in section 7.2). A valid question would be: what makes a beautiful problem? A comprehensive list of attributes can be found in Sinclair (2011), or in Sinclair, Pimm, and Higginson (2006; 2007), and there is substantial overlapping between these attributes and the characteristics of a "flow-friendly" problem discussed above: surprise, curiosity, seeing the old encapsulated in the new, finding that a succession of answers unlocks a succession of new challenges, fitting together pieces of a puzzle, etc. But just like with flow, not everybody sees beauty in the same way, or is attracted to the same things: April and Lucy's comment that "they didn't see the point [in the problem]" reveals that the problem of the four 4s, which proved to be such a rich playground for Glen and Linus, failed to excite their aesthetic sensitivities.

6.6. Textbooks – a convenient scapegoat

A surprising theme that emerged from discussions with the students was not necessarily that they disliked working from the textbook, but rather how great their dislike was. Time and time again, students mentioned as one of the factors in their enjoyment of an activity the fact that "it wasn't from the textbook". A clear disinclination to work from the textbook was exposed in the questionnaires, in the interviews, and even in their reactions in the classroom. Indeed, my experience has been that whenever I distribute practice sheets on any topic what I get from the students is a congenial "merci, Madame", and whenever I utter the words "the practice problems are in the textbook", I get groans and protestations. Chloe and Tamara, for instance, comment on the fact that they check the time "when they're working from the book", because the book is just "reviewing the same thing". James, Marie, Nadyia, and Bianca are much blunter about it: they all regard

[5] However, the *beautiful* problems certainly did not require as much effort on my part to elicit flow.

the exercises in the textbook as repetitive, and admit to being bored when working from it, although accept it as another unpleasant fact of school life. All these students understand the importance of practice in mathematics, so their opposition was not to *work*, but specifically to *work from the textbook.*

Taking into consideration how consistently the students associated the noun “textbook” with the adjective “boring” any time their opinion on the matter was sought, I thought it worthwhile to explore this paradox. It is a paradox because schooling and education in general (and mathematics education in particular) depend on textbooks the way a carpenter depends on a hammer: it is a sine qua non, a tool seen so important that society makes a special effort to provide it for free to students, and whose absence is oft lamented by many teachers, parents and students as a significant impediment to learning. However, when looked at in detail, students regard textbook use, at best, as a necessary evil, and, at worst, as an instrument for inflicting boredom and disengagement.

I note here that my students’ attitude towards textbooks is not exceptional. Textbooks often contain “formidable” text which students have difficulty in understanding and responding to (Guthrie & Davis, 2003) and minimize student choice (Mac Iver, Young & Washburn, 2002). On the first count, most of the mathematics textbooks I use are clearly guilty. My students’ French is seldom as good as the authors think, and the rich expository writing in the word problems is overwhelming for many of them. On the second count, one also has to accept that textbooks are by their very nature temperate documents, purposefully written to be useful to the largest number of people possible. I, for one, cannot fault them for not being mathematically exciting enough.

There is however another aspect of textbooks that arises from my research: although textbooks offer many of the right flow ingredients: feedback, clear goals and opportunity for finding the balance between the skills and the challenge, they are almost flow-proof. Indeed, their feedback is at the end of the book, and consists of answers and, quite often, some explanations. In addition to this, textbooks make the goals of practice very clear: mastery of a certain piece of mathematics. Lastly, all textbooks make an effort to increase the complexity of the exercises they propose, and to separate them by difficulty.

Why then are students so dismissive of their mathematics textbook? The answer to that question is limpidly captured by the students' very words: the tedium of routine and repetition they describe is characteristic of the lower part of the flow graph, where the skills are higher than the difficulty of the task. In other words, the students interviewed perceived textbooks as not enough of a challenge, monotonous and uninteresting. Their perception was unshakeable, even when they were presented with interesting information, novel mathematical facts and challenging mathematical tasks contained in the textbook they so disdained. From informal conversations with students, the negativity shown towards textbooks is not reserved for mathematics only. For some students, their relationship with textbooks – of any subject – has been so damaged by years of educational ennui that even a brilliant book with captivating content may not spark but the merest flicker of interest. It could also be that due to the rise of "edutainment", the gap between what the students "consume" in their life outside school, and what they have to learn in school is ever-widening.

Hence, a better option for mathematics might be an electronic resource, consisting of videos, simulations, digital collections of interesting problems, and in-depth information on the connections between mathematics and real life, in addition to the necessary practice and the explanations. Such a resource could be a more flexible tool for teaching, better adapted to the way students learn today. Last but not least, such a resource would also solve another issue associated with typical textbooks, i.e. the lack of upformatory feedback for the students. Once a student has gained new skills or knowledge from the textbook, who or what alerts the student to move up, to a new challenge? Students do not always act in their best interest, and they are much more likely to stay with the easy problems for too long, rather than move to the more challenging problems too early. The ideal electronic resource described above would automatically increase the difficulty of the problem when appropriate, without the students having to make a conscious decision, thus avoiding both the anxiety caused by being faced by a task too difficult, and the negative emotions engendered by redundant repetitions.

6.7. Boredom and antiflow

Pursuing the discussion about boredom, it needs to be pointed out that the boredom the students talk about is not at all the tormented-yet-creative kind of boredom of the Proustian variety. In my research, most students have singled out the feeling that much of school work is meaningless as a main contributor to disengagement. This sentiment was best captured by Glen, who, when asked details about his enjoyment of school, stated that:

> I'm bored about 80% of the time… I don't know why I have to learn this stuff…I mean, who picked it and said 'you have to learn this'? […] Every class it's the same thing, take notes… yeah, and only about 20% of it is actually interesting…

This distinguishing aspect of the boredom observed in school is better captured in the quasi-equivalent term of "antiflow", introduced by Allison & Carlisle Duncan (1988, p. 118). Antiflow is defined as "the antithesis of flow", and it is the result of having to perform a tedious, irrelevant activity, which is not perceived as leading to personal growth. Thus, the term antiflow combines the traditional meaning of boredom – a state in which opportunities for action are limited, and the challenge is inferior to the skills – with the alienation and disenchantment that occur when the actions available are not congruent with what the person wants to do. In this situation, the *merging of action and awareness* does not occur, leading to a state of antiflow.

Naturally, we cannot know whether Glen's reports of such a severe case antiflow are justified or not. However, in my research, no student has ever stated that they feel school work habitually makes them anxious, or that they feel confused for any length of time, let alone 80%. In an informal setting, during class, they might have complained that having a project due was causing them some stress. Nevertheless, when asked to give a general description of school experience, all mentioned boredom as its enduring theme, rather than anxiety. This opens the possibility that the students do not perceive anxiety and antiflow in the same way: anxiety is a fleeting, short-lived experience, whereas antiflow is a "sticky", persistent state, which seems to take much more space in their consciousness than anxiety.

This is an important observation, because a first glance at the flow diagram may lead to the impression that anxiety and boredom, the two negative states that envelop the flow channel, are equally detrimental to learning. However, that is not so: of these two negatives, boredom emerges as the negativest. Research carried out by Baker, D'Mello, Rodrigo and Graesser (2010) suggests that antiflow and low arousal are longer lasting emotions, more difficult to transform into a pleasant, productive state. In contrast, confusion and anxiety are easier to overcome and shift into flow – at least, there is a localized challenge present, waiting to be resolved. Moreover, the experience of antiflow, characterized by repetitiveness, lack of challenge and of meaning, tends to seep into other areas of life (Allison & Carlisle Duncan, 1988), which may explain why students are so unanimous in describing school as mostly boring, rather than mostly anxiety-inducing. Upon further exploration of the dissimilarity between the areas surrounding the flow channel, perhaps a recalibration of the flow diagram could be effected to highlight this asymmetry.

6.8. Teachers in flow, students in flow?

The question of whether there was any correlation between my flow as a teacher and the flow of the students proved to be very vexatious. It was not something that I intended to examine, until one day when I had to teach a class in another room where I had neither the vertical nor the individual whiteboards. Suddenly, I felt like a bat whose echolocation system was off-line. I realized then that without the feedback I receive from the students, feedback coming not only from their solutions, but also from their discussions, and even their posture, I was floundering in the anxiety zone of the flow graph. Try as one might, without feedback, it was impossible to attend to students' flow components in an efficient manner. Based on this experience, it occurred to me to wonder whether there might be a connection between my flow and the students' flow.

In the case of feedback, it is entirely uncontroversial to assert that any productive exchange between human beings relies on each attending to the state of the other, and that the company of enthusiastic people who love what they're doing is extremely pleasant. Equally uncontroversial are the findings that enjoyment of teaching is more likely to beget enjoyment of learning (Bakker, 2005; Patrick, Hisley & Kempler, 2000), via a process of "emotional transmission", mediated by teacher enthusiasm (Frenzel, Goetz, Lüdtke, Pekrun & Sutton, 2009). Csíkszentmihályi holds such enthusiastic teachers in high regard, as "holy fools", whose sacerdotal duty is to uphold the worthiness of knowledge for knowledge's sake (1982). He notes however that even an enthusiastic teacher in flow may not be able to move students who are not good at mathematics into the flow channel, especially after fourth grade (Csíkszentmihályi, 1997c).

Keeping a journal of my flow and comparing with the tracking notes of flow in students, I am able to report with certainty that there is one activity during which students were not in flow, although I was: lecturing. This was a pivotal discovery: once I noticed that, I was obliged to minimize the time spent lecturing to mere minutes, if at all. This was a happy decision, very well received by the students. Not surprisingly, students rate listening to teachers talk as a low challenge, passive, tedious activity (Freeman, McPhail & Berndt, 2002; Guthrie & Davis, 2003; Hickey, 1997; Mac Iver et al., 2002). At other times, however, both my students and I were in flow at the same time, for instance when students were practicing on the individual whiteboards. My challenge was to find appropriate practice questions, and navigate the delicate frontier between escalation of difficulty and necessary repetition, while at the same time giving informatory feedback to every member of the class. Their challenge was to learn something new, act on my feedback, and then give me salient feedback in response. During these intervals, I felt I was weaving threads into a fabric with a certain pattern, and my mind was like a flying machine, zooming a path through the students, unknotting, looping, twisting, adding substance to the fabric, until everybody felt they had an intimate knowledge of its texture. Based on students' responses to questionnaires, they too felt focused and productive, consistently lost track of time, and considered working the whiteboards as the most enjoyable way to practice.

Life in the mathematics classroom is not only practice – and this is where the issue of the relationship between the flow of the teacher and the flow of the students becomes more perplexing. There were times when the students were problem solving so well, and the groups worked so harmoniously, that there was little for me to do, and I would find myself falling into boredom. I would walk among the students, hoping for a fragment of student disquiet to come my way, to relieve me of my misery. I often felt the need to look at the clock, fighting the perverse desire to interrupt the flow of the class. I would watch the students' excitement and absolute involvement with pride mixed with a small, entirely non-commendable dose of annoyance that I was not part of the game, and I was annoyed at myself for being annoyed.

As soon as I would become aware of my inner disequilibrium, I gave myself a mental challenge to force my consciousness into flow again – a gambit which Csíkszentmihályi called "snatching flow from the jaws of boredom" (Csíkszentmihályi & Csíkszentmihályi, 1988). A favourite challenge of mine was to force time to slow down, through the act of noticing in silence. There were also times when nobody was in flow: the feedback loop was fractured, the challenge was too high, my words didn't come out right. I would then become keenly aware of the passing of time, and both the students and I felt frustrated. Just like in happier times enjoyment enveloped the class, the anxiety of those moments was palpable, too. I made adjustments and sometimes flow was re-established. At other times, it was just a learning experience, and I would try again the following class.

It would have made me happy to be able to report a relationship between the flow of the teacher and that of the students. Unsurprisingly, however, discrepant experiences often occurred during my research. In confirmation, research conducted by Culbertson et al. (2015) has found that although their data supports the hypothesis that there is flow contagion amongst students, it does not support the hypothesis that there is an association between student flow and teacher flow. The authors do not offer a possible explanation, however. To explore this issue, it is worth looking beyond the triad of clear goals, feedback, and balance skills-challenge – which could indeed be occurring at the same time both for students and for the teacher. In my view, the answer is rather in two of the other components of flow, i.e. *loss of self-consciousness* and *control*. Firstly, a teacher can never fully "forget themselves", or allow themselves to cruise throughout the day. A teacher's attention is always needed at the helm: to deal with innumerable, unwanted distractions from learning which may (do) appear, and to monitor closely the state of students to ensure they learn while they stay in flow. Secondly, control in the classroom operates like a seesaw: the more the teacher holds it, the less is there for students, and vice versa. Happy moments, in which the two sides are optimally – and improbably – poised, cannot exist unless the teacher makes an effort to encourage them, in the most deliberate and conscious manner (and there goes the loss of self-consciousness!).

Evidently, a situation when the students are in flow, and the teacher isn't, is innocuous. A teacher who is merely roaming with nothing urgent to do will hopefully feel free to occupy his or her mind with pleasant musings on student learning and its subtleties. Not so when the reverse is true: a teacher in flow, oblivious (or resigned) to the boredom descending on the students during a lecture, is a much more negative experience. Remarking on the fact that some teachers turn to "performer mode" to alleviate the boredom or anxiety of such moments, Csíkszentmihályi (1982) quipped that although students may be impressed, they are however not in flow, but merely observing a captivating teacher in flow. He expressed concern that students might then draw the erroneous conclusions that the teacher's goal is to be the center of attention, or that teaching is simply about being entertaining. That is not to say that educators cannot be entertaining. Famous cynic philosopher Diogenes Laertius remarked long ago on the capricious nature of human allocation of attention:

> discourse on virtue and they pass by in droves; whistle and dance the shimmy, and you've got an audience. (as cited in Tal, 2015, p. 3)

Hopefully, teachers do not feel obliged to dance the shimmy in order to capture the attention of the students. However, if education is to be about transmission of meaning (Nehari & Bender, 1978) rather than transmission of knowledge, it may be worthwhile to reconsider the ways of teaching that do not attend to the flow of the students, even if they might be conducive to flow in the teacher.

Chapter 7. Conclusion

> *I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.*
>
> Isaac Newton

Little did I know, when I set out to research flow in my mathematics classroom, that I was embarking on such a long journey, towards such an uncertain goal. The flow experience occurs most frequently in leisure and in work environments, where most of the research has been conducted, and, in general, schools are not noted for their flow-friendly attributes (Csíkszentmihályi interviewed by Beard, 2015; Csíkszentmihályi & LeFevre, 1989; Csíkszentmihályi, Rathunde & Whalen, 1993; Shernoff, 2013). Furthermore, mathematics classrooms frequently seem to be places where negative emotions engendered by schools (Gumora & Arsenio, 2002; Valiente, Swanson & Eisenberg, 2012) are unhappily welded to anxiety or boredom engendered by mathematics itself (Ashcraft, 2002; Tulis & Fulmer, 2013). Nonetheless, I kept on stumbling over experiences like the one described in the introduction to this research, which gave me reason to hope that it was possible to elicit flow in students in a mathematics classroom, if only the conditions were *just right*. Thus, the focus of my research became a need to discern, refine, and broaden this initial narrow path of perfect fit, so that the flow experience may become commonplace, rather than a wished-for anomaly.

7.1. Flow as praxis

Csíkszentmihályi's flow theory contends that a triad of clear goals, steady feedback, and a balance between skills and demands is necessary for flow to emerge. This assertion has the weight of so much scientific evidence behind it so as to be almost axiomatic. In practice, in addition to goal clarification, feedback and task choice, also in power of the teacher are the choice of incentives, and the structure of the classroom environment.

The quest was therefore to determine what these look like, and how to weave them together in my own practice. Not unexpectedly, how much of each ingredient was required varied. A wide spectrum of flow-needs emerged: students like Chloe and Tamara needed a lot of clarification and feedback in the beginning, but slowly became more independent; others, like Karl and Brandon, required occasional punctual interventions; and further, there were those who, like Glen and Linus, needed mere nudges.

Whenever the goals and feedback were appropriate, there was quasi-unanimity in describing tasks as the right level, neither too easy, nor too hard. The exception: the pair consisting of April and Lucy, who quietly suffered from a lack of clear goals, and therefore felt the task was too hard. Nor did April and Lucy have sufficient feedback – either from me, or from each other. Their failure to enter the flow channel, despite my expectations, sensitized me to the subtleties of group-work: since the teacher is not (and should not be) the only source of feedback, a flow-rich collaboration depends on the "interactional synchrony" (Sawyer, 2006) of a group, that is, how much feedback can the members give each other. Thus, for the purposes of flow, the collaboration need not be focused on the exchange of accurate information, but rather, on mutual resonance, on being attuned to one another.

There was also a third source of feedback: the task itself. A good task contains, in addition to the allurements of play bounded by clear rules, the implicit promise that it won't allow you to get dangerously lost. For instance, embedded within the task of creating numbers using only certain digits, or deciphering a secret message, there are obvious clues which not only help the student along, but also whet the desire to move upwards on the flow channel, via the discovered complexities (Williams, 2000). Moreover, delving deeper, we can distinguish further reasons for preferring tasks with little or no lag between the doing and the knowing how you are doing: firstly, since there is no waiting for feedback, the student is more likely to experience that sense of temporal distortion that is the hallmark of flow. When actions and decisions swiftly and congruously succeed one another, time passes unnoticed. From the testimonies of the students, this absence from time is no less surprising than it is liberating. Secondly, tasks with inherent feedback have a positive effect on the student's sense of control and self-efficacy: when they can interact with mathematics without the teacher as mediator, they are more likely to see themselves as engaged protagonists in their own learning story.

One should not, however, discard tasks in which feedback is of a different nature. I'm referring here to the fact that *being stuck* is in itself clue, perhaps the most valuable in mathematics (and, to be honest, the most plentiful). My students Lonnie and Nikolaus, stuck on the problem of the fractal, nevertheless demonstrated the behaviour of true mathematicians: they persisted on their way, delighting in being mystified, until there was no more benefit to be extracted from that state; only then were they ready to move on. Their prolonged – and self-imposed – sojourn in the perseverance border of the flow channel (Liljedahl, 2017) set them up for the big "payoff" in terms of surprise and enjoyment. Lonnie and Nikolaus' experience is the perfect illustration of an idea that bears repeating: the flow experience needs obstacles in order to come into being and to flourish. In other words, there can be no flow in an activity if one cannot equally be bad at that activity: no risk, no flow.

Of course, students vary in their willingness to take risks. For instance, Nadyia and Bianca chose the safety of repeating a similar task, while James and Marie loudly demanded an increase in complexity. Where do these differences arise from, and can all students get to a state of comfort with discomfort? Before answering these questions, let me be the first to acknowledge that there exist differences between students which have their origin in the family context, and which make individuals vary in their ability to welcome "optimal frustrations" as a natural way of life (Rathunde, 1988). However, my research data cannot elucidate these more obscure, but no less substantive connections: I could not ask students about whether they felt their family life prepared them adequately for autotelic experiences. Early on, I had to make peace with the fact that such considerations, albeit relevant, could not fall within the purview of my research, and thus my answers could only be incomplete. I will therefore limit myself to what happens at the school, as this is the only aspect I have some control over.

And, alas, many undesirable things happen at the school. Shernoff et al. (2003) and Shernoff (2013) list but a few: “teaching to the middle”, ignoring the fact that students have widely different backgrounds and levels of preparation; not giving students enough time to achieve mastery; adopting an instructional format focused on lecturing; insufficient or negative feedback; meaningless tasks and lack of autonomy; or, finally, overemphasising grades. The consequences for students are dire, and go well beyond “being bad at math”: after repeated experiences of anxiety or boredom in school, they lose joy, interest, and motivation, and become chronically engulfed by these two negative states (Cheyne, Carriere & Smilek, 2006; Wood, 2006). Moreover, extended periods of flow deprivation have been shown to have deleterious effects on individuals’ mental state (Csíkszentmihályi, 1975a) and contribute to anhedonia, a state in which people cannot feel the pleasure of activities which they usually enjoy (Csíkszentmihályi, 1978).

My contention, supported by Csíkszentmihályi, who sees flow as a highly desirable state and human beings’ very raison d’être (Massimini, Csíkszentmihályi & Delle Fave, 1988), is that, similar to the progression of negative states, one can also observe a progression of perseverance and enjoyment, which occurs when students are repeatedly exposed to flow experiences. The fruit of this commonplaced flow habit is the “flow mindset” distinguished in section 6.4., where I noted its effects on students who sought, and achieved, repeat flow experiences. That the students had mastered the art of ascending on a spiral of complexity was obvious even long after the research had finished, when the students came back to my class for one last time, in grade 10. The sossometer, this serendipitous metaphor, continued to be used by students as a humorous way to communicate their need for higher challenges. And, at the end of the year, I was touched to receive a good-bye “sossometer” (see Figure 13) from a group of students who had taken part in the research:

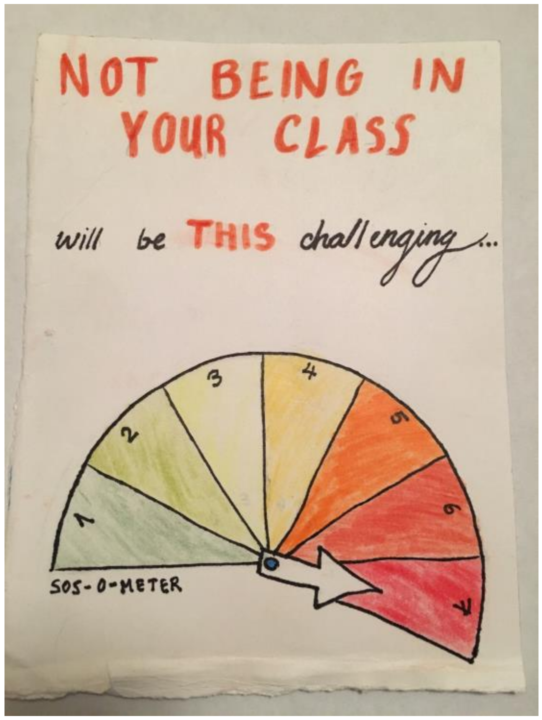


Figure 13. **The good-bye "sossometer".**

This was the most gratifying aspect of the research, as I could not help but entertain a timid, possibly vain hope that once my research would be over, once the students would not be in my class anymore, they would still be left with something useful that would serve them well through life, something that would go beyond mathematics. For much as I cherish the subtleties of mathematics, it is possible to forget them. What is impossible is to un-enjoy them.

7.2. Flow as sustainable axiological principle for teaching

I will approach this issue somewhat obliquely, by first stating that I am wholeheartedly of the opinion that teachers need to collaborate with other teachers in order to refine their practice. I am also of the opinion that such collaborations cannot be successful if there is any trace of dogma or undue persuasion. Finally, I am of the opinion, too, that teachers have an obligation to be both skeptical of, and open to, new ideas, treading a fine line between confidence in their professionalism and the Socratic humility of knowing that they don't know. It is not surprising, therefore, that teachers, when introduced to new research, are always performing a cost-benefit analysis: if I adopt this, what then do I have to give up? Or: will this work for my students? Or: how can I adapt this to my particular circumstances? To those teachers I address myself now, and entreat them to keep in mind, when considering the matter, that flow is a natural state for any human being.

I repeat: flow is not an "educational fad", nor is it a luxury. Flow is in the fabric of the lived experience, and it happens, whether we like it or not – and often, we do not. We could be watching students in flow when they are deep in conversation during class, when they play games on their phones, when they draw cars or intricate mandala patterns, or right before they break an arm during a ball game. Students seek flow experiences assiduously, just like adults do – or even more than adults, considering that they are still in their childhood, and the experiences of freedom and play are still fresh in their minds. We can no more stop them from seeking outlets for flow than we can stop them from breathing.

Thus, I feel that the issue of flow experience in a classroom is different, for instance, from the issue of textbook choice, or manipulatives, or homework, or any other number of things we care about in education – which are, I hasten to add, all worthy of careful consideration. At the risk of belabouring the point, I distinguish flow not because it's necessarily *good*, but because it's a fundamental, all-encompassing, unstoppable energy. By analogy with Dewey's (1913) advice about *catching and holding* students' attention, we may talk about the need to seize and redirect students' innate pursuit of flow, and about the importance of teaching them how to channel this energy for growth and happiness. A few words of caution, though: we cannot get flow in mathematics "for free"[6]. Both teacher and students will have to work for it, especially in the beginning, and, unless the efforts of both parties persist, flow will remain but a rare and random

[6] Contrary to what some think, the flow experience has nothing to do with the common expression "going with the flow". Csíkszentmihályi refers to this confusion in his writings, not without some asperity:

> [flow] has little to do with the widespread cultural trope of "going with the flow." To go with the flow means to abandon oneself to a situation that feels good, natural, and spontaneous. The flow experience […] is something that requires skills, concentration, and perseverance […] the evidence suggests that it is this second form of flow that leads to subjective well-being. (1999, p. 825)

occurrence. To borrow a phrase from philosopher John Kekes, flow is *a grace to be cultivated*.

In no small measure, the value of cultivating flow is also due to the aesthetic experiences organically cultivated alongside. Indeed, it even seems odd to separate flow and aesthetic experience: some consider them “the same state of mind” (Csíkszentmihályi & Robinson, 1990). As Dewey puts it,

> […] the esthetic is no intruder in experience from without, whether by way of idle luxury or transcendent ideality, but […] it is the clarified and intensified development of traits that belong to every normally complete experience. (1980, p. 48)

And, just like the enjoyment of flow begets complexity, growth, and more flow, aesthetic experiences initiate the virtuous circle of “liking, wanting, learning” (Chatterjee, 2014, 2013). Thus, in an ideal classroom, the two would happily co-occur, and nobody would give it a second thought. However, I am unconvinced that this mutually reinforcing symmetry of *grace* has always manifested in my own study. This is because, as I have confessed, I am unable to see any beauty, not even a sliver of prettiness, in polynomials. In that particular instance (James and Marie), although I could recognize the flow experience, I could not perceive the aesthetic experience.

This brings me to my last point in this discussion about value: the ethics of coaxing the flow experience. Would it be possible, for instance, to keep the people in flow, while they practice *only* mundane skills? I argue it is: there were numerous instances during my study when I felt with certainty that I could engineer flow in the students during the most inconsequential activity. In my view, this is what makes the flow experience such a formidable tool. While Jackson and Csíkszentmihályi put it poetically, “practice the skills to the point that you can forget you have them, then abandon yourself to the performance” (1999, p. 51), the reality of flow is sometimes less poetic. Encouraging the students to abandon themselves, to welcome that state of loss of self-consciousness, is an act that should raise some ethical questions for a teacher. Utmost care has to be taken so that students are not inadvertently led towards engaging in meaningless, but flow-inducing busywork. The same mechanism by which flow becomes a rewarding experience can lead to it becoming addictive (Schüler, 2012). While it may be difficult to envisage students becoming addicted to mathematics, it is not so difficult to envisage them becoming addicted to computer or phone games, to surfing the internet, or to taking inordinate physical risks.

7.3. Study limitations

The results and conclusions may be slightly contaminated, a fact unavoidable when conducting research with one's own students. Furthermore, the number of participants was limited, due to the tension between the need for immediacy and the time-consuming nature of administering interviews and questionnaires. In my view, another limitation – and a great frustration – has to do with my lack of skill and experience in leveraging the antecedents of flow. My research was, at first, beleaguered by various mistakes and omissions: misinterpreted cues from students, lack of finesse in clarifying goals, ill-timed (or lack of) feedback, all of which added up to numerous lost opportunities. It was not all about being a novice, either: further introspection about the underlying cause of these oversights reveals that I may have been blinded by a hedonic drive to preserve and advance my own state of flow. Hence, apologies are owed to students like April and Lucy: had I been not only more experienced, but also more self-aware, we could have made it work.

Other limitations have nothing to do with my shortcomings: firstly, without trying, there is no way to tell how much any person needs of the flow antecedents in order to enter the flow channel, or to maintain course once in flow. Secondly, even for the same person, particularly if that person is an adolescent, there are variations depending on the context of the day, the person with whom they collaborate, etc. The vast number of combinations and possibilities make it hard to occasion flow without many trials and errors. Throughout the research, I found myself in near-total agreement with neurobiologist Mark Changizi, who quipped "No matter how badly I teach, I still learn something" (as cited in Stafford, 2014). My only minor quibble is that, in accordance with flow principles, teaching badly is in fact a prerequisite for learning something. Thus, if only by this standard, my research is a resounding success.

7.4. Suggestions for further research

I mentioned in the previous section that increasing the number of participants would no doubt generate results that would be more nuanced and thus yield more predictive power. Moreover, my personal interest, were I able to gratify it, would be to follow the participants for a longer period of time, after they leave my classroom. While I have no doubts about the emergence of flow mindset, I wonder how it would unfold and develop in the following years, and how the students would be using this meta-skill throughout their education. Equally interesting in a school context would be to explore the relationship between achievement and flow, provided that such a study gives due attention to the requisite separation of external incentives usually connected with measures of achievement from the experience itself.

A further question running along the current research is whether, and in what measure, students' self-esteem is connected to their ability to find and maintain the flow experience. My interest in self-esteem and flow comes, partially, as a consequence of observations I made with respect to the experience of girls. On the one hand, I am, just like Csíkszentmihályi, persuaded that students of all genders can have flow experiences that are equally deep and frequent. On the other hand, the girls included in this research did express more anxiety about their lives in general and their performance in particular. An interesting thread to follow may come from Moneta, Schneider and Csíkszentmihályi (2001), who suggest, first, that self-esteem is a predictor of the flow experience, and, second, that adolescent girls have lower self-esteem than adolescent boys, in part because they are more concerned about living up to their own expectations and to the expectations of others. It would be interesting to explore whether this eagerness to please actually harms girls' flow experience, or makes it more resilient.

An adjacent issue which speaks to the inclusivity in the classroom is: can all students experience flow in mathematics? Research on this topic is extremely limited, possibly non-existent. Knowing the importance of attentional mechanisms for the flow experience, Csíkszentmihályi hypothesized that students who suffer from deficits of attention may be unable to mobilize sufficient concentration so as to enter the flow channel (as cited in

Scherer, 2002). A study focused on students with learning disabilities could perhaps cast some much-needed light on their unique experiences at school.

7.5. If teachers can't have flow, what can they have?

When delving into the epiphenomena associated with flow in my classroom, I had a surprising revelation: that just because I was in flow, it did not follow that the students were in flow, too. It was an unwelcome discovery at the time, as I had almost taken it for granted that there was a correlation between the two, perhaps as an extension image of the colloquial – and disingenuous – expression: "If ~~person in charge~~ mother is happy, everybody is happy". However, there seems to be no positive correlation between teacher flow and student flow; in fact, in section 6.8, I discuss why the flow of the teacher may end up having an adverse effect on the flow of the students. On the other hand, flow researchers suggest a teacher who has had flow experiences is more likely to know how to foster flow experiences in students (Rathunde, 2015). It may thus appear, in an ironic twist, that teachers must have flow experiences, only not while teaching.

There is a solution, however: I return to the octant model of flow, described in section 2.3.1, and suggest that the teachers would be wiser to situate themselves in the channels adjacent to the flow channel, namely, the control channel, in which the challenge is moderate, and the skills are higher than the challenge, or the arousal channel, which is characterized by satisfaction and excitement, but in which the challenge is higher than the skill (Delle Fave et al., 2011; 2010). Several considerations recommend these states: first, the teacher cannot completely forget herself in the task at hand, particularly when the task is that of occasioning flow in the students. As discussed, in my experience, complete absorption in the experience of a student or a group could prevent the teacher from attending to the needs of others. Secondly, both the control and the arousal channels offer close approximations of the flow experience and thus will keep the teacher engaged – for teachers, too, cannot risk apathy.

7.6. Where I've been, and where I'm going

I began my thesis with a story of discovery from my childhood. It was not a memory I plucked at random from my reminiscences playlist, for it serves as a dual-purpose allegory: firstly, it is a miniature of the very same process I went through when carrying out and writing my research. In many respects, my modus operandi has not changed: I start out not knowing how to read, and, after a long and arduous process, I complete the mystery book. Secondly, the story is an illustration of how flow operates, and why it is such an inescapable force: the pursuit of growth and ever-increasing complexity are innate in human beings, forever pushing us into precarious and, at the same time, exhilarating circumstances. Rock climbers, chess players, my younger self, my students, we are all bravely jettisoning what Virginia Woolf calls *the cotton wool of daily life*, in order to go and meet challenges halfway, and go on meeting them. We all seek a way to give meaning and substance to reality, through sharpening our strengths, upping our game, honing nerve and sinew, and, in the process, refining and enriching our lives.

Lastly, my story is also a measure of my enduring interest in the flow experience, which for most of my life I knew by its common symptom of complete absorption which made me lose track of time. Because of it, I've missed trains, burned meals to cinders, flooded houses and, more than once, failed to finish my homework. Equally, through it I also learned to how to enjoy learning and stretching my limits, how to flourish and be at home in the world. As a teacher, I've tried to elicit in my students the same experiences because I consider them worthwhile, and, in order to stay true to myself, I have to transmit what I see of value, just like a bee transmits through its DNA the message that collecting pollen is worthwhile. In that respect, the answer to the question "where am I going next" is very much "where I've always been going": the journey is not over, and it will never be. For, to paraphrase Robert Frost, there are no endings, and no beginnings. Everything is middle.

And so, it seems, I've reached the end of one of the middles.

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