Chapter 7
The Affordances of Using Visibly Random Groups in a Mathematics Classroom

Peter Liljedahl

Abstract Group work has become a staple in many progressive mathematics classrooms. These groups are often set objectives by the teacher in order to meet specific pedagogical or social goals. These goals, however, are rarely the same as the goals of the students vis-a-vis group work. As such, the strategic setting of groups, either by teachers or by students, is almost guaranteed to create a mismatch of goals. But, what if the setting of groups was left to chance? What if, instead of strategic grouping schemes, the assignment of groups was done randomly? In this chapter, I explore the implementation of just such a strategy and the downstream effects that its implementation had on students, the teacher, and the way in which tasks are used in the classroom. Results indicate that the use of visibly random grouping strategies, along with ubiquitous group work, can lead to: (1) students becoming agreeable to work in any group they are placed in, (2) the elimination of social barriers within the classroom, (3) an increase in the mobility of knowledge between students, (4) a decrease in reliance on the teacher for answers, (5) an increase in the reliance on co-constructed intra- and inter-group answers, and (6) an increase in both enthusiasm for mathematics class and engagement in mathematics tasks.

Keywords Collaboration • Group work • Social barriers • Integration • Mobilization of knowledge • Randomization

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Introduction

Group work has become a staple in the progressive mathematics classroom (Davidson & Lambdin Kroll, 1991; Lubienski, 2001). So much so, in fact, that it is rare to not see students sitting together for at least part of a mathematics lesson. In most cases, the formation of groups is either a strategically planned arrangement decided by the teacher, or self-selected groups decided by the students—each of which offers different affordances. The strategically arranged classroom allows the teacher to maintain control over who works together and, often more importantly, who doesn't work together. In so doing she constructs, in her mind, an optimal environment for achieving her goals for the lesson. Likewise, if the students are allowed to decide who they will work with, they will invariably make such decisions strategically in the pursuit of achieving their goals for the lesson. In either case, the specific grouping of the students offers different affordances in the attainment of these, often disparate, goals.

But, what if the selection of groups was not made strategically—by either party? What if it was left up to chance—done randomly—with no attention paid to the potential affordances that specific groupings could offer either a teacher or a learner? In this chapter, I explore a different set of affordances that result from the use of randomly assigned collaborative groupings in a high school mathematics classroom.

Group Work

The goals for strategically assigning groups can be broken into two main categories: educational and social (Dweck & Leggett, 1988; Hatano, 1988; Jansen, 2006). Each of these categories can themselves be broken into sub-categories as displayed in figure 7.1. When a teacher groups her students for pedagogical reasons, she is doing so because she believes that her specific arrangement will allow students to learn from each other. This may necessitate, in her mind, the need to use homogenous groupings or heterogeneous groupings where the factor that determines homo- or heterogeneous groupings can range from ability to thinking speed to curiosity. When she groups students in order to be productive, she is looking for groupings that lead to the completion of more work. This may, for example, require there to be a strong leader in a group for project work. It may also mean that friends or weak students do not sit together, as such pairings may lead to less productivity. Groupings designed to maintain

peace and order in the classroom would prompt the teacher to not put 'trouble-makers' together, as their antics may be disruptive to the other learners in the class[1]. Interestingly, students may self-select themselves into groupings for the same aforementioned reasons (Cobb, Wood, Yackel, & McNeal, 1992; Webb, Nemer, & Ing, 2006; Yackel & Cobb; 1996).

More commonly, however, students group themselves for social reasons (Urdan & Maehr, 1995) – specifically to socialize with their friends. Teachers too, sometimes form their groups to satisfy social goals. They may feel that a particular group of students should work together specifically because of the diversity that they bring to a setting. Sometimes, this is simply to force a gender mix onto the collaborative setting. Other times, it is more complex and involves trying to get students out of their comfort zone; to collaborate with, and get to know, students they don't normally associate with. A teacher may choose to create a specific grouping to force the integration of an individual student into a group that they are not yet a part of—for example, the integration of an international student into a group of domestic students. Finally, and less likely, a teacher may specifically wish for their students to work with their friends—often as a reward for positive performance or behaviour in the classroom.
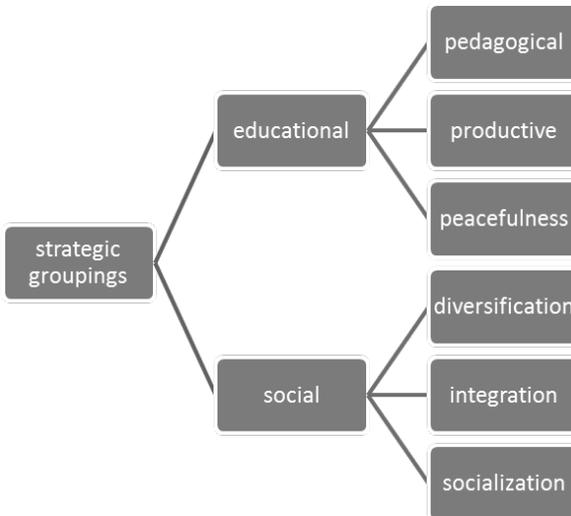
Fig. 7.1 Goals for strategic groupings

Regardless of the goals chosen, however, there is often a mismatch between the goals of the students and the goals of the teacher (Kotsopoulos, 2007; Slavin, 1996). For example, whereas a teacher may wish for the students to work together for pedagogical reasons, the students, wishing instead to work with their friends, may begrudgingly work in their assigned groups in ways that cannot be considered collaborative (Clarke & Xu, 2008; Esmonde, 2009). These sorts of mismatches arise from the tension between the individual goals of students concerned with themselves, or their cadre of friends, and the classroom goals set by the teacher for everyone in the room. Couple this with the social barriers present in classrooms and a teacher may be faced with a situation where students not only wish to be with certain classmates, but also disdain to be with others. In essence, the diversity of potential goals for group work and the mismatch between educational and social goals in a classroom almost ensures that, no matter how strategic a teacher is in her groupings, some students will be unhappy in the failure of that grouping to meet their individual goals. How to fix this? One way would be to remove ANY and ALL efforts to be strategic in how groups are set.

Random Groupings

Over the last six years I have done research in a number of classrooms where I have encouraged the teachers to make group work ubiquitous, where new groups are assigned every class, and where the assignment of these groups is done randomly. In every one of these classrooms the lesson begins with the teacher generating random groups for the day. The specific method for doing this varies from teacher to teacher. Some give out playing cards and have students group themselves according to the rank of the card they have drawn. Others have students assigned a permanent number and then draw groups of 3 or 4 numbered popsicle sticks or numbered disks randomly from a jar. In other classes, the students watch the teacher randomly populate a grid with numbers wherein each row of the grid then forms a group. One teacher I worked with had this grid placement done automatically by a program displayed on an interactive whiteboard. Another teacher I worked with had laminated photographs of all of the students and distributed these into groups by shuffling and then randomly drawing 3 or 4 photos at a time. Regardless of the particulars of the method, however, the norm that was established in each of the classes that I worked in was that the establishment of groups at the beginning of class was not only random, but visibly random. Once in groups, students were then universally assigned tasks to work on, either at their tables or on

the whiteboards around the room. The students stayed in these groups throughout the lesson: even if the teacher was leading a discussion, giving instructions, or demonstrating mathematics.

Although often met with resistance in the beginning, within three to four weeks of implementation, this approach has consistently led to a number of easily observable changes within the classroom:

- Students become agreeable to work in any group they are placed in.
- There is an elimination of social barriers within the classroom.
- Mobility of knowledge between students increases.
- Reliance on the teacher for answers decreases.
- Reliance on co-constructed intra- and inter-group answers increases.
- Engagement in classroom tasks increase.
- Students become more enthusiastic about mathematics class.

Ironically, these are often the exact affordances that teachers' strategic groupings of students is meant, but often fails, to achieve. How is this possible? What is it about the use of visibly random groups that allows this to happen? Drawing on data from one classroom this chapter looks more closely at these aforementioned observed changes as well as what it is about visibly random groupings that occasions these changes.

Methodology

The data for this study was collected in a grade 10 (ages 15-16) mathematics classroom in an upper-middle class neighbourhood in western Canada. The students in the class were reflective of the ethnic diversity that exists within the school at large. Although there are students from many different cultures and backgrounds in the school, and the class, the majority of students (> 90%) are either first or second generation immigrants from China or Caucasian Canadians whose families have been in Canada for many generations. These two dominant subgroups are almost equal in representation. This, almost bimodal, diversity is relevant to the discussion that will be presented later.

The classroom teacher, Ms. Carley (a pseudonym), has eight years of teaching experience, the last six of which have been at this school. In the school year that this study took place, Ms. Carley decided to join a district run learning team facilitated by me. This particular learning team was organized around the topic of group work in the classroom. As the facilitator, I encouraged each of the 13 members of the learning team to start using visibly random groups on a daily basis with their classes. Ms. Carley had joined the team because she was dissatisfied with the results of group work in her teaching. She knew that group work was important to

learning, but, until now, had felt that her efforts in this regard had been unsuccessful. She was looking for a better way, so when I suggested to the group that they try using visibly random groups she made an immediate commitment to start using this method in one of her classrooms. This, in turn, prompted me to conduct my research in her class.

The data was collected over the course of a three month period of time from the beginning of February to the end of April. The time frame is significant because it highlights that this was not something that was implemented at the beginning of a school year when classroom norms (Yackel and Cobb, 1996) are yet to be established and students are more malleable. The fact that the change occurred mid-year allowed me the unique opportunity to compare classroom discourse, norms, and patterns of participation before and after implementation. Initially, I was present for every class. This included three classes prior to implementation as well as the first three weeks (8 classes) after initial implementation. After this, I attended the classes every two or three weeks until the end of the project.

I became a regular fixture in the classroom and acted, not only as an observer, but also as a participant (Eisenhart, 1988), interacting with the students in their groups and on the tasks set by the teacher. The data consists of: field notes from these observations, interactions, and conversations with students during class time: interviews with Ms. Carley: and interviews with select students. Interviews were conducted outside of class time and audio recorded. Over the course of the study, Ms. Carley was interviewed, if only briefly, after every observed lesson. During this time frame 12 students were also interviewed, with two of them being interviewed twice. These data were coded and analysed using the principles of analytic induction (Patton, 2002). "[A]nalytic induction, in contrast to grounded theory, begins with an analyst's deduced propositions or theory-derived hypotheses and is a procedure for verifying theories and propositions based on qualitative data" (Taylor and Bogdan, 1984, p. 127 cited in Patton, 2002, p. 454). In this case, the *a priori* proposition was that the changes that I had observed in other classrooms were linked to the use of a visibly random grouping scheme. This proposition became the impetus for the collection of data in that it drove what I was looking for and how I was looking. It became the lens for my observations and it motivated my interview questions. It also pre-seeded the themes that I was looking for in the coding of the data.

This is not to say that my data collection and analysis was blind to the emergence of new themes. As a participant/observer in the classroom I was aware of, and deliberately looking at, a great many things going on around me. During the coding and analysis of the data I was looking for nuances in the relationship between visibly random grouping schemes and the changes I had observed. So, despite the fact that I had a priori themes in

mind, I still coded the data using a constant comparative method (Creswell, 2008). This recursive coding allowed for the emergence of not only nuanced themes, but also new themes.

Results and Discussion

Similar to the other classes wherein I have observed the implementation of random grouping schemes Ms. Carley's class exhibited the same observable changes. In what follows I explore each of these changes more thoroughly, illuminating the nuances of each with results from the data.

Students become agreeable to work in any group they are placed in

Group work is not something that is foreign to the students in Ms. Carley's class. From time to time she allows the students to sit in pairs or threes to work on their homework and the class had already done one group project on graphing where the students were allowed to self-select who they worked with. When Ms. Carley decided to implement a more ubiquitous approach to group work in general and the use of random groups in particular she chose to use a standard deck of playing cards to generate the groups. She had 30 students in the class and she had decided to have the students work in groups of three. So, she selected from the deck 3 cards of each rank (ace – ten). These were shuffled and then the students were allowed to each select one card. Although she experimented with the number of students per group, and had to make adjustments based on absences, this is a grouping scheme that she stayed with for the duration of the study.

On the first day the students were not told what was going on but just presented with the cards as described. Later, I learned that many of the students had thought that "it was a magic trick". When every student had a card Ms. Carley announced that these would be the groups that they would be working in and assigned a "station" for each group depending on their card. This was an interesting time. Many of the students went dutifully to their stations. However, there were a few students who I observed were trying to fix it so that they were with their friends. I will elaborate on two of these cases in particular.

Hunter, despite his card, went directly to the station where his friend Jackson was sitting. This did not go unnoticed as Ms. Carley immediately noticed that this group now had four members instead of three. When she dealt with this she immediately challenged Hunter to see his card. When I asked her about this later she said that "it had to be Hunter. It is always

Hunter. He is a bit of a scammer and he likes to be with Jackson". In the flurry of the first few minutes of class Ms. Carley had to perform a similar check on one other group of four.

Unnoticed by Ms. Carley, however, was the situation that unfolded immediately in front of me. Jasmine approached a group of three and took the card out of one of the group members' hand replacing it with her own card and said, "you're over there", gesturing towards one of the corners of the room. From my initial observations of the class and my conversations with Ms. Carley I knew that Kim, Samantha, and Jasmine, are very close friends, are part of the "in" crowd within their grade, and tend to stick very close together during free time and when allowed in their other classes. The group that Jasmine approached had Samantha in it.

In general, this sort of jockeying behaviour was observed for the first three classes after implementation. Hunter did try it again but Ms. Carley intervened even before he got to Jackson and on the third day Hunter and Jackson legitimately ended up together—much to the chagrin of Ms. Carley. Jasmine, however, was successful each time she tried to switch groups using the same strategy. After the first week, however, the behaviour stopped for both Hunter and Jasmine. At this point I interviewed both Hunter and Jasmine about their antics.

Researcher	So, I noticed that last week you tried a few times to sit with Jackson. Are you still trying to do so?
Hunter	No.
Researcher	Why not?
Hunter	At first I thought that the teacher was trying to keep us apart. Then, on Friday, we got to work together.
Researcher	So, do you still think the teacher is trying to keep you apart?
Hunter	No. I don't think she likes us working together, but when the cards came up the way they did she didn't change it. I guess it's up to the cards now.

Researcher	I saw what you did last week.
Jasmine	What do you mean?
Researcher	I saw how you switched groups.
Jasmine	Oh that. That's nothing.
Researcher	But you didn't do it this week. Why not?
Jasmine	I guess it doesn't matter so much. I mean, it is just for one class and then the groups change again.
Researcher	What does that have to do with it?
Jasmine	At first I was worried that I was going to be stuck with that group for a long time, like when we worked on the project or in my other classes.

| Researcher | What happens in the other classes? |
| Jasmine | My English teacher changes the seating plan every month and then you're stuck there forever. |

For Hunter, the defining quality of Ms. Carley's grouping scheme was the random nature of it. Once he came to see that it was both random and that the random outcomes would be respected he became more relaxed about it. For Jasmine, however, the defining quality was the short term commitment that the grouping strategy demanded. When I had first observed Jasmine's antics I had assumed that it had to do with trying to be close to her friends when, in reality, she was trying to avoid being "stuck" with a group she didn't like. Once she became confident that the groups were temporary she stopped trying to manipulate the groups.

I also interviewed Jennifer in the third week after implementation. I selected Jennifer because she had shown no overt objections to the grouping schemes used in the class.

Researcher	I'm wondering what you think about all this grouping stuff that is going on.
Jennifer	Its ok I guess. It doesn't matter what I think though, it looks like it's here to stay.
Researcher	What do you mean by "it's here to stay"?
Jennifer	Well, when the teacher started class on last Monday the same way I knew that this is the way it was going to be. When she started class today [Monday of the third week] I was sure of it.

Jennifer's observation coincides perfectly with the subsiding of any residual visible opposition to the random groupings. Although she was not overtly opposed to the groupings in the first week, her mention of the practice continuing in the second and third weeks and how that was a sign that "it's here to stay" indicates a resignation to the new classroom norm (Yackel and Cobb, 1996) that is likely shared by many of her peers. This is a different phenomenon from Hunter, who saw the randomness in the cards or Jasmine, who focused on the temporariness in each grouping. These three themes occurred and reoccurred in many of the conversations I overheard, conversations I was part of, and in interviews. Sometimes they occurred in isolation as in the excerpts presented above. Other times they were present in combination with each other.

More interestingly, resignation to a new norm became the only thing commented on by the third week. That is, regardless of what the students thought about the introduction of visibly random groups the residual effect was that "it is how we do things in this class". Even in the last week of the study when I asked specific students to recall the early days of the use of

the playing cards, their recollections of it was that it was just the introduction of a new way to do things. That is, although the randomization being visible, the cards being respected, and the groups being only for one class were of great importance in the first weeks, what endured to the end was just the norm. This is in alignment with Yackel and Cobb's (1996) observation that norms are not something that are imposed on a class, but are negotiated between the teacher and the students. The fact that the grouping scheme was visibly random and that the groups were only for one period were important elements in these negotiations.

There is an elimination of social barriers within the classroom

As mentioned earlier, there is an almost bimodal diversity in both the class and the school where the study took place. My observations of this "split" are exemplified in the conversations that I had with Ms. Carley prior to her implementation of random groups.

Researcher	Can you think of any problematic situations that you think will prevent this [random groupings] from being successful?
Ms. Carley	The obvious one is the split between the Asian and Caucasian students.
Researcher	What do you mean – *split*?
Ms. Carley	It's almost as though we have two distinct cultures in this school with almost no overlap. The Caucasian students have their own social groupings, not all together. And the Asian students have their own. And there is almost no mixing between the two. In fact, it's almost as though they aren't even aware of each other.
Researcher	I have noticed that. Is that normal you think?
Ms. Carley	I don't know about normal but it is certainly not unique to this school. I have a good friend who teaches in Surrey and she has seen the same thing but with different groups of students. We talk about it often and what we can do about it.

What Ms. Carley describes is a situation that is easily observable in both the hallways and in the classroom. When Ms. Carley allowed the students to self-select who they wanted to work with, the selections were always guided by this "split". This is not to say that there were any racial tensions in the group. I observed no evidence of dislike or disdain for each other. It really was just as Ms. Carley had described – two distinct social groupings.

We both saw this as a formidable challenge and were simultaneously anxious and hopeful about how the random groupings would play out.

It is quite possible that some of Jasmine's antics (described in the previous section) were motivated by this social dichotomy. On both the first and second day of implementation she was randomly assigned to a group that had two Asian students in them. The second time that she "stole" someone else's card she took it from the sole Asian girl in the group where she wanted to be in. But, as stated in the previous section, these sorts of behaviours by Jasmine and others in the class ceased after the first two weeks of implementation as the students settled into the new norm. This is not to say that the social divide had disappeared – yet.

After three weeks of implementing visibly random groups, some interesting phenomena began to emerge. Whereas in the first few days after implementation there was an awkwardness present in the first few minutes of group work, now there was an "at ease*ness*" about the way the students came together. This was more than comfort with a process, however. It was more akin to a familiarity between students. This can be seen in the interview with Melanie.

Researcher	Tell me about how your group work went today?
Melanie	Fine.
Researcher	Who were you with?
Melanie	I was with Sam and … um … the guy … I don't know his name.
Researcher	Frank?
Melanie	That's it. Frank!
Researcher	Can you tell me a little bit about Sam and Frank?
Melanie	Ok. Sam is smart. I worked with her one time before. She really knows what is going on so I try to listen carefully to her when she has something to say. She's in my Science class as well and her sister is in my English class.
Researcher	How do you know that Sam's sister is in your English class.
Melanie	Sam told me today.
Researcher	What about Frank?
Melanie	I don't know Frank that well, but my friend worked with him last week and he said that Frank is a really nice guy.

To help orient this conversation it is useful to know that Melanie is Caucasian and that both Sam and Frank are Asian. What is remarkable about this is that there is an awareness about each other that is forming. Sam is aware that Melanie is in her sister's English class and Melanie is aware that her friend worked with Frank last week. These are both strong indicators that the two groups are now seeing each other—aware of each

other—in a way that Ms. Carley (and I) had observed was not happening prior to implementation. Further, Melanie's interview reveals that the two groups are not only talking to each other, they are talking about each other.

This is not to say that race was the only social barrier at play within this classroom prior to implementation. As in any school, there was also a more subtle, but very real, social hierarchy at play. There were students who were "in" and students who were "out". As already mentioned, Jasmine, Kim, and Samantha were part of the "in" crowd. Prior to implementation they always sat together, and as seen, Jasmine worked hard to maintain this together*ness* at initial implementation. For Jasmine, this was eased by the realization that the groups were short lived. For Samantha, it was eased by the fact that the nature of the group work had changed.

Researcher	It's been six weeks now since Ms. Carley started moving you around. What do you think about it?
Samantha	It's ok.
Researcher	I know that you used to like to sit with Jasmine and Kim a lot. How is it being away from them?
Samantha	I'm not away from them. I still see them all the time and I did sit with Kim and Charles the other day. But it's different now. Before we would just sit and talk. Now we are working on stuff at the boards and stuff. There isn't a lot of time to just socialize anyway.
Researcher	How do you think Jasmine and Kim feel?
Samantha	Jasmine is ok with it now. She wasn't at first. And Kim never cared. She is really easy going.

It is obvious from this transcript that Kim is also at ease, and always was, with the random grouping scheme. More subtle, however, is the mention of Charles. Charles is an Asian boy definitely not in the "in" crowd. I'm pretty sure that prior to implementation Samantha did not know his name. Now she mentions him in passing. This points to what I was observing at this point in the study – Ms. Carley's class had jelled into a cohesive whole, absent of any social divides.

There is a lot to be seen and to be discussed in regards to the breaking down of social barriers, both racial and non-racial, and my naïve treatment of it is not meant to diminish the rich traditions of such research (c.f. DeVries, Edwards, & Slavin, 1978). I merely wanted to highlight the role that the visibly random grouping scheme played in the breaking down of some of these barriers.

Mobility of knowledge between students increases

As mentioned, prior to implementation, group work in Ms. Carley's class was something students did as they worked on their homework or on a project. After implementation, group work became ubiquitous. The main activity in these groups was to work through a series of tasks that Ms. Carley set during her lessons. These were originally "try this one" tasks that followed direct instruction. But as the study went on, Ms. Carley began to also use tasks as a way to initiate discussions. The tasks also became more challenging, requiring the students to do more than just mimic the examples already presented on the boards. This "ramping up" of the use of tasks was accompanied by a number of easily observable changes in the way in which the groups worked, with the most obvious of which was the way in which the knowledge moved around the room.

Immediately after implementation, group work looked very much like it did prior to implementation – the students worked largely independent of each other, interacting only to check their answers with their group members, or to ask one or another to explain how to do something. After four weeks, however, group work looked very different. Students now spent almost no time working independently. Instead, they spent their time working collaboratively on the tasks set by Ms. Carley. This collaboration consisted of discussion, debate, and the sharing and demonstration of ideas. In part this was due, of course, to the increasing demand and frequency of the tasks set by the teacher. But it was also due to the coalescing of the groups into collaborative entities.

Researcher	So, the students seem to be working well together.
Ms. Carley	Yes … I'm still amazed at exactly how well.
Researcher	We've talked a lot about the tasks you are using and how you are using them. Do you think the tasks are responsible for the group work we are seeing now?
Ms. Carley	You know, I've thought a lot about that lately. At first I thought it was all due to the tasks. In fact, I was talking to a colleague who was asking about my class. She was asking for a copy of the tasks so she could start using them with her students and that's when I realized that it's sort of a chicken and egg thing. If we spring the tasks on the students before they know how to work in groups then it won't work. At the same time, if we try to teach them how to work in groups without having something to work on then it won't work either.
Researcher	So, how did you manage it in this class? What came first?

Ms. Carley   I think the random groups came first. That broke the mould on what group work had looked like in the past and gave me room to introduce a new way of working.

Ms. Carley's synopsis aligns well with my observations. Prior to implementation, group work had a well-defined set of actions and behaviours associated with it. These norms were not conducive to the collaborative skills and affordances necessary to increase the demand on students vis-à-vis the ubiquitous use of tasks. The introduction of random groups into the classroom shattered the existing norm and allowed for a new set of classroom norms to be established that were more conducive to collaboration.

The collaboration now visible in the room went beyond the intra-group activity, however. Inter-group collaboration also became a natural and anticipated part of every class. This often took one of three forms: (1) members of a group going out to other groups to "borrow an idea" to bring back to their group, (2) members of a group going out to compare their answer to other answers, (3) two (or more groups) coming together to debate different solutions … or a combination of these as exemplified in my observations of Kevin's group in week four of the study.

Researcher   Good problem today, huh? I didn't get a chance to sit with you today. Can you tell me how you guys solved it?

Kevin   Yeah, that was a tough one. We were stuck for a long time.

Researcher   We were too [referring to the group I was working with]. What did you eventually figure out?

Kevin   Well, we saw that the group next to us was using a table to check out some possibilities and we could see that there was a pattern in the numbers they were using so we tried that. That sort of got us going and we got an answer pretty quickly after that.

Researcher   Was it the right answer?

Kevin   It was, but we weren't so sure. The group next to us had a different answer and it took a long time working with them before we figured out which one was correct.

Kevin's recollection of the day's activities is reflective of what I observed between these two groups and, in fact, many groups on a daily basis. When I asked Sam (who was in the other group) about this, she had some interesting insights about why this coming together of the two groups worked so seamlessly.

Researcher   Your group worked pretty closely with another group today. How did you feel about the fact that they copied from you?

Sam	Did they? I didn't notice. But it isn't really copying. We are all just working together.
Researcher	In other classes I have been in I don't see that happening. You know, groups sharing with each other.
Sam	That's probably because they don't work together as much as we have. I mean, we are always together with different people. I think I have worked with everyone in this room now. If you asked me who I worked with yesterday I'm not sure I could tell you. And if you asked the teacher to tell you who was in which group today I don't think she could tell you either. When we were trying to figure out which answer was correct we were like one big group.

What Sam is describing is what I have come to call the *porosity* of groups. Although group boundaries are defined for the period, these boundaries are clearly temporary and arbitrary. This allows for them to also be seen as open and allowing for the free movement of members from one group to another to extend the collaborative reach of the group. When asked about this, many students mention that they feel that they are free to move around the room as necessary to "get the job done".

Along with this mobility of groups and group members comes *mobility of knowledge* – the movement of ideas, solution strategies, and solutions around the room. In fact, it is the need to move knowledge that prompts the movement of individuals as they go out "to borrow an idea". The free and easy mobility of knowledge results in a marked decrease in the students' reliance on the teacher as the knower.

Researcher	Have you noticed anything else that has changed over the last five weeks?
Ms. Carley	I've noticed that I'm not answering as many questions anymore?
Researcher	Are you not answering them or are you not being asked them?
Ms. Carley	Both, I think. I know there was a point where I was deliberately trying to not answer questions, trying to push the students back into the groups to figure it out. But now that is not a problem. They just don't ask me questions as much anymore. It's like that chicken and egg thing again.

Similar to the relationship between the use of random groups and the use of more challenging tasks, the relationship between the teacher not answering questions and the students not asking questions seems to be in some sort of symbiosis. That is, in order for the group work to become effective and meaningful the teacher needs to stop answering questions

and, as the group work becomes effective and meaningful, the students stop needing to ask questions. Ms. Carley's class has become a collective making use of both intra and inter group collaborations.

This is not to say that the role of the teacher is diminished. Ms. Carley still sets the tasks, the groups, and the expectations. More importantly, however, she monitors the flow of knowledge around the room.

Researcher	I noticed that you were forcing some groups together today. What were you trying to achieve?
Ms. Carley	It depends. Sometimes I am trying to crash ideas together. Other times I am trying to help a group get unstuck. Which groups do you mean?
Researcher	I mean when you sent one whole group from over there to over here.
Ms. Carley	Ah. Well, that group over there had gotten an answer pretty quickly. As it turned out, it was the right answer, but I didn't think they had done enough work checking their answer so I sent them over to that group to shake their confidence a little bit.
Researcher	How so?
Ms. Carley	Well, that group had a different answer and that would force the two groups to figure out what was going on.

Not only is Ms. Carley monitoring the flow of knowledge in the room, she is manipulating it – forcing it move in certain directions and moving it for a variety of different reasons. In so doing, her role in the classroom has changed.

Researcher	So, how are you liking your classroom these days?
Ms. Carley	I'm loving it. I feel like the students are completely different. I'm completely different. It's like I have a new job and its WAY better than my old one.

Students become more enthusiastic about mathematics class

Ms. Carley is not the only one who is enjoying her new role, however. Many of the students I either talked to as part of my classroom participation or in interviews alluded to the fact that Ms. Carley's mathematics class is *now* an enjoyable place to be.

Frank	I like this class *now*.
James	Math is *now* my favourite subject.

In the fifth week of the study I spoke with Jasmine about how she was enjoying this class.

Researcher	So, it's been a while since that day where you were trying to switch groups. How are you enjoying things now?
Jasmine	I love this class. I mean, math isn't my favourite subject. But I love coming here.
Researcher	Why is that? What is it about this class that you love?
Jasmine	I'm never bored. There is always something going on and time passes so quickly.
Researcher	I looked at Ms. Carley's attendance book. For the last four weeks you have never missed a class or even been late. I only looked at four weeks, what would I have seen if I looked further back?
Jasmine	You would have seen some absences and lots of lates. I mean, it's not like I skipped class. I don't skip. It's just that there were reasons to be away. I guess I now try not to let there be reasons.
Researcher	What about lates?
Jasmine	I'm often late for my classes. Not just math.
Researcher	But you haven't been late at all lately.
Jasmine	Hmm … I guess I don't want to be.

Jasmine didn't like mathematics the subject, but she loved mathematics the class. The changes that had occurred, which began with the random groupings, had transformed the Ms. Carley's class into something that she didn't want to miss out on.

This was a trend that I observed in many students. In terms of attendance, absences and lates were down across the board. Prior to implementation, Ms. Carley had an average of 3.2 absences per class and an average of 6.7 lates per class. Between week four and week seven after implementation, the averages were 1.6 and 2.2 respectively. Ms. Carley's class became a place where students wanted to be. Conversations with other students echoed Jasmine's sentiments. In my conversations with Chad, Stacey, and Kendra I decided to push a little further by asking them to draw comparisons.

Researcher	So, how is this class different from other classes?
Stacey	I like this one.
Researcher	Ha … do you not like other classes?
Stacey	I do. But not like this one. This one is way more dynamic. We are always doing something new and …
Kendra	And the beginning of every class is a bit of an adventure when we get to find out who we work with.
Researcher	It's been six weeks. Hasn't it gotten old yet … the thing with the random groups.
Chad	No. It's still fun.

Researcher	I want to continue with Stacey's comments. In what ways is this class different form other classes?
Chad	Hmm … we need to think in this class. There really is no other way around it. In other classes you can sort of just tune out, but not in here.
Kendra	And you have to collaborate. There is no way I could get by just doing it on my own, even if Ms. Carley would let us.
Researcher	It sounds like a lot of hard work.
Stacey	It is, but in a good way. I mean, like I'm never bored.

These comments speak not just to enjoyment, but also engagement. The students need to be engaged in Ms. Carley's class and they seem to enjoy this engagement. The comments of these students confirm what both Ms. Carley and I had observed in the class as a whole.

| Researcher | So, what do you think? How is it going? |
| Ms. Carley | My sense is that it is going really well. This week all of the students really seem to be into it. Everyone shows up ready to go, and then we go. There are no complaints, everyone is smiling, and we get a lot done. |

Conclusions

I stated at the outset that the changes that I observed in Ms. Carley's class are reflective of the changes I had seen in many of the other classes in which I had been privileged to participate as teachers made the decision to start using visibly random grouping schemes. But, in the past, these had just been observations. My more focused approach to studying Ms. Carley's class confirmed my prior (and subsequent) observations, and also informed and enlightened them. As in the other classroom, I had observed that the introduction of random groupings were pivotal in producing broad changes in the classroom. However, these changes were more than just changes to the way the class was run. The introduction of random groupings led to, and allowed for, changes in the students, the teacher, and what was possible in this new setting.

The students became open to working with anyone. The social barriers that existed in the classroom came down and the classroom became a collaborative entity that was not defined by, or confined to, the boundaries set by the teacher. As these barriers came down and the class coalesced into a community, their reliance on the teacher as the knower diminished and their reliance on themselves and each other increased. Their enjoyment

of mathematics (the class, if not necessarily the subject) increased as well as their engagement.

Figure 7.1 (above) showed how neatly the strategic educational and social goals could be partitioned. When non-strategic grouping methods were used, the resulting behaviours cannot be so easily partitioned into educational and social affordances. For example, the increased mobility of knowledge is a direct result from the students' increased reliance on intra- and inter-group generated results. However, this cannot be separated out from the fact that social barriers in the room have come down. Taken together, the data showed that the use of visibly random groupings produces student behaviour that can be seen as being both educational and social in nature (see figure 2). As such, the non-strategic use of visibly random groupings turned out to be a better strategy than the aforementioned strategic grouping schemes.
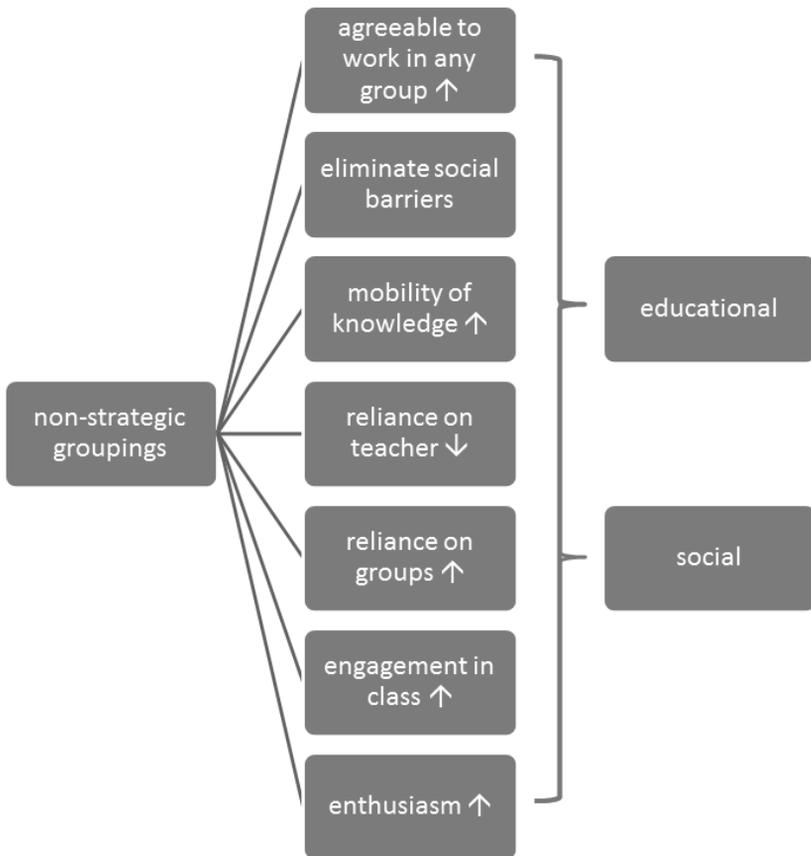
Figure 2: Results of non-strategic groupings

Student change aside, Ms. Carley altered the way she used tasks as well as the way she answered questions. She found that she no longer needed to be the knower or the teller in the room. She changed the timing and the method of her direct instruction and she began to rely much more on her ability to manipulate groups and move ideas around the room. Tasks, too, took on a new life in the class. Their role changed from "try this one" to objects around which group work was organized. They increased in frequency and difficulty and they became the objects and objectives of lessons.

The introduction of visibly random groupings was the impetus that both allowed for and necessitated the many other changes that I observed. Through the renegotiation of classroom norms (Yackel & Cobb, 1996) the students could not continue to behave as they had earlier, Ms. Carley could not continue being the same teacher she had been prior to implementation, and tasks could not have avoided evolving. Change begot change.

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