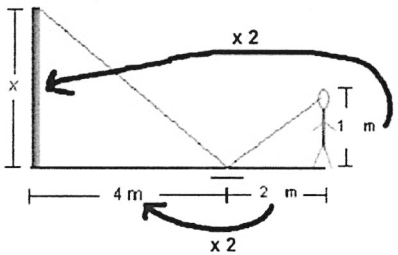
MFM2P Course Pack

Name: _____

MT: Measurement & Trigonometry

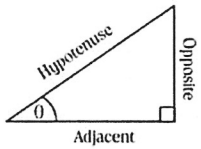
MT1: Similar triangles

$$\frac{1}{x} = \frac{2}{4}$$
$$1 \cdot 4 = x \cdot 2$$
$$4 = 2x$$
$$\frac{4}{2} = \frac{2x}{2}$$
$$2 = x$$
$$x = 2$$

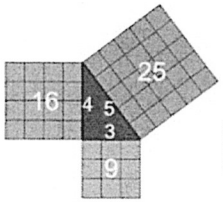


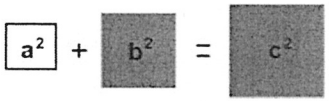
MT2: Trig ratios & Pythagorean Theorem

$leg^2 + leg^2 = hypotenuse^2$

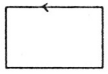
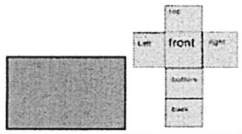
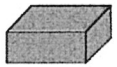


$$\sin(\theta) = \frac{opp}{hyp}$$
$$\cos(\theta) = \frac{adj}{hyp}$$
$$\tan(\theta) = \frac{opp}{adj}$$





MT3: Surface area & Volume

Perimeter	Area / Surface Area	Volume
(around)	(on)	(in)
1-dimensional units	2-dimensional units²	3-dimensional units³
		

LR: Linear Relations

LR1: Solving equations

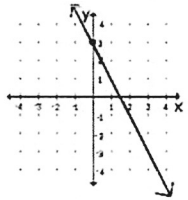
$$6x + 15 = 3x + 8$$
$$-3x \qquad -3x$$
ditch the smallest x guy

$$3x + 15 = 8$$
$$-15 \quad -15$$
ditch the 15

$$3x = -7$$
$$\frac{3x}{3} = \frac{-7}{3}$$
ditch the 3

$$x = \frac{-7}{3}$$

LR2: Creating Graphs & Equations

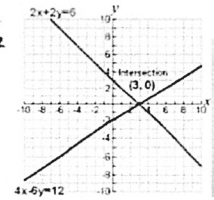


$$y = mx + b$$
$$m = -2 \qquad b = 3$$
$$y = -2x + 3$$

slope = m = −2
y-intercept = (0, b)
= (0, 3)

LR3: Systems of linear equations

x	$2x+2y=6$, y_1	$4x-6y=12$, y_2
0	3	−2
1	⋮	⋮
2		
3	0	0



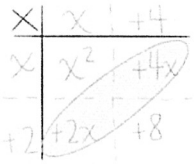
$$2x + 2y = 6$$
$$4x - 6y = 12$$
$$0x + 10y = 0$$
$$\frac{10y}{10} = \frac{0}{10}$$
$$y = 0$$

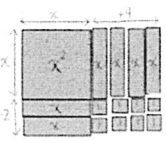
$$4x - 6(0) = 12$$
$$4x - 0 = 12$$
$$\frac{4x}{4} = \frac{12}{4}$$
$$x = 3$$

P.o.I: (3, 0)

QR: Quadratic Relations

QR1: Factor & Expand quadratics

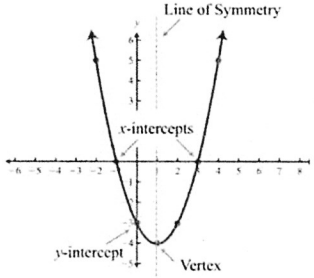


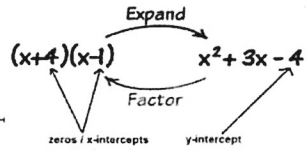


(x+4)(x−1) $\xrightarrow{\text{Expand}}$ $x^2 + 3x - 4$ $\xrightarrow{\text{Factor}}$

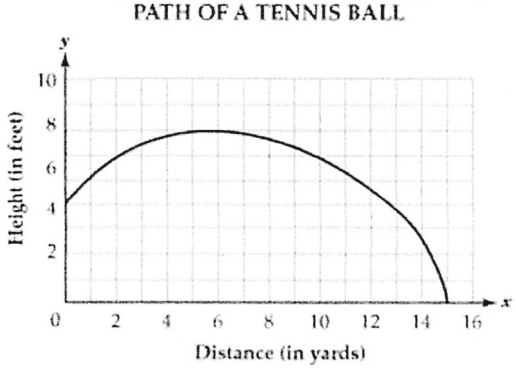
$$(x+4)(x+2) = x^2 + 2x + 4x + 8$$
$$= x^2 + 6x + 8$$

QR2: Characteristics of quadratics





QR3: Interpret quadratic graphs



MFM2P: Grade 10 Applied Mathematics Course Notes

MT1: Similar Triangles **MT = Measurement & Trigonometry**

Similar triangles:

Corresponding sides:

Corresponding angles:

Scale factor:

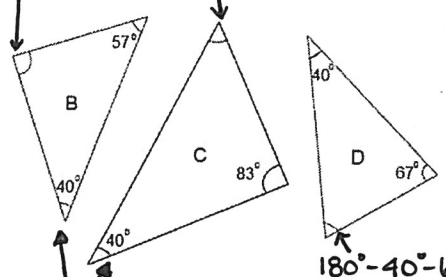
Proportion:

Determine whether or not two triangles are similar:

3 equal angles:

180°-57°-40° = 83°

180°-83°-40° = 57°

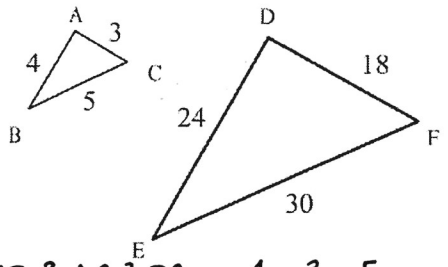


180°-40°-67° = 73°

∴ NOT similar

∴ Similar triangles (3 = angles)

Proportional side lengths:

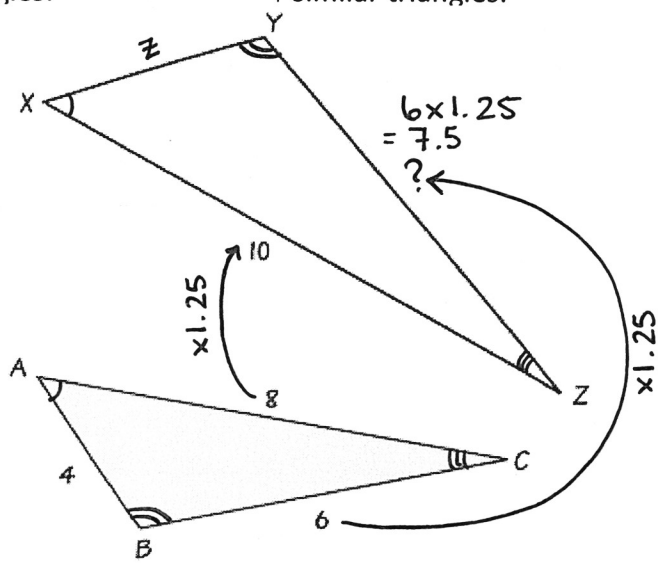


$\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{24} = \frac{3}{18} = \frac{5}{30}$

$\frac{1}{6} = \frac{1}{6} = \frac{1}{6}$

∴ similar (proportional)

Determine the scale factor (ratio) of similar triangles:



Scale factor $= \frac{XZ}{AC}$

$= \frac{10}{8}$

$= 1.25$

∴ ΔXYZ is 1.25 times larger than ΔABC.

Determine missing side lengths in similar triangles:

$\frac{XZ}{AC} = \frac{XY}{AB}$

$\frac{10}{8} = \frac{Z}{4}$

$1.25 = \frac{Z}{4}$

$4 \times 1.25 = \frac{Z}{\cancel{4}} \times \cancel{4}^{1}$

$5 = Z$

$Z = 5$

∴ The missing sides measure 7.5 and 5.

MT2: Trigonometric Ratios & Pythagorean Theorem **MT = Measurement & Trigonometry**

w/ SOH CAH TOA

Sine:	Cosine:	Tangent:	SOH CAH TOA:
$\sin\theta = \frac{opp}{hyp}$	$\cos\theta = \frac{adj}{hyp}$	$\tan\theta = \frac{opp}{adj}$	
Theta (Θ):	**Opposite side:**	**Adjacent side:**	**Hypotenuse:**

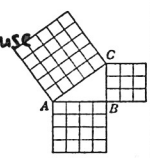
Pythagorean Theorem:

a.k.a sum of squares

$\underbrace{a^2 + b^2}_{\text{"legs"}} = \underbrace{c^2}_{\text{always hypotenuse}}$

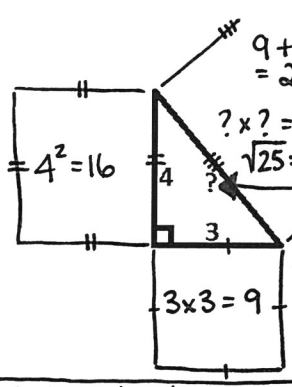
$a^2 = c^2 - b^2$

$b^2 = c^2 - a^2$



Use Pythagorean Theorem to find missing sides in right triangles:

Missing the hypotenuse:



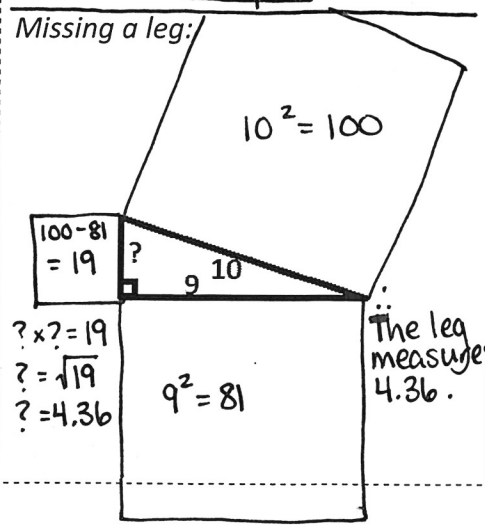
$9 + 16 = 25$

$? \times ? = 25$

$\sqrt{25} = 5$

∴ The hypotenuse measures 5.

Missing a leg:



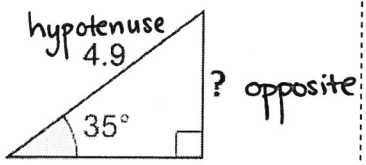
$? \times ? = 19$

$? = \sqrt{19}$

$? = 4.36$

∴ The leg measures 4.36.

Use trig ratios to find missing sides in right triangles:



$\sin\theta = \frac{opp}{hyp}$

$\sin 35° = \frac{x}{4.9}$

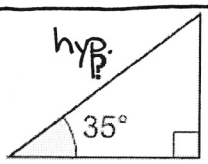
$0.5736 = \frac{x}{4.9}$

$4.9 \times 0.5736 = \frac{x}{\cancel{4.9}} \times \cancel{4.9}$

$2.81 = x$

$x = 2.81$

∴ The side measures 2.81.



4.0 adj

$\cos\theta = \frac{adj}{hyp}$

$\frac{\cos 35°}{1} = \frac{4.0}{x}$

$\frac{1}{\cos 35°} = \frac{x}{4.0}$

$\frac{1}{0.8192} = \frac{x}{4}$

$1.22 = \frac{x}{4}$

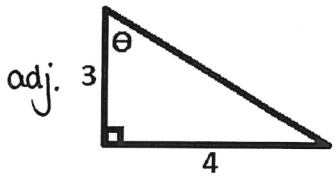
$4 \times 1.22 = \frac{x}{\cancel{4}} \times \cancel{4}$

$4.88 = x$

$x = 4.88$

∴ The hypotenuse measures 4.88.

Use trig ratios to find missing angles in right triangles:



opp.

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{4}{3}$

$\tan\theta = 1.3333$

find on trig. table

$\theta = 53°$

∴ The angle measures 53°.

Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175
2°	.0349	.9994	.0349
3°	.0523	.9986	.0524
4°	.0698	.9976	.0699
5°	.0872	.9962	.0875
6°	.1045	.9945	.1051
7°	.1219	.9925	.1228
8°	.1392	.9903	.1405
9°	.1564	.9877	.1584
10°	.1736	.9848	.1763
11°	.1908	.9816	.1944
12°	.2079	.9781	.2126
13°	.2250	.9744	.2309
14°	.2419	.9703	.2493
15°	.2588	.9659	.2679
16°	.2756	.9613	.2867
17°	.2924	.9563	.3057
18°	.3090	.9511	.3249
19°	.3256	.9455	.3443
20°	.3420	.9397	.3640
21°	.3584	.9336	.3839
22°	.3746	.9272	.4040
23°	.3907	.9205	.4245
24°	.4067	.9135	.4452
25°	.4226	.9063	.4663
26°	.4384	.8988	.4877
27°	.4540	.8910	.5095
28°	.4695	.8829	.5317
29°	.4848	.8746	.5543
30°	.5000	.8660	.5774
31°	.5150	.8572	.6009
32°	.5299	.8480	.6249
33°	.5446	.8387	.6494
34°	.5592	.8290	.6745
35°	.5736	.8192	.7002
36°	.5878	.8090	.7265
37°	.6018	.7986	.7536
38°	.6157	.7880	.7813
39°	.6293	.7771	.8098
40°	.6428	.7660	.8391
41°	.6561	.7547	.8693
42°	.6691	.7431	.9004
43°	.6820	.7314	.9325
44°	.6947	.7193	.9657
45°	.7071	.7071	1.0000

Angle	Sine	Cosine	Tangent
46°	.7193	.6947	1.0355
47°	.7314	.6820	1.0724
48°	.7431	.6691	1.1106
49°	.7547	.6561	1.1504
50°	.7660	.6428	1.1918
51°	.7771	.6293	1.2349
52°	.7880	.6157	1.2799
53°	.7986	.6018	1.3270
54°	.8090	.5878	1.3764
55°	.8192	.5736	1.4281
56°	.8290	.5592	1.4826
57°	.8387	.5446	1.5399
58°	.8480	.5299	1.6003
59°	.8572	.5150	1.6643
60°	.8660	.5000	1.7321
61°	.8746	.4848	1.8040
62°	.8829	.4695	1.8807
63°	.8910	.4540	1.9626
64°	.8988	.4384	2.0503
65°	.9063	.4226	2.1445
66°	.9135	.4067	2.2460
67°	.9205	.3907	2.3559
68°	.9272	.3746	2.4751
69°	.9336	.3584	2.6051
70°	.9397	.3420	2.7475
71°	.9455	.3256	2.9042
72°	.9511	.3090	3.0777
73°	.9563	.2924	3.2709
74°	.9613	.2756	3.4874
75°	.9659	.2588	3.7321
76°	.9703	.2419	4.0108
77°	.9744	.2250	4.3315
78°	.9781	.2079	4.7046
79°	.9816	.1908	5.1446
80°	.9848	.1736	5.6713
81°	.9877	.1564	6.3138
82°	.9903	.1392	7.1154
83°	.9925	.1219	8.1443
84°	.9945	.1045	9.5144
85°	.9962	.0872	11.4301
86°	.9976	.0698	14.3007
87°	.9986	.0523	19.0811
88°	.9994	.0349	28.6363
89°	.9998	.0175	57.2900

MT3: Volume & Surface Area **MT = Measurement & Trigonometry**

Imperial system of measurement:

Metric system of measurement:

Volume:

1 foot ≅ 0.3048 m
1 mile ≅ 1.61 km
1 inch ≅ 2.54 cm

Area:

Surface area:

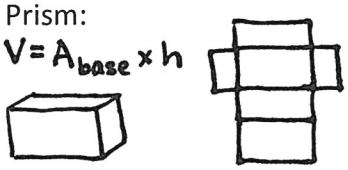
Sphere:

$V = \frac{4\pi r^3}{3}$

$SA = 4\pi r^2$

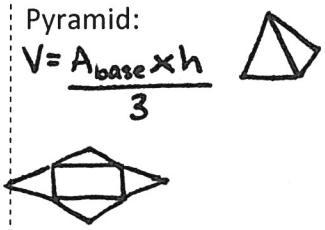
Prism:

$V = A_{base} \times h$



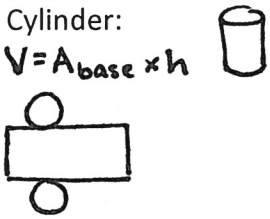
Pyramid:

$V = \frac{A_{base} \times h}{3}$



Cylinder:

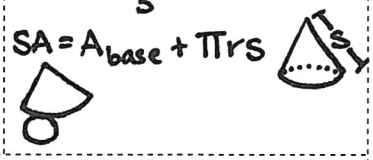
$V = A_{base} \times h$



Cone:

$V = \frac{A_{base} \times h}{3}$

$SA = A_{base} + \pi r s$



Convert units:

12 inches = ? centimeters
1 inch = 2.54 cm

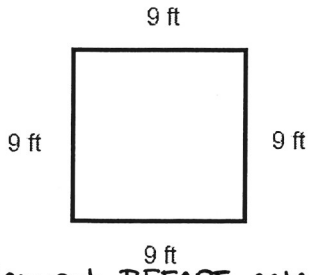
$2.54 \times \frac{12}{1} = \frac{x}{\cancel{2.54}} \times \cancel{2.54}$

$30.48 = x$

$x = 30.48$ cm

∴ 12 inches equals 30.48 cm.

Calculate area in m^2:



① Convert BEFORE calculating area.

9 ft = ? m
1 ft = 0.3048 m

$0.3048 \times \frac{9}{1} = \frac{x}{0.3048} \times \cancel{0.3048}$

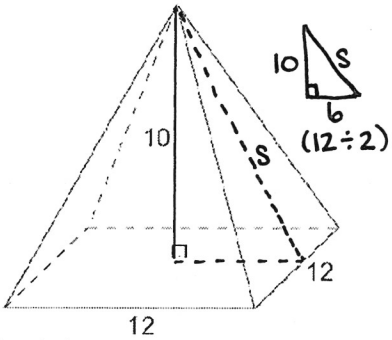
$2.74 = x$

$x = 2.74$ m

② $A = s \times s$
$A = 2.74 \times 2.74$
$= 7.56 m^2$

∴ The area is $7.56 m^2$.

Calculate surface area:



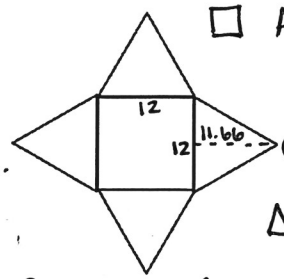
(12 ÷ 2)

① Calculate s:

$s^2 = 10^2 + 6^2$
$s^2 = 100 + 36$
$\sqrt{s^2} = \sqrt{136}$
$s = 11.66$

② Area of base:

□ $A = s \times s$
$= 12 \times 12$
$= 144$



③ Area of 1 lateral face:

△ $A = \frac{b \times h}{2}$
$= \frac{12 \times 11.66}{2}$
$= \frac{139.92}{2}$
$= 69.96$

④ Total surface area:

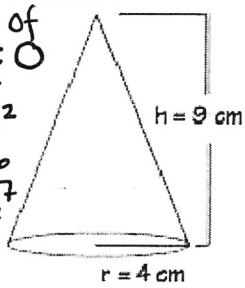
$SA = \square + 4\triangle$
$= 144 + 4 \times 69.96$
$= 144 + 279.84$
$= 423.84$

∴ The surface area is 423.84.

Calculate volume:

① Area of base: ○

$A = \pi r^2$
$= \pi \times 4^2$
$= \pi \times 16$
$= 50.27 cm^2$



② $V = \frac{A_{base} \times h}{3}$

$= \frac{50.27 \times 9}{3}$

$= \frac{452.39}{3}$

$= 150.80 cm^3$

∴ The volume is $150.80 cm^3$.

LR1: Solving Equations **LR = Linear Relations**

Equation:	Variable:	Substitute:	Solve:

Solve Equations:

A taxi's costs are modeled by the equation $C = 0.25d + 3.00$ (where C = cost in \$ & d = distance in km). If a taxi ride cost you \$15, what distance did you travel?

$C = 15\$ \longrightarrow C = 0.25d + 3.00$

$15 = 0.25d + 3$

$-3 \qquad -3$

$12 = 0.25d$

$\frac{12}{0.25} = \frac{\cancel{0.25}^{1}d}{\cancel{0.25}}$

$48 = d$

$d = 48$ km

$\therefore$ I travelled 48 km

If a sphere has a volume of 18 cm^3, what is the measure of its radius?

$V = \frac{4\pi r^3}{3}$

$18 = \frac{4 \times \pi \times (r^3)}{3}$

$3 \times 18 = \frac{4 \times \pi \times r^3}{\cancel{3}} \times \cancel{3}^{1}$

$54 = 4 \times \pi \times r^3$

$\frac{54}{4} = \frac{\cancel{4}^{1} \times \pi \times r^3}{\cancel{4}}$

$\frac{13.5}{\pi} = \frac{\cancel{\pi}^{1} \times r^3}{\cancel{\pi}}$

$4.3 = r^3$

$\sqrt[3]{4.3} = \sqrt[3]{r^3}$

$1.63 = r$

$r = 1.63$

$\therefore$ The radius is 1.63 cm

Rearrange an equation into $y=$ format:

$6x + 3y = 5$

$-6x \qquad -6x$

$0 + 3y = 5 - 6x$

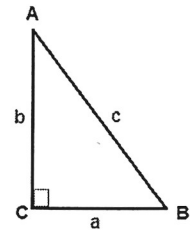
$\frac{\cancel{3}^{1}y}{\cancel{3}} = \frac{5}{3} - \frac{\cancel{6}^{2}x}{\cancel{3}}$

$y = \frac{5}{3} - 2x$

OR

$y = -2x + \frac{5}{3}$

Rearrange the Pythagorean Theorem to help you find the missing leg of a triangle (instead of a missing hypotenuse).



$a^2 + b^2 = c^2$ OR $a^2 + b^2 = c^2$

$-b^2 \quad -b^2 \qquad -a^2 \quad -a^2$

$a^2 + 0 = c^2 - b^2$

$a^2 = c^2 - b^2$

$\sqrt{a^2} = \sqrt{c^2 - b^2}$

$a = \sqrt{c^2 - b^2}$

$0 + b^2 = c^2 - a^2$

$b^2 = c^2 - a^2$

$\sqrt{b^2} = \sqrt{c^2 - a^2}$

$b = \sqrt{c^2 - a^2}$

LR2: Graphing & Writing Equations of Lines **LR = Linear Relations**

Rate of Change / Slope:	Initial Value / y-intercept:	First differences:	Line of best fit:
$m = \frac{rise}{run}$ or $\frac{y_2 - y_1}{x_2 - x_1}$			
Rise:	**Run:**	**Linear:**	$y = mx + b$

Determine whether or not a relationship is linear:

x	y
0	5
1	10
3	20
4	25

RUN: +1, +2, +1 RISE: +5, +10, +5

$m = \frac{rise}{run}$

$\frac{+5}{+1} = 5$, $\frac{+10}{+2} = 5$, $\frac{+5}{+1} = 5$

∴ linear

x	y
0	0
1	1
2	4
3	9

RUN: +1, +1 RISE: +1, +3

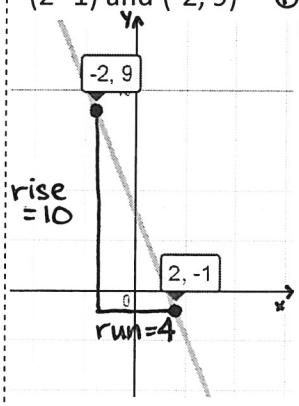
$m = \frac{rise}{run}$

$\frac{+1}{+1} = 1$, $\frac{+3}{+1} = 3$

∴ non-linear

Given 2 points, write an equation for the line that connects them:

(2 -1) and (-2, 9)



① Slope:

$m = \frac{rise}{run}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 - (-1)}{-2 - 2}$

$= \frac{9 + 1}{-4}$

$= \frac{10}{-4}$

$= \frac{5}{-2}$

$= -2.5$

② Sub. $m = -2.5$:

$y = mx + b$

$y = -2.5x + b$

③ Sub. (2, -1) for x, y. Find b.

$y = -2.5x + b$

$-1 = -2.5(2) + b$

$-1 = -5 + b$

$+5 \quad +5$

$4 = 0 + b$

$b = 4$

y-intercept

④ Sub. $m = -2.5$ & $b = 4$:

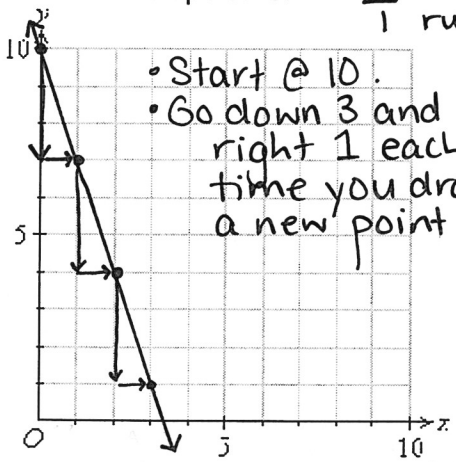
$y = mx + b$

$y = -2.5x + 4$

or

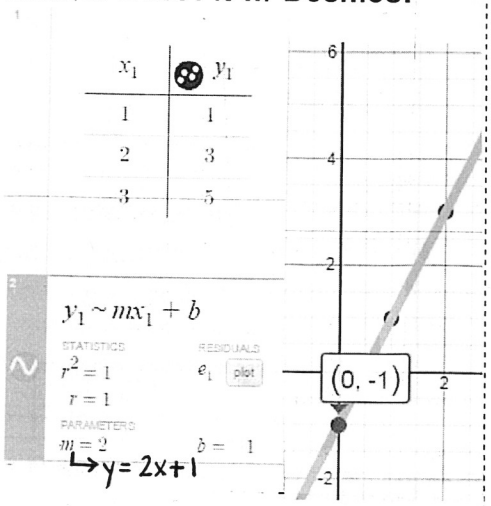
$y = -\frac{5}{2}x + 4$

Graph a line that has a y-intercept of 10 and a slope of -3. $m = \frac{-3}{1}$ $\frac{rise}{run}$



- Start @ 10.
- Go down 3 and right 1 each time you draw a new point.

Line of best fit w/ Desmos:



$\hookrightarrow y = 2x + 1$

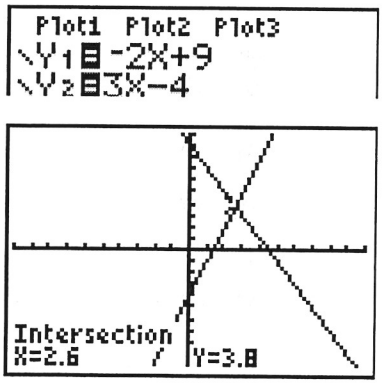
LR3: Systems of Equations **LR = Linear Relations**

Linear System of Equations:	Point of Intersection:
Elimination method:	Substitution method:

Solve a system by graphing:

$$y = -2x + 9$$
$$y = 3x - 4$$

Using a graphing calculator:

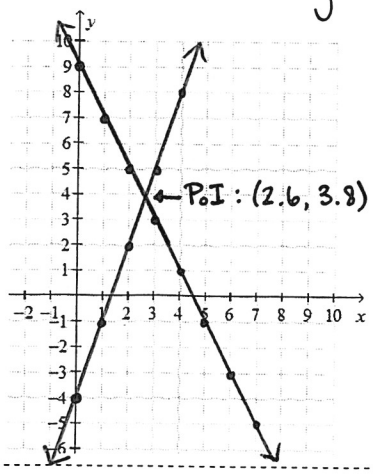


∴ The point of intersection is (2.6, 3.8)

By hand:

$y = -2x + 9$
Start @ 9.
Move down 2 & right 1.

$y = \frac{3}{1}x - 4$
Start @ −4
Move up 3 & right 1.



Solve a system using elimination:

$$2x + 3y = 5$$
$$[3x + y = 11] \times 3$$

① $2x + 3y = 5$
② $9x + 3y = 33$

Subtract ② − ① to eliminate y.

② $9x + 3y = 33$
① $-\ 2x - 3y = -5$

$7x + 0y = 28$

$\frac{7x}{7} = \frac{28}{7}$

$x = 4$

Substitute $x = 4$ into equation ① or ②

① $2x + 3y = 5$
$2(4) + 3y = 5$
$8 + 3y = 5$
$-8 \quad -8$

$3y = -3$

$\frac{3y}{3} = \frac{-3}{3}$

$y = -1$

∴ P.I : (4, −1)

Solve a system using substitution:

① $d = 0.2t$
② $d = 0.5(t - 6)$

Substitute ① $d = 0.2t$ into:

② $d = 0.5(t - 6)$
$0.2t = 0.5(t - 6)$
$0.2t = 0.5t - 6 \times 0.5$
$0.2t = 0.5t - 3$
$-0.5t \quad -0.5t$
$-0.3t =$

$\frac{-0.3t}{-0.3} = \frac{-3}{-0.3}$

$t = 10$

Sub $t = 10$ into:
① $d = 0.2t$
$d = 0.2(10)$
$= 2$

∴ P.I : (10, 2)

① $3x + 2y = 19$
② $x + y = 8$

Isolate x or y in an equation:

② $x + y = 8$
$-x \quad -x$
$y = 8 - x$

Sub $y = 8 - x$ into
① $3x + 2y = 19$
$3x + 2(8 - x) = 19$
$3x + 2(8) - x(2) = 19$
$3x + 16 - 2x = 19$
$3x - 2x + 16 = 19$
$1x + 16 = 19$
$-16 \quad -16$
$x = 3$

Sub $x = 3$ into
② $x + y = 8$
$3 + y = 8$
$-3 \quad -3$
$y = 5$

∴ P.I : (3, 5)

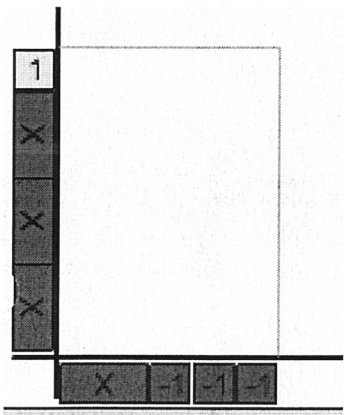
QR1: Expanding & Factoring **QR = Quadratic Relations**

Expand:	Factor:	Simplify:

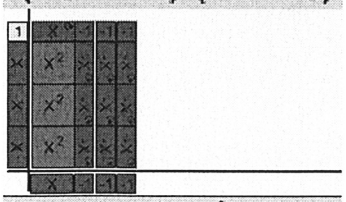
Factored form:	Standard form:

Expand & simplify:

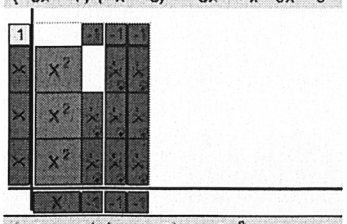
$y = (3x+1)(x-3)$



(3x + 1) (x - 3)



(3x + 1) (x - 3) = $3x^2 + x - 9x - 3$



(3x + 1) (x - 3) = $3x^2 - 8x - 3$

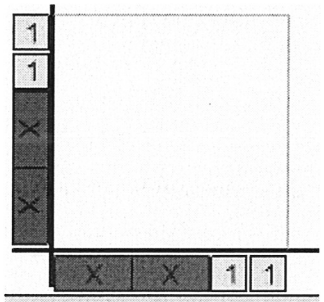
x	3x	+1
x	$3x^2$	+1x
-3	-9x	-3

$y = 3x^2 + 1x - 9x - 3$

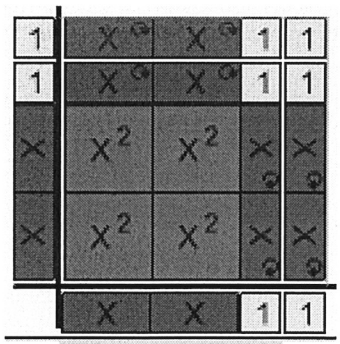
$\therefore y = 3x^2 - 8x - 3$

Expand & simplify:

$y = (2x+2)^2$



(2x + 2) (2x + 2)



$4x^2 + 8x + 4$

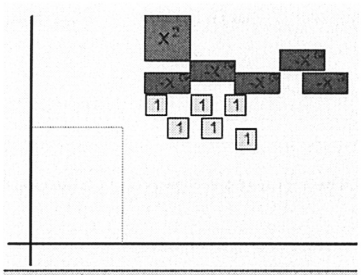
x	2x	+2
2x	$4x^2$	+4x
+2	+4x	+4

$y = 4x^2 + 4x + 4x + 4$

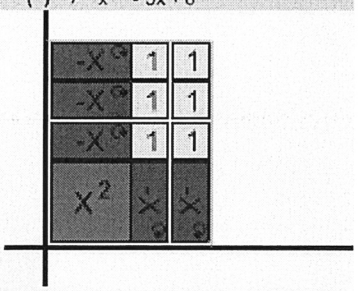
$\therefore y = 4x^2 + 8x + 4$

Factor:

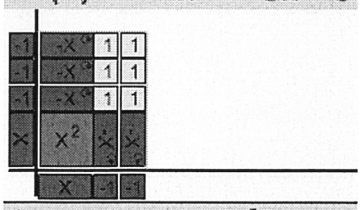
$y = x^2 - 5x + 6$



( ) ≠ $x^2 - 5x + 6$



( ) ≠ $x^2 - 5x + 6$

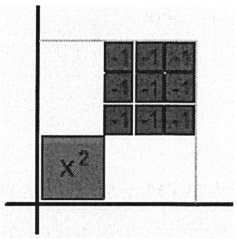


(x - 3) (x - 2) = $x^2 - 5x + 6$

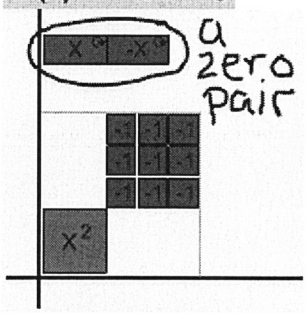
$\therefore y = (x-3)(x-2)$

Factor:

$y = x^2 - 9$

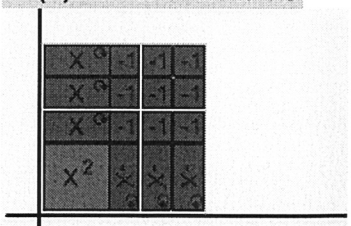


( ) ≠ $x^2 - 9$

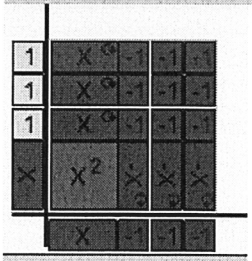


a zero pair

( ) ≠ $x^2 + x - x - 9$



( ) ≠ $x^2 + 3x - 3x - 9$



(x + 3) (x - 3)

$\therefore y = (x+3)(x-3)$

QR2: Characteristics of Quadratic Relations **QR = Quadratic Relations**

Quadratic:	Parabola:	Second differences:	Axis of symmetry:

Vertex/Maximum/ Minimum:	y-intercept:	x-intercepts/zeros:	
			$y = ax^2 + bx + c$ $y = (x-r)(x-s)$

Determine whether or not a relationship is quadratic:

x	y	1st diff	2nd difference
-1	16		
		-14	
0	2		+10
		-4	
1	-2		+10
		+6	
2	4		+10
		+16	
3	20		+10
		+26	
4	46		

+1 +1 +1 +1

∴ quadratic

x	y
0	4
	+8
1	12
	+24
2	36
	+72
3	108
	+216
4	324

+1 +1 +1 +1

+16 +48 +144

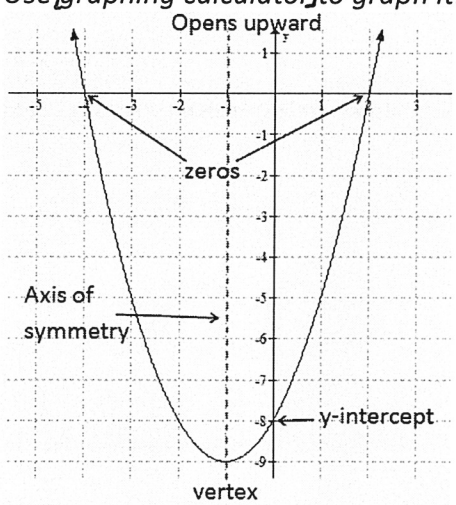
∴ non-linear

∴ not quadratic

Identify the characteristics of a quadratic relation:

$$y = x^2 + 2x - 8$$

Use [graphing calculator OR Desmos] to graph it:



Opens: upward

Axis of symmetry: $x = -1$

Vertex: $(-1, -9)$

Maximum or minimum? min.

y-intercept: -8 OR $(0, -8)$

x-intercepts/zeros: -4 and 2

$(-4, 0)$ and $(2, 0)$

Find the intercepts using the equation:

① Factored form:

$$y = (x-3)(x+5)$$

x · int. OR zeros

flip signs

+3 and −5

② Expand → standard form

x	x	−3
x	x^2	−3x
+5	5x	−15

$y = x^2 - 3x + 5x - 15$

$y = x^2 + 2x - 15$

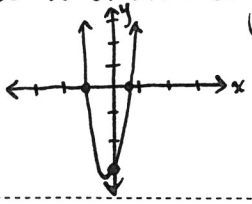
y · int. same sign

−15

∴ zeros: +3 and −5

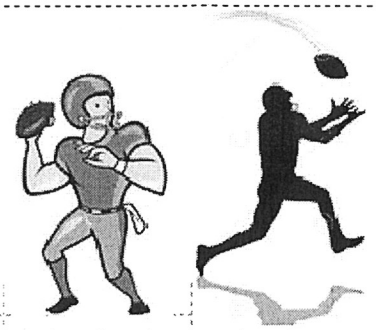
y-intercept: −15

Can use to sketch a graph:



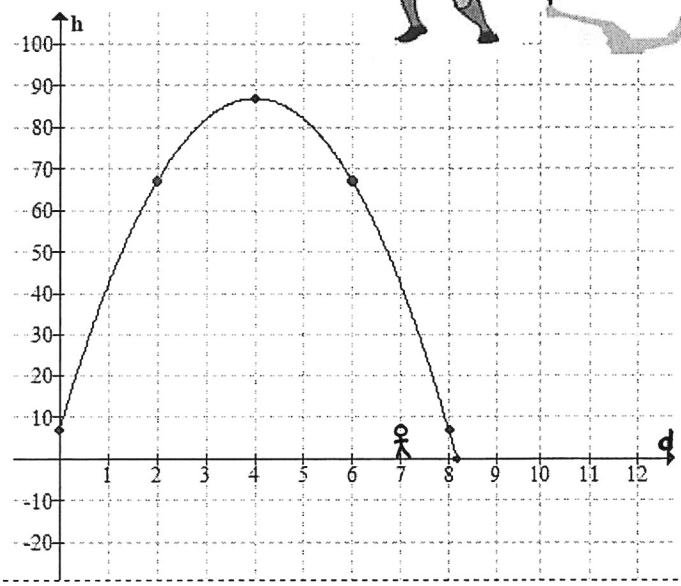
Scenario:
A football is thrown.

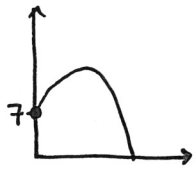
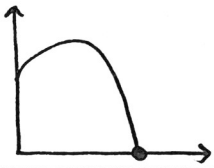
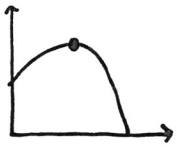
Its path can be modelled by the relation $h = -5d^2 + 40d + 7$,
where *h* is the height of the football in feet
and *d* is the distance the ball travels in meters.



Graph it (can use a graphing calculator):

d	Sub in *d* to calculate *h*	*h*
0	$-5(0)^2 + 40(0) + 7$	7
2	$-5(2)^2 + 40(2) + 7$	67
4	$-5(4)^2 + 40(4) + 7$	87
6	$-5(6)^2 + 40(6) + 7$	67
8	$-5(8)^2 + 40(8) + 7$	7



oint:	*Interpretation of the point:*
y-intercept: 7 	The point from which the football is thrown. It is thrown from a height of 7 m.
x-intercept: 8.2 	The point at which the football lands on the ground, 8.2 m away from the thrower. The football travels a distance of 8.2 m.
Vertex: (4, 87) 	The heighest point the ball reaches is 87 m. The ball is 4m away from the thrower at this point.
A second player is standing 7 m away from the thrower. Could the 2nd player catch the football?	When the ball is 7m away from the thrower, it is approximately 42 feet up in the air. 42 m is too high for player 2 to catch (who's hands would reach about 8ft high). ∴ No, player 2 could <u>not</u> catch the football.

Communicating

Explanations Conventions
Notations Terminology

- *Can you explain this another way?*
- *Have you organized your work step by step? Are the steps numbered? Did you label each step?*
- *Did you explain your thinking along the way?*
- *Have you used symbols, keywords & formatting that everyone in your class would understand?*

Selecting Tools & Computational Strategies

Tools:

- Calculator
- Graphing calculator or software (e.g. Desmos)
- Databases & statistical programs (e.g. Excel)
- Dynamic geometry software (e.g. Geogebra)
- Manipulatives (e.g. algebra tiles, linking cubes)

Computational Strategies:

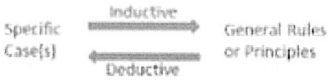
- Mental math
- Estimation
- Algorithms (+, -, x, ÷, ...)

Which one(s) did you use? Why?

Reasoning & Proving

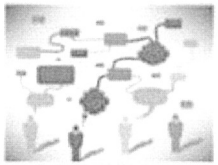
explore phenomena
develop ideas
make a hypothesis
investigate → generalize
extend the pattern
justify results
formulate a proof

Have you verified your answer?



Connecting

- *Describe a similar/simpler problem you've solved before & how you solved it.*
- *What prior knowledge do you have that can help help you solve this problem?*
- *How do the different representations I used connect with each other?*
- *Which math concepts or procedures are related to this problem?*
- *When/How would we use this math in daily life?*



REFLECTING

- *Is your answer reasonable? How do you know?*
- *How close was your estimate?*
- *Is there another way of solving?*
- *Which method is most effective/efficient?*
- *What did you find easy/hard?*
- *Did you make any errors in calculation or thinking?*
- *How would your answer change if . . . ?*

Representing

concrete	algebraic
materials	geometric
pictures	dynamic
diagrams	software
graphs	charts
tables	words
numbers	symbols

- *Have you used as many representations as you can?*
- *Which representation seems best? Why?*

PROBLEM SOLVING

What do I know?
What do I need to find out?

What is my plan?

Solve it!

Answer and Explain

Strategies:

physical model	list
diagram	logical reasoning
pattern	table
guess & check	work backwards
make an assumption	chart