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RESEARCH ON AFFECT IN MATHEMATICS EDUCATION: A RECONCEPTUALIZATION

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Affective issues play a central role in mathematics learning and instruction. When teachers talk about their mathematics classes, they seem just as likely to mention their students' enthusiasm or hostility toward mathematics as to report their cognitive achievements. Similarly, inquiries of students are just as likely to produce affective as cognitive responses; comments about liking (or hating) mathematics are as common as reports of instructional activities. These informal observations support the view that affect plays a significant role in mathematics learning and instruction. Although affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field. If research on learning and instruction is to maximize its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers. One theme of this chapter is that all research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction.

Current efforts to reform the mathematics curriculum place special importance on the role of affect. The National Council of Teachers of Mathematics has reaffirmed the centrality of affective issues in its recent publication of the standards for curriculum and evaluation (Commission on Standards for School Mathematics, 1989). Two of the major goals stated in this document deal with helping students understand the value of mathematics and with developing student confidence. In its Standard on mathematical disposition, the assessment of student confidence, interest, perseverance, and curiosity are all recommended. Similarly, the National Research Council's (1989) report on the future of mathematics education (*Everybody Counts*) puts considerable emphasis on the need to change the public's beliefs and attitudes about mathematics. In the United States, there is a tendency to believe that learning mathematics is a question more of ability than effort. Adults are willing to accept poor performance in school mathematics, but they are not so willing to accept poor performance in other subjects. Both adults and children often proclaim their ignorance of mathematics without embarrassment, treating this lack of accomplishment in mathematics as a permanent state over which they have little control. The improvement of mathematics education will require changes in the affective responses of both children and adults.

As these reports show, the U. S. reform movement in mathematics education clearly takes affective factors as an important area where substantial change is needed. This emphasis on affective issues is related to the importance that the reform movement attaches to higher-order thinking. If students are going to be active learners of mathematics who willingly attack nonroutine problems, their affective responses to mathematics are going to be much more intense than if they are merely expected to achieve satisfactory levels of performance in low-level computational skills.

A variety of large-scale studies provide a substantial amount of data that indicate there is good reason to be concerned about affective factors. The Second International Mathematics Study (Robitaille & Garden, 1989) indicates that there are large differences between countries on measures of mathematical beliefs and attitudes, just as there are large differences in achie-

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vement. Various national assessments have also included data on affective issues. Dossey, Mullis, Lindquist, and Chambers (1988) report that students in the United States become less positive about mathematics as they proceed through school; both confidence about and enjoyment of mathematics appear to decline as students move from elementary through secondary school. Students in other countries (Foxman, Martini, & Mitchell, 1982; McLean, 1982) also show little enthusiasm for mathematics as they progress through school.

Although efforts to evaluate mathematics programs and to promote the reform and improvement of mathematics education pay considerable attention to the affective domain, these efforts usually take a very practical approach, using questionnaires to gather common-sense data on beliefs and attitudes toward mathematics. This kind of evaluation usually does not attempt to present a theoretical framework for the assessment of affect, nor does it include data from small-scale, qualitative studies that could provide a more detailed picture of students' affective responses to mathematics. The improvement of theory and the use of a variety of research methods are two additional themes that will recur throughout this chapter.

The chapter begins by considering alternative theoretical foundations for research on affect. Mandler's (1984) theory, an approach to research on affect that is based on cognitive psychology, is selected for further discussion, particularly because it illustrates how affect can be incorporated into cognitive studies of mathematics learning and teaching. The chapter then presents a framework for research on affect that reorganizes the literature into three major areas: beliefs, attitudes, and emotions. Next, research on a number of topics from the affective domain is summarized, and linked to the proposed framework. Finally, the chapter explores how qualitative as well as quantitative research methods can be used in research on affect, and discusses the implications of the chapter for future research.

TERMINOLOGY AND GENERAL BACKGROUND

For the purposes of this discussion the affective domain refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition. H. A. Simon (1982), in discussing the terminology used to describe the affective domain, suggests that we use affect as a more general term; other terms (for example, beliefs, attitudes, and emotions) will be used in this chapter as more specific descriptors of subsets of the affective domain. In the context of mathematics education, feelings and moods like anxiety, confidence, frustration, and satisfaction are all used to describe responses to mathematical tasks. Frequently these feelings are discussed in the literature as attitudes, although that term does not seem adequate to describe some of the more intense emotional reactions that occur in mathematics classrooms. For example, the "Aha!" experience in mathematical problem solving is usually recognized as a joyful event, generally of limited duration; such an event does not fit with the definitions of attitude used by most researchers.

As Hart (1989b) and H. A. Simon (1982) indicate, describing the affective domain is no easy task. Terms sometimes have different meanings in psychology than they do in mathematics education, and even within a given field, studies that use the same terminology are often not studying the same phenomenon. For example, Hart (1989b) notes that anxiety is sometimes described as fear, one of the more intense emotions, and in other studies as dislike or worry. Clarification of terminology for the affective domain remains a major task for researchers in both psychology and mathematics education.

Moreover, affect is generally more difficult to describe and measure than cognition. H. A. Simon (1982) notes that there may be different kinds of sadness or fear, distinctions and gradations that our ordinary language is ill-equipped to make in a dependable way. As Simon suggests, working on cognition seems relatively simple compared to the difficulties of dealing clearly with affect. As an example of these difficulties, one might consider the work on building taxonomies of educational objectives that took place over 30 years ago. The work on the cognitive domain (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956) has had a major impact on curriculum and evaluation. The taxonomy for the affective domain (Krathwohl, Bloom, & Masia, 1964) has not had such an impact, and even its authors acknowledged that they found the task to be exceedingly difficult. There are, as Tittle and her colleagues point out (Tittle, Hecht, & Moore, 1989), a number of reasons for the difference in impact of these two documents, but surely one of the reasons for the differences is the complexity and difficulty of dealing adequately with the affective domain.

There have been many reviews of the literature related to affective factors and mathematics education, including those by Aiken (1970, 1976), Kulm (1980), Reyes (1980, 1984), and Leder (1987). These reviews have generally focused on attitudes toward mathematics as their major concern, rather than trying to describe and analyze all components of the affective domain. Also, they have in general worked from within the traditional paradigm of educational research, with its emphasis on quantitative methods, paper-and-pencil tests, and the positivistic perspective of behaviorist or differential psychology. This discussion will attempt to broaden the view of both the theories and the methods that might be useful in the study of affective factors in mathematics education.

PSYCHOLOGICAL THEORIES AND AFFECT

Changes in psychological theories have had a major impact on how affect is treated in mathematics education research. Frequently researchers have treated affect as an avoidable complication of modest significance; students have been viewed in rather mechanistic terms. The researcher's model of the student has a major impact on how the research is conducted, particularly in terms of the affective domain. If we believe that the learner is someone who only receives knowledge rather than someone who is actively involved in constructing knowledge, our research program could be entirely different in terms of both the affective and the cognitive domain.

The influence of behaviorism on educational psychology in this century has been an important factor in the neglect of the affective domain. Skinner (1953), for example, described the emotions as examples of imaginary constructs that were commonly used as causes of behavior. As Mandler (1984, 1985) indicates, the behaviorists have generally been unwilling to look closely at the underlying processes that are related to affective responses, particularly if data on those processes are gathered through introspection or verbal reports (Ericsson & Simon, 1980). As a result, behaviorism has much to say about stimulus and response in learning, but it has little interest in the influence of affective factors on learners.

In more recent times, experts in cognitive science and artificial intelligence have taken a serious interest in the study of mathematics learning (Schoenfeld, 1987a). However, these researchers have also tended to exclude affective factors from their considerations. As Norman (1981) has pointed out, the researcher's task would be much simpler if the emotions were superfluous; the desire to avoid complexity has been a major reason for the lack of attention to affective issues in cognitive science (Gardner, 1985). However, Norman's (1981) recommendation that cognitive science needed to focus on more than just "pure" cognition has had an impact, and current research on cognitive issues pays increased attention to affective and cultural factors.

In contrast to the behaviorists and some advocates of cognitive science, researchers in differential psychology and social psychology have given substantial attention to the notion of affective issues, especially to the study of attitudes. This work is characterized by its emphasis on definitions of terms, its preoccupation with measurement issues, and its reliance on questionnaires and quantitative methods. This approach can be characterized as the traditional paradigm for research on affect. Books by Ajzen and Fishbein (1980) and Rajecki (1982) provide extensive summaries of this work in general psychology. Research on the application of these ideas to mathematics education has been reviewed by Aiken (1970, 1976), Kulm (1980), Reyes (1984), and Leder (1987), among others. There has been considerable dissatisfaction with this traditional paradigm, both in psychology (Abelson, 1976; Berscheid, 1982; Mandler, 1972, 1989) and in mathematics education (Kulm, 1980; McLeod, 1985, 1988), particularly because of the lack of a strong theoretical foundation for the work. Nevertheless, most of what we know about affective factors in mathematics education comes from work within this traditional paradigm (Meyer & Fennema, 1988; Reyes, 1984). Fennema (1989) provides a spirited defense of the methods and contributions of this quantitative approach, noting that the traditional paradigm of differential psychology, rigorously applied, has produced a substantial amount of knowledge about affect and mathematics education. Moreover, this knowledge has been particularly useful in attacking problems of gender-related differences in participation in mathematics (Fennema, 1989; Fennema & Leder, 1990).

Although traditional quantitative approaches provide substantial information on some issues, there are many other areas (for example, emotional responses to mathematical problem solving) that are not susceptible to this approach. There are also a number of topics (for example, anxiety) where the research is confusing and contradictory. As a result, it seems useful to try to develop a new paradigm for research on affect that would be more comprehensive and more closely integrated with current research on cognition. There are always dangers in following a fad, and the new emphasis on cognitive theories in psychology does show signs of narrow-mindedness and an easy enthusiasm for new terminology rather than critical thinking. And as Messick (1987) suggests, the "hypercognitization" of affect by cognitive psychologists could result in the omission of important issues that should be central to our research agenda, rather than leading to progress in the field. In particular, there is no need to forget what has been learned in the past as we examine new approaches to old problems. Nevertheless, new approaches can lead to new progress in research, and new paradigms can lead to helpful reconceptualizations.

An alternative paradigm for research on affect has grown out of the work of developmental psychology and the rising influence of cognitive psychology in the recent past. This new paradigm for research on affect in mathematics learning can be characterized by its emphasis on theoretical issues, its interest in qualitative methods, its use of interviews and think-aloud protocols, and its attention to beliefs and emotions as well as attitudes. For examples of this kind of work in psychology, see Mandler (1984), Kagan (1978), and Ortony, Clore, and Collins (1988). Snow and Farr (1987) and Case, Hayward, Lewis, and Hurst (1988) present analyses of similar work with more direct connections to education, and Bassarear (1989), Goldin (1988), and McLeod and Adams (1989) present some applications and extensions of these ideas in mathematics education. The elaboration of both the traditional and alternative paradigms as they apply to the affective domain will be a continuing theme throughout this chapter.

COGNITIVE APPROACHES TO RESEARCH ON AFFECT

The emergence of affect as an important part of cognitive theories has been documented recently by Snow and Farr (1987). The beginning of these attempts to incorporate affective factors into cognitive theories can be traced back at least to the work of Schacter and Singer (1962) and H. A. Simon (1967); however, the leading theorists in this area now include cognitive psychologists like Lazarus (1982, 1984) and Mandler (1975, 1984). Overviews of the field (for example, Scherer & Ekman, 1984; Strongman, 1978) give substantial emphasis to the new influence of cognitive psychologists on this area. Although there are several cognitive theorists who are having a substantial impact on the study of affective issues broadly defined (including Beck & Emery, 1985; Bower, 1981; Meichenbaum, 1977; Ortony et al., 1988; Sheffler, 1977; L. R. Simon, 1986; Zajonc, 1984), the one who has done the most to apply his ideas to problems in mathematics education is Mandler (1989).

Mandler's general theory is presented in some detail in his 1984 book, and he has recently described his view of how the theory can be applied to the teaching and learning of mathematical problem solving (Mandler, 1989). At the risk of oversimplification, only a brief summary of his theory will be presented here. Most of what is presented appears to be compatible with other theorists who come from a cognitive point of view (for example, H. A. Simon, 1982).

Mandler's view is that most affective factors arise out of the emotional responses to the interruption of plans or planned behavior. In Mandler's terms, plans arise from the activation of a schema. The schema produces an action sequence, and if the anticipated sequence of actions cannot be completed as planned, the blockage or discrepancy is followed by a physiological response. This physiological arousal is typically felt as an increase in heartbeat or in muscle tension. The arousal serves as the mechanism for redirecting the individual's attention, and has obvious survival value which presumably may have played some role in evolutionary development. At the same time the arousal occurs, the individual attempts to evaluate the meaning of this unexpected or otherwise troublesome blockage. The evaluation of the interruption might classify it in one of several ways: a pleasant surprise, an unpleasant irritation, or perhaps a major catastrophe. The cognitive evaluation of the interruption provides the meaning to the arousal.

There are several important parts to the analysis of the meaning of the interruptions. First, the meaning comes out of the cognitive interpretation of the arousal. This meaning will be dependent on what the individual knows or assumes to be true. In other words, the individual's knowledge and beliefs play a significant role in the interpretation of the interruption. The role of the culture that shapes these beliefs would seem to be particularly important.

Second, the arousal that leads to the emotion is generally of limited duration. Normal individuals adjust to the unexpected event, interpret it in the context in which it occurs, and try to find some other way to carry out their plan and achieve their goal. The emotion may be intense, but it is generally transitory in normal individuals, at least initially.

Third, repeated interruptions in the same context normally result in emotions that become less intense. The individual will reduce the demand on cognitive processing by responding more and more automatically, and with less and less intensity. The responses in this situation become more stable and predictable and begin to resemble the kinds of attitudes that have been the emphasis of past research on affect in mathematics education.

To help clarify the situation, consider the affective responses of a sixth-grade student to a typical story problem. Suppose that the student believes that story problems should make sense and should have a reasonable answer that can be obtained in a minute or two. Suppose also that the student has had some success in other areas of mathematics. If the student is unable to obtain a satisfactory answer in a reasonable time, the failure to solve the problem (an interruption of the plan) is likely to lead to some arousal. The interpretation of this arousal is likely to be negative and is often reported as frustration by students who are able to verbalize their feelings. If the students are able to overcome the blockage and find a solution to the problem, they may report positive reactions to the experience. If negative reactions to story problems occur repeatedly, the response to story problems will eventually become automatic and stable. In this situation the student would have developed a negative attitude toward story problems.

In summary, there appear to be at least three major facets of the affective experience of mathematics students that are worthy of further study. First, students hold certain beliefs about mathematics and about themselves that play an important role in the development of their affective responses to mathematical situations. Second, since interruptions and blockages are an inevitable part of the learning of mathematics, students will experience both positive and negative emotions as they learn mathematics; these emotions are likely to be more noticeable when the tasks are novel. Third, students will develop positive or negative attitudes toward mathematics (or parts of the mathematics curriculum) as they encounter the same or similar mathematical situations repeatedly. These three aspects of affective experience correspond to three areas of research in mathematics education which we will now examine.

BELIEFS, ATTITUDES, AND EMOTIONS IN MATHEMATICS LEARNING: RECONCEPTUALIZING THE AFFECTIVE DOMAIN

Snow and Farr (1987), in their discussion of affect and cognition, point out that new research on affect must find ways to come to terms with the cognitive revolution in psychology. In particular, any reconceptualization of the affective domain should attempt to be compatible with cognitive-processing models of the learner. In this context, the work of Mandler (1984) should provide a useful general guide. The theoretical analyses of Mandler (1984) and the practical analyses of mathematics classrooms suggest that beliefs, attitudes, and emotions should be important factors in research on the affective domain in mathematics education. Table 23.1 provides a brief outline of these major constructs.

Beliefs, attitudes, and emotions are used to describe a wide range of affective responses to mathematics. These terms vary in the stability of the affective responses that they represent; beliefs and attitudes are generally stable, but emotions may change rapidly. They also vary in the level of intensity of the affects that they describe, increasing in intensity from "cold" beliefs about mathematics to "cool" attitudes related to liking or disliking mathematics to "hot" emotional reactions to the frustrations of solving nonroutine problems. Beliefs, attitudes, and emotions also differ in the degree to which cognition plays a role in the response, and in the time that they take to develop.

TABLE 23–1. The Affective Domain in Mathematics Education

Category	Examples
Beliefs	
About mathematics	Mathematics is based on rules
About self	I am able to solve problems
About mathematics teaching	Teaching is telling
About the social context	Learning is competitive
Attitudes	Dislike of geometric proof
	Enjoyment of problem solving
	Preference for discovery learning
Emotions	Joy (or frustration) in solving nonroutine problems
	Aesthetic responses to mathematics

For example, beliefs are largely cognitive in nature, and are developed over a relatively long period of time. Emotions, on the other hand, may involve little cognitive appraisal and may appear and disappear rather quickly, as when the frustration of trying to solve a hard problem is followed by the joy of finding a solution. Therefore, we can think of beliefs, attitudes, and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability. A review of some of the relevant literature provides support for the importance of these three constructs.

Beliefs

Research on beliefs in mathematics education has become an important thread linking a number of studies of both teachers and students. Thompson's (1984) work on teacher conceptions of mathematics and Schoenfeld's (1985) studies of the belief systems of problem solvers are important examples. Data have typically been gathered through observations of students and teachers, as well as through questionnaires and interviews. Researchers have generally not used a consistent framework; instead, the data have been organized in rather different ways in different studies, with each researcher trying to explain the influence of beliefs in each particular context. People who come from research on problem solving (for example, Schoenfeld, 1985; Silver, 1985) have tended to emphasize the role of student beliefs about mathematics as a discipline. For example, many students believe that problems can be solved quickly or not at all, that only geniuses can be creative in mathematics, and that proof just confirms the obvious (Schoenfeld, 1985). Other researchers, particularly those who investigate gender-related differences (for example, Fennema & Peterson, 1985) have emphasized students' beliefs about themselves as learners. In this category we find beliefs about students' ability to do mathematics or about the importance of effort to success in learning mathematics.

Although each of these approaches has contributed to our knowledge of how beliefs are important to mathematics learning and teaching, little emphasis has been given to providing an overall structure for the study of beliefs in mathematics education. There are, however, some examples of broader approaches to research on beliefs. Lester, Garofalo, and Kroll (1989) describe beliefs in terms of the subjective knowledge of students regarding mathematics, self, and problem-solving activities. Underhill (1988) discusses beliefs in terms of several dimensions, including whether mathematics is primarily rule-oriented or concept-oriented, and whether mathematics is learned by having knowledge transmitted to students or constructed by students. Fennema and Peterson (1985) suggest connections between beliefs and autonomous learning behavior, with the subsequent impact on higher-order thinking in mathematics, particularly in terms of gender-related differences in mathematics achievement.

There are a variety of ways to organize research on beliefs. Rokeach (1968), for example, organizes beliefs along a dimension of centrality to the individual. Those beliefs that are most central are those on which there is complete consensus; beliefs about which there is some disagreement would be less central. Beliefs that are imposed on individuals by authority figures would be still less central. One alternative for mathematics education would be to develop a taxonomy of beliefs like Rokeach provides for more general settings.

D'Andrade (1981) suggests that beliefs develop gradually through a process much like "guided discovery" where children respond to the situations in which they find themselves by developing beliefs that are consistent with their experience. Certainly cultural factors play an important role in this process of developing beliefs. In mathematics education most researchers seem to assume that the development of beliefs about mathematics is heavily influenced by the cultural setting of the classroom (Schoenfeld, 1989).

In this chapter beliefs related to mathematics education are discussed in terms of the experiences of students and teachers. Students' beliefs are categorized in terms of the object of the belief: beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the contexts in which mathematics education occurs. The discussion of these categories will also include comments on teachers' beliefs about mathematics and instruction.

Beliefs about Mathematics. Research on students' beliefs about mathematics has received considerable attention over recent years. The National Assessment of Educational Progress has included items related to beliefs about mathematics for some time. The most recent assessment (Brown et al., 1988) indicates that students believe that mathematics is important, difficult, and based on rules. These beliefs about mathematics, although not emotional in themselves, certainly would tend to generate more intense reactions to mathematical tasks than beliefs that mathematics is unimportant, easy, and based on logical reasoning. Some researchers may not see any need to include such beliefs as part of the affective domain, and it is true that these beliefs are mainly cognitive in nature. However, the role of beliefs is central in the development of attitudinal and emotional responses to mathematics, and thus beliefs are included in this description of the affective domain.

A variety of major evaluation studies have dealt with beliefs about mathematics. Dossey et al. (1988) report that students in the United States (grades 3, 7, and 11) believe that mathematics is useful, but involves mainly memorizing and following rules. McKnight et al., (1987) found similar results in the U.S. data from the Second International Mathematics Study (grades 8 and 12).

Research on beliefs has been highlighted by the results of research on problem solving. As Schoenfeld (1985) and Silver (1985) have pointed out, students' beliefs about mathematics may weaken their ability to solve nonroutine problems. If students believe that mathematical problems should always be completed in five minutes or less, they may be unwilling to persist in trying to solve problems that may take substantially longer for most students. Nevertheless, this kind of belief has been generated out of the typical classroom context in which students encounter mathematics. There is nothing wrong with the students' mechanism for developing beliefs about mathematics (D'Andrade, 1981; Schoenfeld, 1988); what needs to be changed is the curriculum (and beyond that, the culture) that encourages such beliefs.

Another important area of research on beliefs comes mainly out of the work on gender differences in mathematics education. Most of the data has come from studies that used the Fennema and Sherman (1976) scales, especially the scale on the perceived usefulness of mathematics. Fennema (1989), in summarizing this research, notes that males in general report higher perceived usefulness than females. Other scales (for example, mathematics as a male domain) also deal with beliefs about mathematics. These kinds of beliefs are important both for gender differences in mathematics achievement and for the related differences between females and males in affective responses to mathematics (see Leder, Chapter 24 on gender, this volume).

Students' beliefs about mathematics do change as students grow older. Kouba and McDonald (1987) report that students in the elementary school grades tend to think that mathematics cannot be easy. In the view of these students, if it is easy, it is not mathematics. As the children grow older, and as the material that they have learned (e.g., counting) seems increasingly elementary, they change their beliefs about what mathematics is to accommodate the notion that mathematics must be hard and unfamiliar. These students also see mathematics primarily as doing something, usually something algorithmic; the connections to the typical elementary school mathematics curriculum are relatively direct.

Stodolsky (1985) describes how beliefs about mathematics influence how students (and teachers) perform in elementary school mathematics classrooms, especially as compared to social studies classrooms. In social studies, students are much more likely to work in groups, to develop their research skills, and, in general, to work on tasks that are compatible with the development of higher-order thinking skills. In contrast, in mathematics classrooms students spend a lot of time alone doing "seatwork." Other writers have noted how students view mathematics as a skill-oriented subject, and how such limited views of the discipline lead to anxiety about mathematics (Greenwood, 1984) and more generally interfere with higher-order thinking in mathematics (Garofalo, 1989).

Most research (not including evaluation projects) on beliefs about mathematics as a discipline have relied primarily on classroom observations and interviews with students. Kloosterman and Stage (1989), however, have developed a questionnaire to measure students' beliefs about mathematics and about themselves. Data gathered through this instrument (and others) should provide a useful complement to data from qualitative studies that use interviews and observations.

Beliefs about Self. Research on self-concept, confidence, and causal attributions related to mathematics tends to focus on beliefs about the self. These beliefs about self are closely related to notions of metacognition, self-regulation, and self-awareness (Corno & Rohrkemper, 1985). Some aspects of beliefs about self have been researched quite thoroughly, especially in the area of research on gender differences. Other aspects are only beginning to be investigated. For example, a substantial amount of data has been gathered on differences between males and females in levels of confidence in doing mathematics, but very little on how children develop a belief in themselves as autonomous learners (Fennema & Peterson, 1985).

Major evaluation studies provide useful background data on some beliefs about self. National assessment data from the United States (Dossey et al., 1988) asked children in grades 3, 7, and 11 if they were good at doing mathematics. The percentage of students who responded positively dropped from 65% in grade 3 to 53% in grade 11, providing at least a rough measure of what happens to levels of confidence as students progress through school.

Research on self-concept and confidence in learning mathematics indicates that there are substantial differences between males and females in these areas. Reyes (1984) and Meyer and Fennema (1988) summarize the relevant literature. In general, males tend to be more confident than females, even when females may have better reasons, based on their performance, to feel confident. The interaction of confidence and mathematical performance, especially in the area of nonroutine problem solving, is an important research topic. It seems likely that success in problem solving will engender a belief in one's capacity for doing mathematical problems, leading to an increase in confidence, which correlates positively with achievement in mathematics (Fennema, 1989).

The notion of confidence in oneself as a learner of mathematics has also been investigated under the rubric of self-concept (Shavelson & Bolus, 1982). Literature on the notion of self-concept in relation to mathematics learning has been reviewed by Reyes (1984). The most important implications of research on self-concept will probably come from its connections to metacognition, self-regulated learning, and intrinsic motivation to learn (Corno & Rohrkemper, 1985; Kilpatrick, 1985; Schoenfeld, 1987b). Such research becomes quite complicated to sort out since the general notion of self-concept and the more specific notion of mathematical self-concept appear to be related but distinct (Marsh, 1986; Reyes, 1984).

Another set of beliefs about self has been investigated quite thoroughly in the context of causal attributions for success and failure. Although there are several antecedents of this work and many different applications of the ideas, the central themes are well explicated in a recent reformulation of the theory by Weiner (1986). The three main dimensions of the theory deal with the locus (internal or external), the stability (for example, ability versus effort), and the controllability of the causal agent. For example, a student who fails to solve a mathematics problem could say that the problem was too hard—a cause that is external, stable, and uncontrollable by the student. A student who succeeds in solving a problem might attribute that success to effort—a cause that is internal, unstable, and controllable.

The nature of the attributions of female and male students has been an important theme in recent research in mathematics education, and the results of this research provide some of the most consistent data in the literature on the affective domain. For example, males are more likely than females to attribute their success in mathematics to ability, and females are more likely than males to attribute their failures to lack of ability. In addition, females tend to attribute their successes to extra effort more than males do, and males tend to attribute their failures to lack of effort more than females do. The differences in participation in mathematics-related careers appear to reflect these gender differences in attributions (Fennema, 1989; Fennema &

Peterson, 1985; Meyer & Fennema, 1988; Reyes, 1984; Wolleat, Pedro, Becker, & Fennema, 1980).

Larger issues that are related to beliefs about self include much of the literature on motivational issues (Dweck, 1986). Much of this research is not done in a mathematical context, although there are some studies that relate the motivation and the confidence of mathematics learners (e.g., Kloosterman, 1988). The general literature, however, is full of overlapping concepts like self-efficacy (Bandura, 1977), learned helplessness (Dweck, 1986), and motivation (Covington, 1983, and many other authors). Later in the chapter the literature on each of these areas will be discussed in more detail.

Beliefs about Mathematics Teaching. So far our discussion has concentrated on students' beliefs about mathematics and about themselves. But there is a corresponding set of beliefs that students and teachers hold about mathematics teaching that are also important to the study of affect in mathematics education. There have been a number of important studies of teachers' beliefs about mathematics and mathematics teaching (for example, Barr, 1988; Grouws & Cramer, 1989; Marcilo, 1987; Peterson, Fennema, Carpenter, & Loef, 1989; Sowder, 1989; Thompson, 1984), and current recommendations for a research agenda on mathematics teaching suggest that more work be done in this area (Cooney, Grouws, & Jones, 1988). There are also more general studies of teachers' beliefs (Wittrock, 1986), including teachers' beliefs about instruction (Eisenhart, Shrum, Harding, & Cuthbert, 1988; Peterson & Barger, 1985), as well as teachers' attributions (Prawat, Byers, & Anderson, 1983); these latter investigations are more directly related to affective factors in classroom instruction. However, most of the research along these lines does not deal specifically with mathematics teaching, and we have little information on students' beliefs about mathematics instruction.

Beliefs about the Social Context. Recent research on mathematics learning has given increased attention to the social context of instruction (Cole & Griffin, 1987) and more generally to cultural issues in mathematics education (Bishop, 1988; Lave, 1988; Newman, Griffin, & Cole, 1989; Saxe, 1990). Students' beliefs about the social context appear to be another area that is closely related to affective concerns. For example, Cobb, Yackel, and Wood (1989) found that explicit teaching of social norms in a primary classroom was directly related to the kinds of affective reactions that the students expressed. Similarly, at the secondary level, Grouws and Cramer (1989) found that the classrooms of effective teachers of mathematical problem solving were characterized by a supportive classroom environment where social norms encouraged students to be enthusiastic and to enjoy mathematical problem solving. From a broader perspective, the social context provided by the school and the home can also have an effect on students' beliefs. Parsons, Adler, and Kaczala (1982), in their study of parental influences on students' attitudes and beliefs, noted that the affective reactions of students (particularly females) often reflect social norms as expressed by the parents. Research in crosscultural settings also points out the influence of the broader social context (Stevenson, 1987; Stevenson, Lee, & Stigler, 1986; Stigler & Mao, 1985).

In summary, research on beliefs and their influence on students and teachers has been an important theme in investigations of learning and instruction in mathematics. Some of this research is directly connected with affective issues (for example, confidence), but much of it is not. Since beliefs provide an important part of the context within which emotional responses to mathematics develop, we need to establish stronger connections between research on beliefs and research on emotions in the context of mathematics classrooms. More generally, research in mathematics education needs to develop a more coherent framework for research on beliefs, their relationship to attitudes and emotions, and their interaction with cognitive factors in mathematics learning and instruction.

Attitudes

Research on attitudes towards mathematics has a relatively long history. For recent reviews and analyses, see Haladyna, Shaughnessy, and Shaughnessy (1983), Kulm (1980), Leder (1987), and Reyes (1984). Many of these reviews use attitudes as a general term that includes beliefs about mathematics and about self. In this paper attitude refers to affective responses that involve positive or negative feelings of moderate intensity and reasonable stability. Examples of attitudes toward mathematics would include liking geometry, disliking story problems, being curious about topology, and being bored by algebra. As Leder (1987) and others have noted, attitudes toward mathematics are not a unidimensional factor; there are many different kinds of mathematics, as well as a variety of feelings about each type of mathematics.

Attitudes toward mathematics appear to develop in two different ways. One was referred to earlier—attitudes may result from the automatizing of a repeated emotional reaction to mathematics. For example, if a student has repeated negative experiences with geometric proofs, the emotional impact will usually lessen in intensity over time. Eventually the emotional reaction to geometric proof will become more automatic, there will be less physiological arousal, and the response will become a stable one that can probably be measured through use of a questionnaire. A second source of attitudes is the assignment of an already existing attitude to a new but related task. A student who has a negative attitude toward geometric proof may attach that same attitude to proofs in algebra. To phrase this process in cognitive terminology, the attitude from one schema is attached to a second related schema. Abelson (1976) and Marshall (1989) provide a more detailed discussion of theoretical issues related to the formation of attitudes.

There have been a large number of studies of attitudes toward mathematics over the years; the topic seems to be particularly popular among dissertation writers. The review articles listed at the beginning of this section include more extensive descriptions of these studies. Only a few selected examples of relevant research will be mentioned in this section.

Most major evaluation studies provide data on attitudes toward mathematics. National assessment data (Dossey et al., 1988) illustrate the major results: there is a positive correlation between attitude and achievement at all three grade levels assessed (grades 3, 7, and 11), but the percentage of students who say they enjoy mathematics declines from 60% in grade 3 to 50% in grade 11. Similar results appear in the Second International Mathematics Study (McKnight et al., 1987) and in studies in other countries (Foxman et al., 1982; Leder, 1987; McLean, 1982).

Research suggests that neither attitude nor achievement is dependent on the other; rather, they interact with each other in complex and unpredictable ways. For example, data from the Second International Mathematics Study indicate that Japanese students had a greater dislike for mathematics than students in other countries, even though Japanese achievement was very high (McKnight et al., 1987). Recent studies exhibit a growing appreciation for the complexity of the affective domain; the original attempts to measure attitude toward mathematics seem exceptionally primitive, given our current knowledge and experience in the area (Leder, 1987).

Some studies have assessed attitude toward various subdomains that are part of or related to mathematics. For example, McKnight et al. (1987) report data on how much students like to use calculators, as well as how they feel about checking answers and memorizing rules in mathematics. As expected, students generally like to use calculators, but dislike memorizing; checking answers falls somewhere in between. Corbitt (1984) interviewed students in the eighth grade regarding how much they liked 15 different mathematical topics. The results showed some differences from students' liking of mathematics in general; for example, students reported being bored with the typical review of computation in the eighth grade.

Research on attitude provides a broad and rather indistinct picture of a limited range of affective responses to mathematics. Researchers in the area generally limit themselves to various kinds of questionnaires (Henerson, Morris, & Fitz-Gibbon, 1978), but there are useful examples of studies that use interview data as well (Corbitt, 1984; Marshall, 1989). As the research methodology becomes more flexible and more studies use multiple research methods, including interviews rather than just questionnaires, we can expect research on attitude to make new contributions to the field of mathematics education.

In the literature it is difficult to separate research on attitudes from research on beliefs. If attitudes develop out of emotional responses, as we hypothesize, it should be possible to analyze attitudes in terms of the corresponding emotional responses. For example, if students get frustrated with computer-assisted instruction, they may develop a negative attitude toward computers; if students find it emotionally satisfying to work with their friends on mathematical problems, they may develop a positive attitude toward small-group instruction in mathematics. Further progress in research on attitudes should profit from a more careful analysis of emotional responses in mathematics education. Although Mandler's (1984) theory does not yet include any major attempt to categorize the various emotional responses to mathematics, the efforts by Ortony et al. (1988) to build a general classification system for the emotions may eventually provide some help in this area.

Emotions

The emotional reactions of students have not been major factors in research on affect in mathematics education. This lack of attention to emotion is probably due in part to the fact that research on affective issues has mostly looked for factors that are stable and can be measured by questionnaires. To phrase this observation in another way, most research in the past has looked at products, not at processes, and at beliefs and attitudes rather than emotions. However, there have been a number of studies that have looked at the processes involved in learning mathematics, and these studies have sometimes paid attention to the emotions. Certainly the current trend toward detailed studies of a small number of subjects allows the researcher to be aware of the relationship between the emotions and cognitive processing; such an awareness was not possible in traditional large-scale studies of affective factors. In this section, we review briefly a few of these studies.

One of the early studies of problem-solving processes was conducted by Bloom and Broder (1950). In this work they noted how students' engagement in the task led them into periods of tension and frustration, especially when they felt that their attempts to reach a solution were blocked. Once the block had been overcome, the students would relax and report very positive emotions. This study was conducted before the current focus on cognition became common, and it is justifiably recognized as an early exemplar of research on cognitive processes. It also provides a useful model for integrating research on cognition and affect.

Reports of strong emotional reactions to mathematics do not appear in the research literature very often. An important exception is the work of Buxton (1981). His research deals with adults who report their emotional reaction to mathematical tasks as panic. Their reports of panic are accompanied by a high degree of physiological arousal; this arousal is so difficult to control that they find it disrupts their ability to concentrate on the task. The emotional reaction is described as fear, anxiety, and embarrassment, as well as panic. Buxton interprets these data in terms of Skemp's (1979) views of the affective domain, and suggests a number of strategies to change students' beliefs in order to reduce the intensity of the emotional response.

A number of other researchers have investigated factors that are related to the influence of emotions on cognitive processes in mathematics. Wagner, Rachlin, and Jensen (1984) report how algebra students who were stuck on a problem would sometimes get upset and grope wildly for any response that would get them past the blockage, no matter how irrational. On a more positive note, Brown and Walter (1983) discuss how making conjectures can be a source of great joy to mathematics students. In a similar way, Mason, Burton, and Stacey (1982) talk about the satisfaction of the "Aha!" experience in mathematical problem solving, and make suggestions about how students can be encouraged to savor and anticipate positive emotional experiences related to mathematics learning. Lawler (1981) also documents the positive responses that accompany that moment of insight when a child first sees the connections between two important ideas.

These observations on emotion and cognitive processing resulted from studies that were focused on cognitive rather than affective issues, and reports of students' emotional responses were frequently sidelights rather than highlights of the studies. However, there has been some research that has focused directly on the role of the emotions in mathematics learning.

McLeod, Metzger, and Craviotto (1989) report on the emotional reactions of expert and novice problem solvers, where the experts are research mathematicians and the novices are undergraduate students who are not mathematics majors. This study found that the emotional reactions to the frustrations and joys of solving problems are basically the same for each group. The experts, however, are better able to control their emotions than the novices. In another study that included emotional responses as an important component, Bassarear (1989) conducted extensive interviews with two college students over the course of a semester, observing the interaction of their beliefs, attitudes, and emotions with their performance in mathematics class. The data from these interviews suggest how emotional responses can play a significant role in students' learning of mathematics.

Although comments about emotion do appear in the research literature from time to time, it is fairly unusual for research on mathematics education to include measures of physiological changes that accompany the emotions. However, in a recent study Gentry and Underhill (1987) gathered data on muscle tension along with paper-and-pencil measures of anxiety toward mathematics. As one might expect, there was little correlation between the two measures, suggesting that traditional measures of anxiety may be quite different from the emotional responses that influence students in the classroom. Similar results were obtained by Dew, Galassi, and Galassi (1984), who compared physiological measures of heart rate and skin conductivity with data from paper-and-pencil measures of mathematics anxiety.

In summary, research on emotional responses to mathematics has been conducted, but it has never played a prominent part in research on the affective domain in mathematics. A major problem has been the lack of a theoretical framework within which to interpret the role of the emotions in the learning of mathematics. Mandler's (1984) theory should help to provide such a framework. The recent appearance of the volume by Harré (1986), with its constructivist approach to affective issues, should also provide a suitable theoretical framework for researchers who take a strong constructivist perspective. Similarly, the work of Case et al. (1988), which integrates neo-Piagetian thinking about cognition with a serious attempt to explain the development of emotion, will be of particular interest to researchers who take the perspective of developmental psychology. Although Case and his colleagues have so far concentrated on younger children and the development of jealousy and other emotions that are not specifically related to mathematics education, there is reason to expect that these ideas can be extended to older children and to a mathematical context. In conclusion, the available data from a variety of sources and a variety of theoretical perspectives suggest that careful observation of students, along with detailed interviews, should help researchers in their analysis of the emotional states of mathematics learners (McLeod, 1988)

RELATED CONCEPTS FROM THE AFFECTIVE DOMAIN

The research on beliefs, attitudes, and emotions that was outlined in the previous sections provides a general overview of research on the affective domain. There are, however, a number of other research topics—one could refer to them as minitheories about parts of the affective domain—that have implications for mathematics education. The purpose of this section is to outline several of these areas, and to suggest how they could fit into the general rubric of beliefs, attitudes, and emotions as described above. Sometimes these topics have not been investigated in the context of mathematics education; nevertheless, they seem relevant to the field. We begin with those areas that have the most in common with traditional foci of the affective domain in mathematics education, like confidence and self-concept. Later we shall also deal with relatively new concepts from research in mathematics education, like metacognition and intuition, and suggest how those areas might be integrated more completely with research on affect.

Confidence

Confidence in learning mathematics has been studied at least since the days of the National Longitudinal Study of Mathematical Abilities (Crosswhite, 1972). Reyes (1984) provides an excellent review of the literature in this area. Although different studies have used varying methods to assess confidence, in general it is reasonable to think of confidence as a belief about one's competence in mathematics.

Confidence correlates positively with achievement in mathematics, and the relationship is generally quite strong, with correlation coefficients of greater than 0.40 appearing in studies at the secondary school level (Reyes, 1984). Confidence is also related to elective enrollment in mathematics courses, and has been used frequently in investigations of gender differences in mathematics (Fennema, 1989; Linn & Hyde, 1989). Although some data have suggested that confidence is also related to patterns of classroom interaction between students and teachers, more recent work in this area indicates that the differences are not as consistent as expected (Hart, 1989a).

Instruments to measure confidence vary greatly in complexity. In the National Assessment data, students were simply asked if they were good at mathematics (Dossey et al., 1988); on the other hand, Fennema and Sherman (1976) developed a scale to assess confidence, following approved test development procedures for validating items, assessing reliability, and so forth. Other researchers have fallen in between these two ends of the spectrum; many have developed more general scales, including items related to confidence, but have not separated confidence from other kinds of affective factors.

Some recent studies of confidence continue to produce patterns of results established in earlier research efforts. Kloosterman (1988) investigated the correlation between measures of confidence and other affective variables, like motivation and causal attributions, with grade 7 students. As predicted, these kinds of variables are correlated; however, the usefulness of these kinds of conclusions for theoretical development and for practical applications appears to be limited (Mandler, 1972). Mura (1987) investigated differences in level of confidence, career plans, and gender with college students. In line with other studies, women tended to be less confident than men, and fewer women planned to take advanced mathematics than men. In studies with students at the elementary (Newman & Wick, 1987) and secondary school levels (Newman, 1984), data indicated that boys were more confident than girls in estimation tasks at the secondary school level; however, at the elementary school level, the levels of confidence of boys and girls on these estimation tasks did not differ significantly.

Students' beliefs about their competence in mathematics are an important affective factor in mathematics classrooms. Future research on confidence needs to take into account the complete mosaic of mathematical beliefs, rather than just studying one such belief in isolation. For example, if students feel confident about doing mathematics and believe that mathematics is nothing more than doing computational exercises, their beliefs about mathematics as a discipline provide a different perspective regarding their statements of confidence. Making sense of confidence as a variable in mathematics education will require a more complete picture of the affective domain than is presently found in most studies.

Self-concept

Self-concept can be thought of as a generalization of confidence in learning mathematics (Reyes, 1984). Substantial effort has gone into research on both general self-concept and academic self-concept, and the relation of each to academic achievement (Byrne, 1984). Work by Shavelson and Bolus (1982) is representative of the area. Shavelson and his colleagues assume that students develop a general self-concept, which can be analyzed into various components, including an academic self-concept. The relationships between general and academic self-concepts, and between academic self-concept and self-concept in specific subject-matter areas like mathematics, are still being debated; however, Byrne (1984) does report some support for a hierarchical system through which more specific self-concepts are combined to make up the general self-concept. More recently, Marsh (1986) has gathered data on self-concept by discipline, and found that mathematics self-concept and verbal self-concept were not correlated, but that mathematics self-concept was correlated with achievement in mathematics, just as verbal self-concept was correlated with verbal achievement.

Since the relationship of self-concept to achievement is consistently positive, continued research on such beliefs about self seems appropriate. Since studies of self-concept have generally used only quantitative methods, there is much work that could be done in qualitative studies to further our understanding of how differences in self-concept are related to differences in mathematical performance. For example, students who have a poor self-concept in mathematics may need help in changing their beliefs about mathematics as a discipline, as well as in seeing themselves as competent learners of the subject.

Self-efficacy

A variation of self-concept is the notion of self-efficacy (Bandura, 1977). As Schunk (1984) points out, notions of self-efficacy are related to decisions about which activities students choose to participate in, how much effort they expend, and how long they persist in those activities. Norwich (1987) investigated the relationship between self-efficacy and performance in mathematics among primary school students in England, as well as the relationship between measures of self-efficacy and self-concept. The data suggest that neither self-efficacy nor self-concept were particularly significant predictors of achievement. In a study of mathematics students at the college level, Hackett and Betz (1989) found that self-efficacy was a good predictor of students' choice of major and that it also correlated positively with achievement and attitudes toward mathematics. In particular, Hackett and Betz (1989) claimed that self-efficacy was a better predictor of college major than measures of achievement. Although the data on self-efficacy are interesting, it is difficult to sort out why self-efficacy as a construct should be much more successful as a predictor than mathematical self-concept or confidence in learning mathematics. Again, a broader and more integrated view of various beliefs about self may help to make research of this type more meaningful to the field.

Mathematics Anxiety

The study of mathematics anxiety has probably received more attention than any other area that lies within the affective domain. Hembree (1990), in a meta-analysis based on 151 studies, confirmed that mathematics anxiety is related to poor performance in mathematics, and that a variety of treatments are effective in reducing mathematics anxiety and in improving performance. Treatments that involve systematic desensitization and relaxation training were found to be most effective. Efforts to change beliefs about mathematics were also of some help. From the data that are already available (Berebitsky, 1985; Gattuso & Lacasse, 1987; Hembree, 1990), it seems reasonable for researchers to propose relatively complete models of how beliefs, attitudes, and emotions are involved in the development of mathematics anxiety, and to develop treatment programs that deal with the issue in a more comprehensive way. Meanwhile, researchers are able to provide helpful suggestions to teachers (Brush, 1981), and some research has tested alternative instructional formats that may be more appropriate for students who report high levels of mathematics anxiety (Clute, 1984).

Although there has been considerable progress in investigating mathematics anxiety, the concepts underlying the research continue to be murky, and the terminology remains unclear. As Hart (1989b) points out, anxiety has sometimes been characterized as fear, a "hot" emotion, and sometimes as dislike, an attitude. Researchers have often failed to distinguish between Spielberger's notions of state and trait anxiety (Spielberger, Gonzalez, & Fletcher, 1979). The relationship of mathematics anxiety to performance in mathematics is sometimes difficult to demonstrate (Gliner, 1987; Llabre & Suarez, 1985; Mevarech & Ben-Artzi, 1987); studies that attempt to clarify the relationships between various measures of mathematics anxiety, test anxiety, and related concepts (Dew et al., 1984; Ferguson, 1986; Hendel, 1980; Richardson & Woolfolk, 1980; Rounds & Hendel, 1980) report only modest success. These kinds of studies continue to emphasize measurement issues rather than theory building. Conceptions of mathematics anxiety are often difficult to separate from test anxiety (Sarason, 1980, 1987) as it applies to mathematics; moreover, test anxiety appears to provide the main source of theoretical support for much of the research on mathematics anxiety.

Current research on mathematics anxiety, with its emphasis on statistical methodology and correlational analyses of related concepts, remains subject to the criticisms that Mandler (1972) made many years ago; significant correlations do not imply significant increases in our knowledge of the field, especially given the difficulties involved in building instruments for the affective domain and the lack of an adequate theoretical foundation for the work. Some studies have attempted to provide a stronger cognitive orientation for research on anxiety (Hunsley, 1987), to build a constructivist foundation for such work (Carter & Yackel, 1989), to investigate the influence of mathematics anxiety on the performance of elementary school teachers (Bush, 1989), and to separate more intense, emotional aspects of anxiety from the less intense, attitudinal facets (Wigfield & Meece, 1988). In spite of these significant efforts, this research area continues to make slow progress.

The difficulties involved in dealing with mathematics anxiety have led some researchers to propose alternative approaches that are based on Freudian psychology. Ginsburg and Asmussen (1988), for example, discuss the extreme anxieties that some people develop regarding mathematics, and argue that the techniques of depth psychology will be needed to treat such problems successfully. Nimier (1977), in an intriguing discussion of the impact of fears and defense mechanisms on mathematics students, provides a Freudian interpretation of certain patterns of behavior that are common in mathematics classrooms. In related work, Legault (1987) discusses how these Freudian interpretations of students' behavior have special implications for gender differences in mathematics education. Although studies of this type have not had a major impact on research in mathematics education, they may in the future yield important insights, especially regarding people who suffer from extremely negative reactions to mathematics (Buxton, 1981).

Causal Attributions

The work on causal attributions related to mathematics learning has been quite extensive. The theory, outlined earlier in this chapter, has a relatively strong theoretical foundation (Weiner, 1986). Again, as with research on confidence in learning mathematics, some of the most interesting results have dealt with gender-related differences. For example, males are more likely to attribute their success to ability than females, and females are more likely to attribute their failure to lack of ability than males. Several summaries of the research on causal attributions that is related to mathematics learning are available in the literature (Fennema, 1989; Meyer & Fennema, 1988; Reyes, 1984).

There are a number of recent studies of causal attributions among mathematics students. Kloosterman's (1988) work on relationships between attributions and confidence, discussed earlier, is a good example of the kind of research that has been done in this area. He found that students who were high in confidence were also likely to attribute success to ability and failure to effort. These interrelationships among concepts suggest new avenues for investigating relevant variables within the context of a well-developed theory. In other studies, Choroszy, Powers, Cool, and Douglas (1987) extended the work on causal attributions to community college students in American Samoa; Heckhausen (1987) analyzed various relationships between attributions and achievement; and Graham (1984) and Prawat et al. (1983) conducted research on teachers' attributions about student performance. None of these researchers provide any unexpected results, but they are all engaged in useful efforts to test and extend the theories of Weiner (1986).

Effort and Ability Attributions

Although Weiner (1986) appears to have the most complete theoretical perspective on issues related to attributions, considerable research has been done on effort and ability that is parallel to but not directly dependent upon Weiner's work. For example, Ames and Archer (1987) found that elementary school students who attributed success to effort were more likely to exhibit a mastery orientation, putting their emphasis on learning and understanding through hard work, meeting challenges, and making progress. Students who attributed success to ability were more likely to be interested in good grades than in understanding. In a review of the research on issues of effort and ability, Holloway (1988) compared data from Japan and the United States. Some of the major findings from this report are that effort is believed to be of primary importance in determining achievement in Japan, but ability is seen as the primary factor in the United States. Apparently, Japanese homes encourage task involvement in ways that promote effort attributions. Related work by Hess, Chang, and McDevitt (1987), Stevenson et al. (1986), and Stigler and Perry (1988) provides further support for the importance of cultural differences in effort and ability attributions.

Learned Helplessness

The psychological literature on learned helplessness is quite extensive, and in recent years the influence of this concept is being felt more directly in mathematics education research. Diener and Dweck (1978), in work with elementary school students, described a pattern of behavior called learned helplessness where students attributed failure to lack of ability. Such students tended to demonstrate a low level of persistence and to avoid challenges whenever possible (Dweck, 1986). The contrasting pattern, referred to as a mastery orientation, was characterized by students who made few attributions, but who concentrated on monitoring their performance. In general, mastery-oriented students saw intelligence as a growing collection of concepts and procedures that they were able to understand. Dweck and Bempechat (1983) link these two contrasting orientations (mastery versus learned helplessness) to students' beliefs about intelligence and the related attributions that the students make. Again, some of the most interesting investigations of these ideas appear in studies of gender-related differences (Parson, Meece, Adler, & Kaczala, 1982).

Motivation

Many of the studies that have been discussed in this paper have something to do with motivation. There are a great many ways to study motivation, and there has been a great deal written on the general topic (Ames & Ames, 1984; Bates, 1979; Boekaerts, 1988; Brophy, 1983; Corno & Rohrkemper, 1985; Covington, 1983; Covington & Omelich, 1985; Hatano & Inagaki, 1987; Maehr, 1984; Malone & Lepper, 1987; Morgan, 1984; Nicholls, 1984; Paris, Olson, & Stevenson, 1983; Pekrun, 1988; Stipek, 1984; Weinert, 1987). Although most of these authors do not deal with mathematics education directly, their work as a whole is certainly applicable to mathematics classrooms.

One of the difficulties with the work on motivation is the diffuse and disconnected nature of the field. In spite of the best efforts of leading researchers (Ames & Ames, 1984; Dweck, 1986; Snow & Farr, 1987), the field appears to be made up of often disconnected components dealing with such topics as achievement motivation, social motivation, extrinsic versus intrinsic motivation, fear of success, need for achievement, and so forth. According to Mandler (1989), part of the problem is that there is still no framework for research on motivation that fits comfortably into current research in cognitive psychology. Norman (1981), in his essay on cognitive science, suggested that motivation could be dealt with as a derived issue in cognitive science, where motivational factors could be explained through research on beliefs and emotions. Whether such an approach is appropriate or whether it would lead to any particular clarification in the field is yet to be determined. But clarity is obviously needed in the chaotic collection of research studies carried out under the general heading of motivation.

In summary, research areas like self-concept, causal attribution, and learned helplessness all help to complete our picture of the affective domain as it influences performance in learning and teaching. However, these research areas can be strengthened if they can be related to the complete realm of research on affect. In particular, researchers need to clarify the level of affective intensity that students are reporting. For example, if students' confidence is being assessed, then the research needs to distinguish as carefully as possible between beliefs about competence and feelings of inadequacy. Similarly, in research on anxiety, we need to distinguish between intense emotional responses (panic or fear) and other negative but less intense responses (dislike or worry). If the research in these areas can be related more directly to research on beliefs, attitudes, and emotions, the level of intensity of the affective response should be more clear, thus contributing to the overall understanding of how the affective domain is related to mathematics learning and teaching.

TOPICS RELATED TO THE COGNITIVE DOMAIN

The position of this paper—and of Mandler's (1984) theory—is that the affective and cognitive domains are intimately linked. Affective responses do not occur in the absence of cognitive evaluations, according to the theory. Nevertheless, some topics seem more closely connected to the cognitive domain than others. This section describes a number of research topics that are closely identified with research on cognition, even though they have a strong connection to the affective domain.

Autonomy

In their analysis of gender-related differences in mathematics education, Fennema and Peterson (1985) developed a model that linked performance on tasks that require higher-order thinking skills in mathematics to beliefs, social influences, and autonomous learning behaviors (ALBs). These ALBs include activities like independent thinking about a problem and willingness to persist in problem solving. Such ALBs are hypothesized to be a mediating link between students' knowledge and beliefs on the one hand and mathematical performance on the other (Fennema & Leder, 1990). Descriptions of successful teachers of mathematics (Cobb et al., 1989; Grouws & Cramer, 1989; Peterson & Fennema, 1985) place considerable emphasis on how teachers can help students develop this kind of autonomy.

Related to autonomy is Witkin's notion of field independence (Witkin & Goodenough, 1981). Although Witkin considered field independence and field dependence to be cognitive styles, rather than affective factors, the pattern of performance of field-independent students is quite similar to Fennema and Peterson's (1985) notion of autonomous learning behaviors. Given the importance of autonomy and independent thinking in the curriculum reform movement that is now so prominent in mathematics education (Commission on Standards for School Mathematics, 1989), it may be wise to return to earlier forms of these concepts and to conduct more detailed investigations of cognitive styles (Messick, 1987). One recent effort in this direction (Kelly-Benjamin, 1990) investigated differences between general and mathematical learning styles among high school seniors; results of a factor analysis suggest that students' mathematical learning styles are different from their general learning styles, and that these differences have implications for mathematics instruction.

Aesthetics

Although mathematicians often remark on the aesthetic dimensions of mathematics, the study of aesthetics has not received much attention in the mathematics education community (Dreyfus & Eisenberg, 1986). Mathematicians like Poincare and Hadamard have been quite interested in how aesthetic factors have influenced the development of mathematical thinking; it seems appropriate that the topic should receive more attention from researchers in mathematics education.

In a study of expert problem solvers who were mathematicians, Silver and Metzger (1989) concluded that expertise involved taste as well as competence. Aesthetic factors clearly influenced the problem solver's emotional reaction to the problem, which frequently involved recognition of the beauty and elegance of the problem or its solution. In addition, aesthetic factors were related to the problem solvers' monitoring of their solution strategies and cognitive processing.

This aesthetic monitoring provided an interesting link between metacognitive processing and affective responses to problem solving. The study of aesthetic influences on mathematical performance appears to be an important issue in the development of expert problem solvers, and it certainly deserves more attention in the curriculum than it currently receives (Dreyfus & Eisenberg, 1986).

Intuition

Intuition, like aesthetics, plays an important role in mathematicians' discussions of mathematical thought. Recently this topic has received considerable attention in two books (Fischbein, 1987; Noddings & Shore, 1984), both of which emphasize mathematics in their analysis. Intuitive knowledge in mathematics is knowledge which is self-evident, which carries with it characteristic feelings of certitude, and which goes beyond the facts that are available (Fischbein, 1987; Noddings & Shore, 1984). Some authors have tried to explicate the role of intuition in rational number learning (Kieren, 1988) and other arithmetic concepts (Resnick, 1986). Clearly there is much more research that could be done, particularly on the teaching and learning of mathematical problem solving, to develop an understanding of the role of intuition and of how students' intuitions could be improved.

Metacognition

Metacognition has received substantial attention in research on mathematical problem solving in recent years (Campione, Brown, & Connell, 1989; Garofalo & Lester, 1985; Schoenfeld, 1987b; Silver, 1985) and the links between metacognition and the affective domain have been duly noted as well (Brown, Bransford, Ferrara, & Campione, 1983; Garner & Alexander, 1989; Lawson, 1984; McLeod, 1988; Prawat, 1989). Lester and his colleagues (Lester et al., 1989) have been the most specific about the relationships between metacognition and affective factors like confidence and interest. In a more general setting, Weinert and Kluwe (1987) have published a volume devoted to the analysis of the relationships between metacognition and motivation. The task of specifying the ways in which metacognitive processing interacts with the affective domain is difficult; however, substantial progress is being made in understanding the area and in deriving implications for instruction, particularly in the area of problem solving (Lester et al., 1989). Some work has also been done on mathematics instruction in more general settings. For example, Newman and his colleagues (Newman, 1990; Newman & Goldin, 1990) have investigated how students regulate their own behavior in the context of seeking help in the mathematics classroom. Their results, which suggest that willingness to seek help is constrained by students' beliefs about self, provide a nice example of how to integrate research on beliefs, attitudes, and metacognition in order to analyze a specific issue related to mathematics instruction.

Social Context

The relationship of affective factors to mathematics learning and teaching is always influenced by the social context. The study of these contextual factors is receiving increased attention, particularly because of their relationship to issues of gender and ethnicity (Cole & Griffin, 1987). The role of the social context is also receiving much more attention from those in cognitive science who would like to see their research have a greater impact on real-world classrooms (Brown, Collins, & Duguid, 1988). This emphasis on the role of contextual factors has come in part from anthropology, where studies of learning in settings outside of school have led to new insights (Lave, 1988). The applicability of these insights to school settings is often unclear; for example, the notion of apprenticeships for mathematics students (as well as for future African tailors) has considerable appeal, but it is not clear how such a notion could be implemented on a broad scale in school mathematics.

The analysis of the social context has been an interest of psychologists as well as anthropologists. Saxe (1990) has studied the development of mathematical thinking with an emphasis on the interaction of culture and cognition; many of his examples are taken from investigations of how some Brazilian children develop mathematical skills through their work as candy sellers. In an example of work from another area, Magnusson (1981) provides an analysis of the characteristics of situations that are important to psychology. He includes complexity, clarity, tasks, rules, roles, physical settings, other persons, expectancies, affective tones, and emotions in his list of situational characteristics. There are many difficulties involved in making sense of a psychology of situations; nevertheless, the area presents an alternative approach to understanding the social context of teaching and learning.

Another broad approach to the issue of social context is provided by the work of Bishop (1988) on mathematical enculturation. His analysis of mathematical culture puts considerable emphasis on beliefs and attitudes, and he argues for a curriculum that puts appropriate emphasis on inducting students into the culture of our discipline. In a related effort to shed light on the role of the social context in learning, Newman et al. (1989) argue that cognitive change is as much a social as an individual process, suggesting that researchers need to focus more on the context and the social interaction among learners. From a somewhat different perspective, Cocking and Mestre (1988) and Orr (1987) look at cultural influences on learning mathematics that cause particular problems for students who are members of linguistic (or other) minority groups. Just as affective factors are particularly important to gender-related differences in mathematics performance (Fennema, 1989), it seems reasonable to hypothesize that affective factors are particularly important to differences in performance between groups that come from different cultural backgrounds.

Research that has been done in this area indicates the significant role of the family in students' beliefs and attitudes toward schooling in general and toward mathematics in particular (Parsons, Adler, & Kaczala, 1982; Stevenson et al., 1986; Stigler & Perry, 1988). The cross-cultural comparisons in these and other studies are especially interesting. Mathematics education in the United States certainly can learn much from careful analyses of classrooms in other countries, and more research on issues related to the social context of instruction should prove to be very helpful (Research Advisory Committee, 1989).

Technology

One aspect of the social context of learning and teaching is the presence of technology in the classroom. Although it is possible to utilize some kinds of technology in the classroom without changing anything in a substantive way (as computer-based drill and practice programs have demonstrated for years), other kinds of technological advances should have a significant impact on the social context and the affective environment. Kaput (1989), in a stimulating analysis of the impact that computers could have on classroom instruction, notes that students who learn in a computer environment could have quite different affective responses to learning tasks than students who do not. For example, students who use computers can discover their own errors and correct them independently, rather than being corrected by a teacher or fellow students.

The computer has become an important object in our culture, and considerable research has been done on our reactions to computers. Turkle (1984) presents an interesting analysis of how children's views of the computer develop, from the early stage where young children are still trying to figure out if the computer is alive or not, to a later stage where the computer is an object to be mastered or an object that can reflect one's own identity. Other investigators have written about attitudes toward computers (Collis, 1987; Gressard & Loyd, 1987; Swadener & Hanafin, 1987) using traditional methods and instruments. Collis and Williams (1987) investigated cross-cultural differences between Chinese and Canadian adolescents in their attitudes toward computers, noting some differences between cultures, as well as between females and males. It seems likely that technology can play an important role in changing beliefs about mathematics and possibly even in improving attitudes toward mathematics; more research and development along these lines seems appropriate, particularly studies that take affective factors into account.

Research on topics that fall in between the purely cognitive and strictly affective areas is especially important to the field of mathematics education. These topics provide a natural link between research on affect and cognition, a link that we explore further in the next section.

THEORIES AND METHODS FOR RESEARCH ON THE AFFECTIVE DOMAIN

Most research on the affective domain has followed the traditional paradigm of quantitative research, and as Fennema (1989) points out, this approach has produced valuable information on the affective domain. In recent years, however, research in the cognitive domain has made successful use of a variety of qualitative as well as quantitative techniques. Such a combination of techniques seems appropriate for the affective domain as well. Howe (1988) notes that many researchers have an ideological commitment to one set of research methods, assuming that purity of method is necessary to development of theory. However, as Howe (1988) points out, there is no convincing evidence that qualitative and quantitative methods are incompatible. Given the nature of research on the affective domain, it seems likely that a variety of reasonable research methods have a chance to make a contribution to the field, as long as the data are interpreted intelligently.

A number of researchers have discussed the strengths and weaknesses of qualitative methods in research in education. For example, Firestone (1987) and Jacob (1987) present a general analysis of the problems, and the classic work of Ericsson and Simon (1980) deals with the use of verbal reports as data. In addition to these general analyses, some authors have dealt specifically with issues related to doing qualitative research in mathematics education. Eisenhart (1988), for example, provides an analysis of ethnographic methods in the specific context of research on mathematics teaching and learning, and Ginsburg and his colleagues (Ginsburg, 1981; Swanson, Schwartz, Ginsburg, & Kossan, 1981) discuss the use of clinical interview methods in mathematics education.

If researchers are to make progress in building theory and gathering relevant data about the role of the affective domain in the learning and teaching of mathematics, they need to provide data on a wide range of issues. Some of these issues (for example, beliefs and attitudes) can be analyzed through the use of traditional quantitative techniques, but qualitative data will add substantially to the completeness of our understanding of these issues. Measures of emotional reactions to mathematics can be done quantitatively (for example, measures of heart rate), but it seems much more natural in the context of mathematics classrooms to investigate such issues through studies that use qualitative techniques. For example, college students who were asked to draw a graph that represented their emotional reactions during a problem-solving episode were able to describe the "highs" and "lows" that they felt at various points, and to specify some of the reasons for their positive and negative feelings (McLeod, Craviotto, & Ortega, 1990); presumably, ways could be found to obtain similar information from younger students as well. Having students keep journals where they write on a regular basis about mathematics can also provide data on affective responses (Adams, 1989). In addition, if research is going to help us understand the role of affect in mathematics learning and teaching, studies of affect must be integrated with studies of cognition. Most research on learning ignores affective issues even when they are quite pertinent. For example, studies of students who are attempting to solve nonroutine problems are very likely to involve fairly intense affective responses (McLeod et al., 1989), and researchers who fail to gather data on these responses are missing an important characteristic of student performance.

Integrating affect into cognitive studies of mathematics learning would improve research on both cognition and affect. The following sections describe a variety of studies of learning and teaching which attempt to use both quantitative and qualitative methods, and which integrate research on affect and cognition.

Integrating Research on Affect and Learning

Research on learning with young children often provides opportunities to include affect in studies that are designed primarily to study cognitive issues. For example, Marshall (1989) reports on the affective reactions of sixth-grade students to mathematical story problems. Although the main purpose of the research was to investigate children's development of schemas for story problems, the interviewer also encouraged students to verbalize their affective reactions to the problems. Given this opportunity to discuss their feelings in a supportive environment, many children responded quite freely. Some of the children had rather intense emotional reactions, including a few who discovered something new about mathematics during the solution of the problems and who were delighted with their new knowledge. A few others demonstrated negative reactions to the problems, including one child who reported a rapid heartbeat as well as general discomfort and fear during the interview. In this case the interviewer ended the questions and spent some time reassuring the child instead. The source of this child's difficulty appeared to be the blockage that the child experienced in attempting to solve a nonroutine problem. Most children, however, used their verbal comments to express well-established attitudes about story problems; these attitudes often revealed negative views toward mathematics or toward themselves as problem solvers. In Marshall's analysis, these emotional and attitudinal responses are attached to various components of the schemas involved in solving story problems. Marshall's procedures and analysis provide a good example of how a study with cognitive objectives can be expanded to include affective issues in a natural way.

In another study involving story problems, this time at the seventh-grade level, Lester et al. (1989) focused mainly on the role of metacognition in problem solving. In order to explain the context in which metacognitive decisions were made, the researchers also gathered data on affective factors, including children's beliefs about themselves as problem solvers and their attitudes toward mathematical problem solving. The data from this study support the view that the social context and the beliefs which it engenders have an important influence on both the students' affective responses as well as their metacognitive acts.

These studies of children's learning indicate that affect plays an important role in students' (i.e., novices') mathematical performance. In another study linking research on affect and cognition, Silver and Metzger (1989) gathered related data on the performance of experts. In their study Silver and Metzger interviewed research mathematicians and asked them to solve non-routine problems while thinking aloud. These interviews provide a rich source of data on the relationship between the affective domain and expertise in mathematical problem solving. A striking result of these data is the important role played by aesthetics in the monitoring and evaluation of expert performance. Rather than viewing problems from a utilitarian perspective, these experts spoke frequently about the elegance, harmony, and coherence of various solutions (or attempted solutions) to problems. The aesthetic aspects of the problem-solving experience were clearly linked to the experts' emotional responses, including their enjoyment of the problem.

In another study of experts, Taylor (1990) investigated the attitudes of mathematicians toward mathematics. This study is interesting due to its use of qualitative methods and its observations about gender-related differences in the development of mathematicians' careers. Through the use of qualitative methods, data are gathered on such factors as the development of confidence and willingness to persist, along with information on causal attributions for the mathematical success of the participants.

A variety of studies with a cognitive orientation have included affective factors as an important part of the research. For example, Peterson and her colleagues (for example, Peterson & Swing, 1982) have completed a series of studies that have included lengthy interviews with students who were asked to comment on affective as well as cognitive matters. Ginsburg and Allardice (1984) provide an intriguing view of how beliefs and emotion can contribute to the difficulties of young children who are unsuccessful in mathematics. These and other studies suggest that the usual methods for research on cognition can be adapted to include appropriate attention to the role of affect in the learning of mathematics.

Integrating Research on Affect and Teaching

Research on teachers and teaching in mathematics education seldom focuses on the affective factors that are frequently so visible in classrooms. This section will discuss several papers that do include affect, and do so in ways that go beyond the traditional attitude measures of previous years.

Cobb et al. (1989) have provided extensive data on how a teacher in a second-grade classroom dealt with emotions in the learning of mathematics. The data were obtained through careful observation of the classroom over an entire school year. The observers were able to document how the teacher worked with the students as they developed beliefs about mathematics. For example, the teacher was very explicit about the need to justify answers to mathematical problems, and about the importance of the justification. She was also explicit in her specifications of the acceptable kinds of behavior regarding solving mathematical problems. For example, she repeatedly emphasized the satisfactions that come with solving problems independently, and instructed students not to tell the answers to those who were still working on the problems. She was very clear in letting the students know that persistence in spite of frustration was important to success in solving problems. Since these classroom norms were a change from what had been expected of the students in other contexts, the changed norms were explicitly taught and practiced, and the teacher worked hard to see that these norms were adhered to. The result was a classroom where students showed a lot of satisfaction and enthusiasm for problem solving, and viewed themselves as autonomous learners.

In a study of pre-service teachers' estimation skills, Sowder (1989) investigated the teachers' tolerance for error, their attributions of success and failure, and other beliefs about mathematics and about themselves. Through extensive interviews with a sample of teachers, Sowder was able to create a profile of the beliefs that characterized good and poor estimators. Good estimators tended to have strong self-concepts with regard to mathematics, to attribute successes to their ability rather than just to effort, and to hold the belief that estimation was important. Poor estimators were more likely to have a weak self-concept in mathematics, to attribute successes to effort, and not to value estimation. The exceptions to these general patterns were interesting cases that showed how individual beliefs about mathematics could have an important impact on individual performance on estimation tasks. The general conclusions of this study provide some indication of the difficulties that will be involved in implementing recommendations to include estimation in the elementary mathematics curriculum. Clearly many teachers who are in the field, as well as many more who are on their way, do not hold beliefs about mathematics or about themselves that are compatible with the goals of the curriculum in terms of estimation skills (Sowder, 1989).

In a third study of teachers, Grouws and Cramer (1989) observed six expert teachers of problem solving at the junior high school level. The focus of this study was to identify the affective characteristics of the classrooms of these teachers during problem-solving lessons. Each teacher was observed five to seven times over the course of a semester. The observations revealed that students enjoyed problem solving, persevered on problem-solving tasks, and worked willingly on problem-solving assignments. Interviews with teachers helped with the identification of strategies that led to this positive affective climate in the classroom. For example, teachers appeared to work hard to establish a good relationship with students. They tended to be friendly rather than formal, and to share personal anecdotes about their own problem solving that illustrated their own strengths and weaknesses as problem solvers. In addition, the teachers established a system that held students accountable for their performance in problem solving. The system itself varies, although most teachers did pay attention to more than just the answer to the problem. Also, the teachers made frequent use of cooperative groups, and noted that small-group work tended to promote independence and to reduce feelings of frustration. Although no single factor appeared to be the cause of the success of these expert teachers, further research should provide indications of how these classroom characteristics contribute to the development of positive affective environments for problem solving.

Other studies of teaching also include affective factors in some detail. Tittle (1987), for example, is investigating how data on students' affective characteristics could be provided to teachers, thus making it possible for teachers to tailor instruction to students' affective as well as cognitive characteristics. This approach could be particularly important for gender differences in mathematics education (Tittle, 1986). Brophy (1983) has also written about strategies for improving the motivational climate in classrooms through strategies for encouraging enthusiasm for learning, reducing anxiety, and inducing curiosity. Thompson and Thompson (1989) discuss how students respond to a teacher's efforts to provide a supportive atmosphere for mathematical problem solving. Further research on affect in classrooms should provide more guidance on these topics.

These studies of teachers and teaching provide useful information on how beliefs, emotions, and attitudes play a significant role in mathematics instruction. They also demonstrate how affective factors can be incorporated in cognitive studies of teaching. If studies of affect are isolated, they do not have a significant impact on researchers who are primarily interested in cognition. If research on affect can be integrated into cognitive studies of teaching and learning, our knowledge of affective factors will be more likely to have an impact on instruction.

SUMMARY

Research on affect has been voluminous, but not particularly powerful in influencing the field of mathematics education. It seems that research on instruction in most cases goes on without any particular attention to affective issues. Similarly, there is little attention to research on affect in most curriculum development efforts, and apart from the topic of gender-related differences in mathematics, research on affect appears to have little impact on curriculum development or teacher education programs in mathematics.

A major difficulty is that research on affect has not usually been grounded in a strong theoretical foundation. When such research did occur within a theoretical framework, there was little connection between that framework and the theoretical foundations of cognitive research in mathematics education. To people who work on cognition, research on affect seemed to be a collection of generally unrelated clumps of studies on issues like motivation, attitude, and causal attributions. With no overriding themes or general framework, affective studies appeared to be unconnected with each other and quite separate from the interests of most cognitive researchers.

There are several things that can be done to improve this state of affairs. For example, researchers who focus on affective issues need to be more aware of how their research can contribute to research on cognition. Similarly, those who focus on cognition need to be more aware of research on affect and to include affective issues in a meaningful way in their studies. Too often researchers who focus on affect rely on measures of achievement (like standardized tests) that would not be acceptable to cognitive researchers. On the other hand, researchers with a cognitive orientation often ignore affect or treat the issue in a cavalier manner, using inappropriate instruments that happen to be convenient. Efforts to encourage the two groups of researchers to work together are just beginning, but the results have been encouraging so far (McLeod & Adams, 1989).

There are a number of research questions where collaboration of researchers with different perspectives is needed. For example, the cognitive and affective domains intersect in the area of beliefs (Schoenfeld, 1985), and researchers need to work together to map out this area more clearly, relating various beliefs to the cognitive processes of learners and teachers. This chapter has suggested one way of organizing research on beliefs in mathematics education, but more detailed and contrasting analyses are needed. Similarly, the domains of attitudes and emotions in the context of mathematics education need to be analyzed and clarified in order for research to proceed in an orderly fashion. Recent theoretical advances (Mandler, 1984; Ortony et al., 1988) should provide some help in this effort, particularly in determining how early emotional responses may be the source of later attitudes toward mathematics.

Another research topic of special importance is the relationship of affective responses to the development of higher-order thinking skills. Current efforts at curriculum reform place special emphasis on solving nonroutine problems, on applying mathematics in new situations, and on communication regarding mathematical problems. The novelty (as well as the difficulty) of such changes in the curriculum will cause more intense affective reactions for many students and teachers; research that investigates these more intense emotional responses is particularly important if the reform movement is to succeed. Those responsible for the changes in the mathematics curriculum during the 1950s and 1960s expected students and teachers to respond as enthusiastically to mathematical abstractions as mathematicians did; those involved in the current reform movement need to know more about the affective implications of the proposed changes for students and teachers, particularly those who think of themselves as being outside the mathematics community.

Another area where research on affect is particularly needed is in studies of the uses of technology to support mathematics instruction. The rapid improvements in technological support for mathematics education are leading to changes in the organization of classrooms and the definition of mathematical tasks. The advent of graphing calculators and symbol manipulation systems, for example, should eventually result in significant changes in what mathematics we teach. These changes in the curriculum will be accompanied by changes in beliefs about mathematics, and by opportunities for more positive emotional experiences in mathematics education. Research should help guide our efforts to increase positive affective responses to mathematics through the creative use of technology.

Finally, research on affective issues in mathematics education should develop a wider variety of methods. The debate over qualitative versus quantitative research methods appears to be almost over, and the time for intelligent use of multiple research methods that fit the research problems is here. The use of clinical interviews and detailed observations should provide the field with a deeper understanding of the role of affective issues in mathematical learning and teaching.

This chapter has presented an overall theoretical framework that is consistent with current studies of cognition in mathematics education. The division of the affective domain into beliefs, attitudes, and emotions seems appropriate for the interests and views of mainline cognitive research in mathematics education. If future research on affect can be linked more closely to the study of cognitive factors in learning, the affective domain should receive more attention in curriculum development, teacher education, and research on teaching and learning in our field.

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