Chapter 5

On the Edges of Flow: Student Problem Solving Behavior

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Abstract. Engagement in mathematical problem solving are aspects of problem solving that are often overlooked in our efforts to improve students' problem solving abilities. In this chapter I look at this constructs through the lens of Csíkszentmihályi's theory of *flow*. Studying the problem solving habits of students within a problem solving environment specifically designed to induce flow I look specifically at student behavior when there is an imbalance between students' problem solving skills and the challenge of the task at hand. Results indicate that students have higher than expected perseverance in the face of challenge and tolerance in the face of the mundane, and use these as buffers while autonomously correcting the imbalance. Emerging from this research is an extension to Csíkszentmihályi's theory of *flow* and support for the teaching methods emerging out of my earlier work on *Building Thinking Classrooms.*

5.1 Flow and the optimal experience

In the early 1970's Mihály Csíkszentmihályi became interested in studying, what he referred to as, the *optimal experience* (1998, 1996, 1990),

> “a state in which people are so involved in an activity that nothing else seems to matter; the experience is so enjoyable that people will continue to do it even at great cost, for the sheer sake of doing it.” (Csíkszentmihályi, 1990, p.4)

The optimal experience is something we are all familiar with. It is that moment where we are so focused and so absorbed in an activity that we lose all track of time, we are un-distractible, and we are consumed by the enjoyment of the activity. As educators we have glimpses of this in our teaching and value it when we see it.

Csíkszentmihályi, in his pursuit to understand the optimal experience, studied this phenomenon across a wide and diverse set of contexts (1998, 1996, 1990). In particular, he looked at the phenomenon among musicians, artists, mathematicians, scientists, and athletes. Out of this research emerged a set of elements common to every such experience (Csíkszentmihályi, 1990):

1. There are clear goals every step of the way.
2. There is immediate feedback to one's actions.
3. There is a balance between challenges and skills.

4. Action and awareness are merged.
5. Distractions are excluded from consciousness.
6. There is no worry of failure.
7. Self-consciousness disappears.
8. The sense of time becomes distorted.
9. The activity becomes an end in itself.

The last six elements on this list are characteristics of the internal experience of the doer. That is, in describing an optimal experience a doer would claim that their sense of time had become distorted, that they were not easily distracted, and that they were not worried about failure. They would also describe a state in which their awareness of their actions faded from their attention and, as such, they were not self-conscious about what they were doing. Finally, they would say that the value in the process was in the doing – that the activity becomes an end in itself.

In contrast, the first three elements on this list can be seen as characteristics external to the doer, existing in the environment of the activity, and crucial to occasioning of the optimal experience. The doer must be in an environment wherein there are clear goals, immediate feedback, and there is a balance between the challenge of the activity and the abilities of the doer.

This balance between challenge and ability is central to Csíkszentmihályi's (1998, 1996, 1990) analysis of the optimal experience and comes into sharp focus when we consider the consequences of having an imbalance in this system. For example, if the challenge of the activity far exceeds a person's ability they are likely to experience a feeling of frustration. Conversely, if their ability far exceeds the challenge offered by the activity they are apt to become bored. When there is a balance in this system a state of, what Csíkszentmihályi refers to as, *flow* is created (see fig. 1). Flow is, in brief, the term Csíkszentmihályi used to encapsulate the essence of optimal experience and the nine aforementioned elements into a single emotional-cognitive construct.

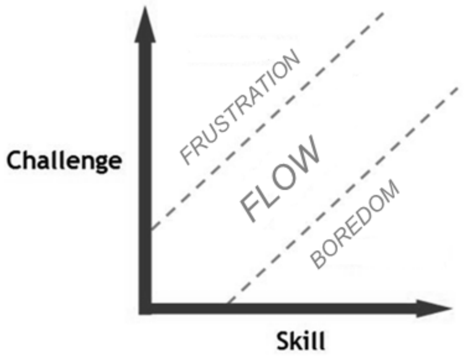


Fig. 5.1 Graphical representation of the balance between challenge and skill

5.1.1 Flow in Mathematics Education

Flow is one of the only ways for us, as mathematics education researchers, to talk productively about the phenomenon of engagement. The nine aforementioned elements of flow gives us not only a vocabulary for talking about aspects of the subjective personal experience of engagement, but it also gives us a way to think about the potential environments that occasion engagement in our classrooms.

Williams (2001) used Csíkszentmihályi's idea of flow and applied it to a specific instance of problem solving that she refers to as discovered complexity. Discovered complexity is a state that occurs when a problem solver, or a group of problem solvers, encounter complexities that were not evident at the onset of the task and are within their zone of proximal development (Vygotsky, 1978). This occurs when the solver(s) "spontaneously formulate a question (intellectual challenge) that is resolved as they work with unfamiliar mathematical ideas" (p. 378). Such an encounter will capture, and hold, the engagement of the problem solver(s) in a way that satisfies the conditions of flow. What Williams' framework describes is the deep engagement that is sometimes observed in students working on a problem solving task during a single problem solving session.

Extending this work, I used the notion of flow to look at situations of engagement extended over several days or weeks wherein students return to the same task, again and again, until a problem is solved (Liljedahl, 2006). The results of this work showed that although flow was present in each of the discrete problem solving encounters, what allowed the engagement to sustain itself across multiple encounters was a series of discovered complexities in each session linking together to form what I referred to as a *chain of discovery*.

5.1.2 Flow as a framework for describing teaching

In prior research (Liljedahl, 2016a), I looked at the practices of two teachers through the lens of flow in general and their ability to set clear goals, provide instant feedback, and maintain a balance between challenge and skill in particular. From this a number of conclusions emerged. First, thinking about flow as existing in that balance between skill and challenge, as represented in figure 5.1, obfuscates the fact that this is not a static relationship. Flow is not the range of fixed ability-challenge pairings wherein the difference between skill and challenge are within some acceptable range. Flow is, in fact, a dynamic process. As students engage in an activity their skills will, invariably, improve. In order for these students to stay in flow the challenge of the task must similarly increase (see fig. 5.2).

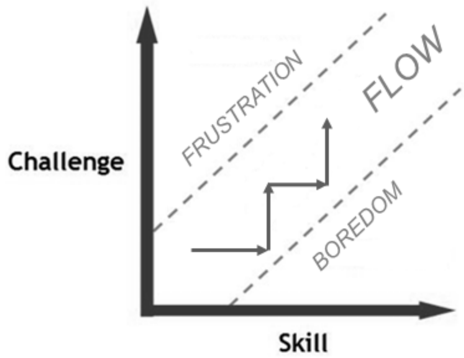
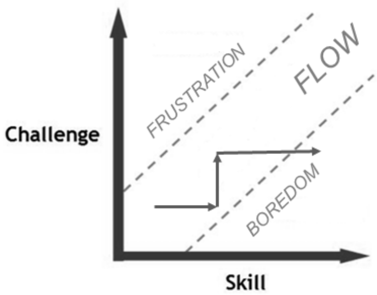


Fig. 5.2 Graphical representation of the balance between challenge and skill as a dynamic process

In a mathematics classroom, these timely increases of challenge often fall to the teacher. But this is not without obstacles. For example, if a student's skill increase either too quickly or too covertly for the teacher to notice that student may slip into a state of boredom (see fig. 5.3). Likewise, when the teacher does increase the challenge if that increase is too great the student may become frustrated (see fig. 5.4).



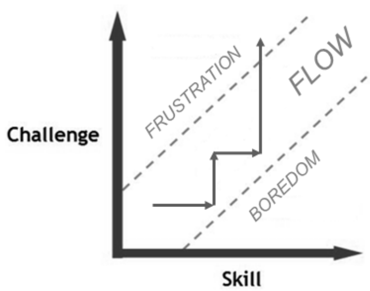


Fig. 5.3 Too fast an increase in skill **Fig. 5.4** Too great an increase in challenge

This leads to the second conclusion from the aforementioned research (Liljedahl, 2016a). How teachers manage these situations of boredom and frustration is important. In the study one of the teachers managed such situations synchronously, either giving hints or extensions to the class as a whole, usually after three groups finished or she got three of the same questions respectively. For most groups the timing of these hints and extensions was off, and not helpful in maintaining flow. The second teacher, however, managed these situations asynchronously, dealing with groups individually as they got stuck or completed a problem. Student engagement in the second teacher's class was visibly higher as he was maintaining flow through the constant and timely maintenance of the balance between ability and complexity for each group. In short, timing matters.

5.1.3 The Next Stage

Given the importance of timing, the reality is that the teacher is not always going to be able to get to every student or every group just as an imbalance between challenge and skill occurs. In the research presented in this chapter I look more closely at this phenomenon in general, and how students cope autonomously with an imbalance between their skills and the challenge of the task at hand.

5.2 Methodology

To answer this question I observed student collaborative and individual problem solving work within two carefully selected classrooms. In what follows I describe this setting as well as the methods I use to capture and analyze the data.

5.2.1 The problem solving setting

To get at the answers to this question I needed to observe students in settings that were natural to them and where their work was visible. To this end I strategically selected two classrooms belonging to two different teachers – Cameron and Charmaine. Both of these teachers conducted their classrooms according to a teaching framework called *building thinking classrooms* (Liljedahl, 2016b, 2016c, 2014)[1].

This framework is predicated on a desire to design "a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion. It is a space wherein the teacher not only fosters thinking but also expects it, both implicitly and explicitly" (Liljedahl, 2016b, p.364).

My earlier empirical work on the design of such spaces emerged a collection of nine elements that both describes a thinking classroom and offers a prescriptive framework for teachers to building such a classroom. For both Cameron and Charmaine, five of these elements are particularly salient for describing their classroom norms (Yackel & Rasmussen, 2002).

1. At the beginning of every class, students are assigned to a visibly random group (Liljedahl, 2016b, 2016c, 2014) of two to four students. These groups will work together on assigned problem solving tasks for the duration of the lesson.
2. Once in groups, the lessons begin with the assignment of tasks to be solved. In the beginning of the school year these tasks are highly engaging, non-curricular, collaborative tasks that drive students to want to talk with each other as they try to solve them (Liljedahl, 2008). After a period of time (usually 2-3 weeks) these are gradually replaced with curricular problem solving tasks that permeate the entirety of the lesson and emerge rich mathematics (Schoenfeld, 1985) that can be linked to the curriculum content to be 'taught' that day[2].
3. The work on these aforementioned problem solving activities are done with groups standing a vertical non-permanent surfaces such as whiteboards, black-boards, or windows (Liljedahl, 2016b, 2016c). This makes visible all work being done, not just to the teacher but to the groups doing the work. To facilitate discussion, there is only one felt pen or piece of chalk per group.
4. Throughout this work, student engagement is maintained through the teacher's judicious and timely use of hints and extensions (Liljedahl, 2016a, 2016b). Csíkszentmihályi's theory of flow (1998, 1996, 1990) is the framework for thinking about this. Hints and extensions need to be given so as to keep students in a perfect balance between the challenge of the current task and their abilities in working on it.
5. At some point within this sequence of tasks the teacher will pull the students together for a debrief of what they have been doing. At this time the teacher

[1] The second teacher in the prior research study (Liljedahl, 2016b, 2016c, 2014) was also teaching according to this framework.

[2] Although the curricula tasks are simply questions from the textbook I characterize them as problem solving tasks because they are often new to the students, present something that is problematic for them, and often cause them to be stuck. However, as will be seen in the presentation of results, in some cases the tasks become rudimentary for the students.

will either go over one or more of the students' solutions or work through a new problem together with the class as a whole. This helps reify the work the students have been doing and is timed so that every group is able to participate in discussion and benefit from the reification. In the *Building Thinking Classroom* framework, this activity is called *levelling to the bottom* (Liljedahl, 2016b)[3].

Both Cameron's and Charmaine's classrooms are guided by these principals of teaching. This is not to say that their classrooms are identical. Charmaine, for example makes regular use of non-curricular tasks for the beginning part of her lesson (2-3 times a week). She finds these motivate the students. Charmaine *levels to the bottom* with about 25-30 minutes left in class after which she assigns 5-8 questions for practice for the students to do as they wish. Generally, the students take advantage of the time allotted (usually 20 minutes) for this to complete them in class. About half of the students do so in self-identified groups working at a board. The other half of the class completes these sitting in their desks either on their own or in small self-determined groups.

Cameron, on the other hand tends to use only curricular problems. And rather than assign practice questions he just keeps the students working on progressively harder curricular problems in their random groups for the whole period. He feels that this is sufficient practice for the students and feels no need to assign further work for them to do. Cameron tends to *level to the bottoml* twice in every lesson – once after all groups have gotten through 2-3 problems and once at the very end of the lesson.

For the purposes of the research presented here, both of these classrooms offered the affordances for me to easily observe students working within and an environment designed to occasion flow. The teachers were both managing engagement through the timely use of hints and extensions to maintain a balance between the challenge of an activity and the ability of each group. The student work was visible and there was enough autonomy afforded in the room that the students would need to take action when they found themselves in a situation where challenge and ability may be out of balance.

5.2.2 The data

Data for the research presented in this classroom were collected in Cameron's grade 12 Pre-calculus class and Charmaine's grade 11 pre-calculus class[4]. Each

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[3] The term *levelling* originally comes from Schoenfled (1985) and refers to that moment when a teacher will go over the solution to a problem or exercise students have been working on. *Levelling to the bottom* specifies when that levelling is to occur.

[4] In the province where this research was conducted these are the academic streams of mathematics laddering towards university level calculus. Grade 11 students are typically 16 or 17 years old and grade 12 student are typically 17 or 18 years old.

class was visited five times over an seven week period in the middle of the second semester[5].

When doing the aforementioned research on teachers' actions vis-à-vis their efforts to occasion flow (Liljedahl, 2016a) I used video recorded data. This was a natural fit as I had only one subject to track. For the research presented here, however, video turned out to be prohibitive for two reasons. First, as I was looking for very specific situations within the classroom I needed to be constantly monitoring student activity around the room. Trying to do so through a digital video recorder narrowed my field of view too much to quickly scan what was emerging around the room. As such, I used instead a variant of the methodology of noticing (Jacobs, Lamb, & Philipp, 2010; Mason, 2011, 2002; van Es, 2011). Rather than scanning the room for emergent phenomena, however, I was looking for one of two very particular phenomena – situations where a group of individual student's abilities exceeded the challenge of the task (see fig. 5.3) or the challenge of the task exceeded the ability of the group or individual student (see fig. 5.4). I call these *moments of imbalance*.

Once a *moment of imbalance* was identified I would focus in on that group or that individual. I initially tried video recording these instances of imbalance but this was too intrusive for the students. I learned early that these instances are very delicate and the slightest disturbance would collapse the moment. Instead, I would observe silently, taking detailed field notes and occasional photographs – both of which the students were used to having their teachers do as a regular part of their teaching. When these moment seemed to be waning I would then conduct short in-the-moment interviews.

There is an efficiency in this methodology in that all of the data that was generated was relevant to the phenomenon of interest.

All three of these forms of data (field notes, photographs, and audio recordings) were accomplished using an iPad app called Notability™. This app not only allows for the simultaneous taking of hand written notes, photographs, and audio recordings, but synchronizes the artifacts of notes and photographs with the audio track. That is, upon playback the selection of any one note or photo will cue the audio to the instance when those artifacts were created.

5.2.3 Analysis of data

Csíkszentmihályi's theory of flow (1998, 1996, 1990) in general and the imbalance between student ability and task challenge in particular were central to the identi-

[5] In the province where this research was done schools are either semestered or linear. Linear schools have students taking eight classes on a rotation schedule over the course of the whole year. Semestered schools have students taking the same four classes every day for the first half of the school year (first semester) and another four classes every day for the second half of the school year (second semester).

fication of moments of interest. The analysis of these moments, however, was focused more on students' actions and reactions in these moments of imbalance. To this end, the data were analyzed using analytic deduction (Patton, 2002). That is, I was looking for the emergence of codes through a constant comparative method in general, and the emergence of similar behaviors under similar moments of imbalance in particular.

5.3 Results and analysis

From this analysis a series of six nuanced moments of imbalance emerged, each marked by a different type of student action or reaction. In what follows I present cases exemplifying each of these moments as well as some general comments about similar such cases. These six nuanced moment of imbalanced are broken into two main categories – moment wherein the skill of the students exceeds the challenge of the task at hand and moments wherein the challenge of the task at hand exceed the skills of the students.

5.3.1 When skills exceed challenge

There were three distinct types of reactions by individual or groups students when faced with a situation wherein their skills exceeded the challenge of the task at hand. Contrary to Csíkszentmihályi's theory of flow (1998, 1996, 1990), not all of these reactions were boredom.

5.3.1.1 The case of quitting

Within the data there were several moments wherein students quit. A portion of these cases were as a result of the students being bored with the activity at hand. To exemplify this I present the case of Mikaela and Allison, two students in Charmaine's class.

As mentioned, at the end of each lesson Charmain assigned 5-8 questions for the students to work on with whomever they wished and on whatever surface they wished. During the five lessons I observed in Charmaine's class I noticed that during this phase of the lesson about half of the students chose to work at their desks and half of the student chose to work on the vertical spaces. At each location, a few students worked on their own while the rest worked in groups of two or three. At the end of one of these lesson I observed in Charmaine's class my attention was drawn to Mikaela and Allison.

These two girls had not been in the same group during the random grouping during the main part of the lesson, but for this portion of the class they had chosen to work together and they had chosen to do so sitting down at their desks. What drew my attention to them was that they seemed to be off task, animatedly talking about something not related to mathematics. As I start to attend to them I notice that they had finished the first two questions assigned to them. After some period of non-mathematical discussion Mikaela says, "Ok. Let's do the next one."

The two girls looked at the text book they were sharing, wrote down the third question and then independently solved it quickly and without difficulty. They compared answers and then resumed their previous off task conversation. I lingered behind them waiting to see if one or the other would prompt the other to do the next question. After ten minutes without a return to the mathematics assigned I decide to interview the pair.

Researcher	I notice you are not working on the assigned questions. What's up?
Mikaela	We did some of them.
Researcher	I saw that. I noticed that you did two very quickly. Took a little break from the math and then went back and did another one. I was sort of waiting to see if you would get back to it.
Allison	This stuff is easy. I'll finish it at home on my own.
Mikaela	Its actually too easy. I don't even think I will bother finishing it at home.
Allison	… Yeah. I probably won't either.
Researcher	It's easy? Is that why you stopped working on it?
Mikaela	Yeah.
Researcher	I saw you two work together before at the end of class. I don't recall seeing you two giving up before.
Allison	We aren't giving up. Sometimes we don't finish all the questions because they are hard and we run out of time. But these are easy.
Researcher	What makes them easy.
Mikaela	They just are. The first three are exactly the same and we could do them no problem.

In my conversation with Charmaine after class she confirmed for me that these girls often work together but was surprised when I told her about my conversation with them.

Charmaine	Allison and Mikaela always work together at the end of class and they are usually very diligent. I'm surprised that they were off-task. That's not their style.

During the ten lessons I observed in Cameron's and Charmaine's classes I only managed to capture three other instances that I would say fall into the same category – quitting because the students were bored by seemingly too easy a collection of tasks. Each of these cases occurred in Charmaine's class, occurred during the last phase of the class where students worked on questions out of the text, involved students who were sitting down, and involved students who were not normally off-task. And in each case the students perceived the questions they were working on to either be easy and/or redundant. I say perceived because in the case of Mikaela and Allison, although the first three questions were easy and redundant the next three questions were not redundant and were quite a bit more challenging.

5.3.1.2 The case of seeking increased challenge

Quitting out of boredom was not the only reaction to a situation where the skills of a group or of an individual exceeded the challenge of the task at hand. Some students opted, instead, to autonomously seek increased challenge. To exemplify this I look at a case from Cameron's class captured while students were working at the whiteboards in randomly assigned groups. During this part of the lesson Cameron was circulating the room helping groups that were stuck (or have made a mistake) and giving more challenging questions to groups that were done. Cameron is very deliberate about how he does this. Before a group can get his help or the next question he engages the group in conversation to assess where the group's thinking is at – both as a collective and individually. This takes time and sometimes groups that are done are left waiting.

During one of my visits to his class my attention was drawn to a group of three boys – Carl, Ameer, and Colton. What I had noticed about them was that these three boys were smoothly moving through all of the questions in Cameron's repertoire, and they were doing so without Cameron once having had visited them to give them the next question. What they were doing was pulling the questions from the visible work of groups that Cameron had visited and given the next question to. I watched them do this for 30 minutes during which I began to discern the *modus operandi* of the group.

In essence, this group of boys used the visible work of others around them to not only pull new questions, but to also check solutions to questions they had already solved. Sometimes they did this remotely, just through observation. Other times, especially when answers didn't match, they engaged their peers in discussions.

After 30 minutes of watching them work like this I decide to ask them about what they were doing.

Researcher	So, I notice that you guys are now on question 5 and your teacher has not visited you once. How are you getting your questions?
Ameer	We just look around and see what the next question is and do that one.
Researcher	What would your teacher say about that?
Carl	Um … he'd probably want to check to see that we got the previous one before giving us the next one …
Ameer	… but we are doing that.
Researcher	Why don't you just wait for your teacher to get here and give you the next question?
Carl	We're on a roll. And sometimes we have to wait a long time.
Researcher	Do you realize that you are doing the problems out of sequence from the order your teacher is giving them?
Colton	Oh really? That’s probably why some were so hard.

It turns out that this was a very common behavior in both Cameron's and Charmaine's classrooms. Rather than wait for their teacher to give them the next questions groups were opting, instead, to move on on their own. For the most part they did this just by pulling the next question from groups that were ahead of them. This was facilitated by the visible nature of the work afforded by the vertical non-permanent surfaces that the classes were working on.

Related to this behavior, but much less common, was the phenomenon of students creating their own extensions to problems they had been working on. In the ten lessons I observed, I only saw this happen twice. In both situations the groups made a change to the problem they had just solved. In one of these cases the group did this every time they were waiting for the teacher. It could be argued that this is a form of problem posing (xxx), but my sense was that the students were more trying to anticipate the teacher's next type of question than pursue a curiosity.

5.3.1.3 The case of tolerance in the face of the mundane

An altogether different reaction to being tasked with doing easy and redundant questions is to just do them – without quitting and without seeking to increase the challenge. I observed such behavior in the case of Jennifer, who always worked at her desk on her own at the end of Charmaine's lessons.

What drew my attention to Jennifer was that she seemed to be moving through the questions with great easy, never asking for help. Upon closer inspection I also noticed that, in addition to the assigned questions, she was also completing questions that were not assigned. Towards the end of one of the class I asked her about this.

Researcher	I have been watching you while I have been here. I notice that you always do a lot of questions. Can you tell me about that?
Jennifer	Yeah. I like to do a lot of questions. It's good practice. It's how I learn.
Researcher	So, are you looking for harder and harder questions to challenge yourself.
Jennifer	Not really. I just do all of them. So, if the teacher asks us to do 4a, I will also do 4bc and d and so on.
Researcher	Do you find them easy.
Jennifer	Yeah..
Researcher	How many do you do?
Jennifer	I just work the whole time at the end of class and then for maybe an hour at home.

Jennifer uses practice as a way to ensure that she is learning the content of the day. Unlike the students who quit in the face of boredom, Jennifer seems to be perfectly content working on questions that she considers to be easy for long periods of time. Her tolerance for the mundane is high.

In my time in Charmaine's class I saw two other girls who I suspect were very much like Jennifer in their approach to learning and their tolerance for the mundane. These girls also worked alone in their desks in the last part of every lesson. I also observed similar behaviours in a pair of boys working at a whiteboard towards the end of one of Charmaine's lessons.

From my conversations with Charmain I learned that these boys, Kirk and Philip, always worked together on a whiteboard to complete the end of class assignment. In the third lesson I observed I saw them exhibiting much of the same behavior I had seen in Jennifer and the other two girls. That is, they were doing questions beyond what was assigned.

Researcher	I don't think you were asked to do that question?
Kirk	We know. Sometimes it is good to check that you really know what you are doing by doing a few more just like it.
Researcher	Do you always do that?
Kirk	Not always. Usually only if we are not sure.

Like Jennifer, Kirk and Philip, are staying within equally challenging tasks to build up their understanding of the mathematics. Unlike Jennifer, however, they seem to be doing so as a way to continue to build their understanding, as opposed to just practicing.

5.3.2 When challenge exceeds skills

There were three distinct types of reactions by individual or groups students when faced with a situation wherein the challenge of the task at hand exceeded their skills. According to Csíkszentmihályi's theory of flow (1998, 1996, 1990), such an imbalance should result in frustration. This was rarely the case, however.

5.3.2.1 The case of quitting

Of course, frustration is a likely result of such an imbalance and, as such, I was on the lookout for such a reaction. I found it in three students in Cameron's classroom work on the first question of the day.

Sometimes at the beginning of a lesson both Cameron and Charmaine begin with a small lesson or a short example. But they both claim that they do this rarely and only in situations where the lesson pertains to entirely new content. As a result, most lessons begin with groups being tasked with the solving of a problem. Often, these problems are similar to questions from a previous lesson (not necessarily the lesson immediately before), or a small extension from a question encountered previously. During one of my visits to Cameron's class I observed a group of three students – Shannon, Katrina, and Robert – who seemed to be lost. After about three minutes they became quite exasperated and quit.

Researcher	I have been watching your group for a bit and I notice that you aren't working?

Robert	We gave up. This question is stupid.
Katrina	We tried, but we weren't getting anywhere. So we gave up.
Researcher	What do you think the problem is?
Shannon	This question is too hard.
Robert	… too hard. We don't get it.
Katrina	And the teacher hasn’t come over to help us.
Researcher	What kind of help are you looking for?
Shannon	You know, a hint or something.
Researcher	What would a hint do for you.
Shannon	Help us understand the question.
Katrina	… or remind us a little bit about how to do it.

For this group the question they have been asked to solve exceeded their abilities and without any help from the teacher they gave up. Interestingly, the type of help they were seeking was either to reduce the complexity of the task (*understand the question*) or increase their ability (*remind us* of what we have done in the past).

Surprisingly, in the ten lessons I observed, I only managed to capture a total of four instances of a group giving up. In each of these cases there was visible exasperation and disengagement present as well as a marked lack of progress on the task. And in each of these cases the groups claimed that the task was too difficult for them.

I also managed to capture two cases of individual students, working on questions at the end of one of Charmaine's lessons, giving up. These were harder to capture as there were many instances of students not working during this autonomous time. As expected, during such freedom some students will choose to not do anything. Most of these students told me that they were taking a break or planning to work on these questions at home. Only two students admitted to me that the questions were too difficult and they were giving up, with one of these stating, "I'll just do them at home with my tutor".

5.3.2.2 The case of getting help

A much more common reaction to facing too great challenge was for students in both classes to seek help. What this looked like, however, was much more subtle than simply asking the teacher for help. Like the case of students seeking increased challenge, many groups who were stuck sought help from the groups around them. This is nicely exemplified by the random group of Mikaela, Lena, and Michael working on a question in Charmaine's class.

I watched this particular group for an entire class. What stands out from these observations was how much they interacted with the groups around them, both passively and actively. This interaction fell into two main categories – checking answers and getting ideas.

Checking answers passively involved simply looking around the room and seeing if any other groups had arrived at the same answer as them. This became much more active if they saw an answer that differed from theirs. This happened twice during the lesson I observed. In the first instance they had a quick conversation with the group next to them. This resulted in their neighboring group changing their answer. In the second instance it involved them crossing the classroom to another group and having a lengthy discussion and then together redoing the problem as a group of six and arriving at an answer that they were all happy with.

Getting ideas passively involved one or more members of the group looking around the room at other groups' work. Interestingly, Lena did this for every problem regardless if they were stuck or not. A much more active approach was used on their last question together. For this task they were stuck for quite a while and Lena's scanning of the class had not helped the group move forward. At this point Mikaela walked over to Allison's group[6] and asked her if she knew what to do. Allison's group was making good progress and was confident about the direction they were going in, but Mikaela didn't understand what they were doing. Eventually, Lena and Michael joined Mikaela in trying to understand what was happening. Michael quickly caught on and urged both Lena and Mikaela to return to their station where he explained it to them.

After 40 minutes of observing this group I interviewed them.

Researcher	I notice that you have been moving about the room a bit. Why?
Michael	Oh. We were just stuck so we went over there to get some ideas.
Researcher	Did it help?
Michael	Oh yeah. We got it now.
Mikaela	Michael got it. It took me a little longer, but I'm good now.
Researcher	You were also moving around a little bit earlier in the class?
Mikaela	Oh, you mean when we were checking answers? Yeah, we thought we were doing something wrong, but we were good.
Researcher	Lena, you like to look around a lot.
Lena	I do? What do you mean?
Researcher	You know, when Michael was working on the board you look around a lot at the other groups.
Lena	Right. I am just making sure we are on the right track.

This sort of behavior was endemic in both classrooms with too many occurrences for me to track. The vertical work spaces facilitated the ability for groups to passively check their answers and get ideas. The random groups created the porosity (Liljedahl, 2014) that made the more active interactions and movement of ideas possible. Together it meant that groups were never wanting of help if they were stuck – thereby avoiding becoming frustrated.

5.3.2.3 The case perseverance in the face of challenge

[6] Recall that Mikaela and Allison regularly worked together when given the opportunity.

But not all groups sought help when they were stuck. In the ten lessons I observed I captured a few instances where a group or an individual opted to not seek help, either passively or actively, from the groups around them. One such group was a pair of boys working on the assignments at the end of one of Charmaine's classes. These boys, Oliver and Connor, worked persistently on one of the questions for 15 minutes without making progress. Even when Charmaine approached them they resisted her offering of help. At the end of class I asked them about this question.

Researcher	Question #5 was a tough one, huh?
Oliver	Yeah, that one took us a while.
Connor	In the end it wasn't that hard though. We were just missing something.
Researcher	Oh really. How did you figure it out?
Connor	We just kept at it and then we saw it.
Researcher	I noticed that your teacher came over to help. Did she help you?
Oliver	No, we wouldn't let her. We knew we knew how to do it and we wanted to figure it out ourselves.

I observed similar behavior in Stephanie, who worked on the assignment by herself at a whiteboard. She spent 10 minutes on the same question before moving on to other questions, and then returned several times to try it again. In the end she never did solve it in class, but she never sought any form of help from the people around her or the teacher. I spoke to her as the bell rang.

Researcher	Did you ever get it?
Stephanie	7a? Not yet. I'll work on it at home.
Researcher	Until you get it?
Stephanie	Until I get it.
Researcher	What if you don't. Will you get some help?
Stephanie	I always get it eventually.

Stephanie showed great perseverance with this task. From the interview it seems like this is a normal occurrence that she is comfortable with. Her confidence in that she will eventually solve it indicates that she is willing to persevere for long periods of time.

There were no such occurrences in Cameron's class. I suspect this is because there were no times in Cameron's lesson where the students were not immersed in an environment saturated with potential help afforded by the vertical spaces and random groups. This is not to say that the students in Cameron's class were not capable of such perseverance, but only that Charmaine's class offered an opportunity for me to observe such perseverance.

5.4 Discussion

The aforementioned six nuanced moments of imbalance show that for different individuals and different groups the transitions from flow to boredom or frustration has variable immediacy. Mikaela and Allison became bored and got off task as soon as their groups abilities exceeded the challenge of the task at hand. Similarly, Shannon, Katrina, and Robert became frustrated and gave up as soon as the challenge exceeded their ability. For these two groups, and the groups and individuals who reacted similarly to an imbalance between ability and challenge Csíkszentmihályi's original representation of flow holds (see fig. 5.5).

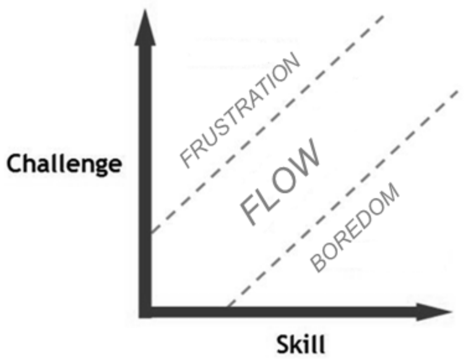


Fig. 5.5 Csíkszentmihályi's representation of the balance between challenge and skill

For Jennifer and Stephanie this transition was not as abrupt. Jennifer spent long periods of time within a space where her ability far exceeded the challenge posed by the tasks she was working on without getting off-task or quitting. The groups and individuals like Jennifer had a great tolerance for the mundane that prevented them from sliding into boredom. Likewise, Stephanie worked persistently without giving up on a task that presented too great a challenge for her ability. Groups and individuals who demonstrated the same tenacity had a great perseverance in the face of challenge that prevented them from becoming frustrated. Taken together, these two cases, and the cases like them, indicate that for some students the boundary between flow and boredom and frustration is not as thin as Csíkszentmihályi's (1998, 1996, 1990) theory of flow would imply (see fig. 5.6).

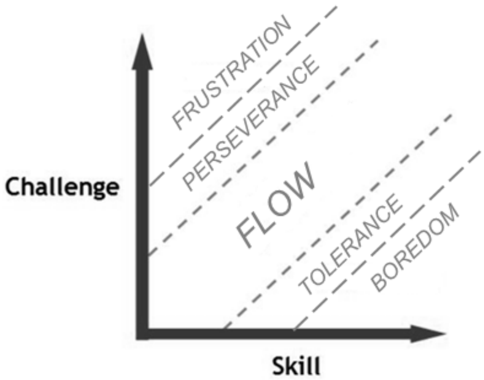
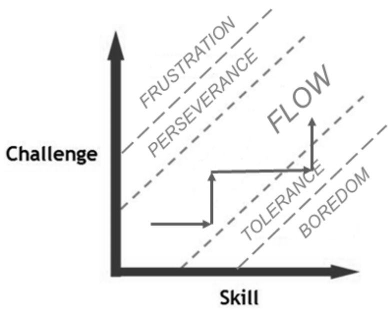


Fig. 5.6 Modified representation of the balance between challenge and skill

Other students used the buffer created by perseverance and tolerance to avoid frustration or boredom as they sought to correct the imbalance between skill and challenge that they were experiencing. Carl, Ameer, and Colton used the groups around them to passively and actively check their own answers and to seek out more challenging tasks when they were done. Similarly, Mikaela, Lena, and Michael used the groups around them to both passively and actively access help when they were stuck. These groups, and the groups and individuals like them, managed to autonomously maintain the balance between challenge and ability. When their ability was too great they autonomously sought to increase the challenge (see fig. 5.7) and when the challenge was becoming too great they autonomously sought to increase their ability or decrease the challenge (see fig. 5.8). To what degree these groups had tolerance or perseverance was not evident from the data as they too quickly managed to right the balance between skill and challenge. What was evident, however, was that when there was an imbalance these groups did not quit out of boredom or give up out of frustration. The highly visible and collaborative environments created by the use of vertical non-permanent surfaces and visibly random groups no doubt facilitated the management of this balance.



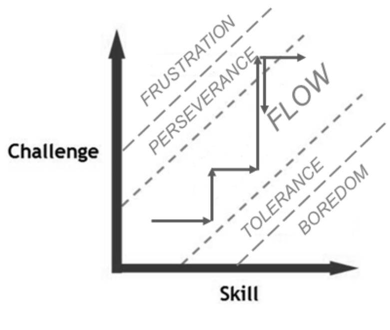


Fig. 5.7 Reaction to too great an ability. **Fig. 5.8** Reaction to too great a challenge

5.5 Conclusions

I began the research presented here as an extension of research wherein I used Csíkszentmihályi's theory of flow (1998, 1996, 1990) as a lens for looking at how teachers used hints and extensions to create and maintain engagement in their classrooms. This prior research showed that flow, as articulated by Csíkszentmihályi (1998, 1996, 1990), is an effective lens for describing effective and ineffective teachers' actions and reactions to student work.

In the research presented here, however, Csíkszentmihályi's theory of flow (1998, 1996, 1990), was insufficient for predicting the majority of student reactions when faced with an imbalance between challenge and skill. Although students did quit out of boredom and frustration, they rarely did so. Instead, they showed resilience to these kinds of imbalances in the form of either perseverance in the face of challenge or tolerance of the mundane. And they often used this resilience as a buffer while they autonomously corrected these imbalances by actively and passively seeking help or increased challenges.

To a great degree this resilience was facilitated by the collaborative and visible structures created by the participating teachers adherence to the Building Thinking Classrooms (Liljedahl, 2016b, 2016c, 2014) framework of teaching. These structures filed the space with opportunities to either access help or increase the challenge as needed.

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