The Transition From High School Mathematics To First Year Calculus

by

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Abstract

Student preparedness for first-year calculus has been an ongoing concern for post-secondary programs and their respective career paths. Researchers have investigated the benefits and pitfalls of prior calculus knowledge, and the general development of academic competence in an effort to improve student success and retention. Most of the literature is survey based, rather than anecdotal, and serves to inform universities about their own student populations, rather than to inform incoming students about how to be successful. This study compares the perspectives of students and lectures based on anecdotal responses to related questions, and identifies the expectations and habits that are different from students' high school experiences. The results showed most differences were in regards to the amount of time students needed to spend on homework problems, their problem-solving skills and level of engagement with their homework; and the level of self motivation and independence that was needed for success.

Keywords: transition to calculus from high school mathematics; student perceptions of university mathematics, university lecturer perceptions of first-year calculus students; student success in first-year calculus.

Dedication

This thesis is dedicated to my mother, Koki, who would have been very pleased to know that I had undertaken it. I also dedicate this to my husband Hans, who supported me throughout the process. I am grateful to have had the opportunity to pursue this task to its' completion.

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List of Acronyms

CUPM	Committee on Undergraduate Program in Mathematics
MAA	Mathematical Association of America
NCTM	National Council of Teachers of Mathematics
PDP	Professional Development Program
SFU	Simon Fraser University
STEM	Science Technology Engineering and Mathematics
U of A	University of Alberta
UBC	University of British Columbia

Chapter 1.

Introduction

What is mathematics?

What does it mean to do mathematics?

What does it mean to teach mathematics?

What does it mean to learn mathematics?

These are some of the questions the professors and instructors in the Master of Mathematics Education program at Simon Fraser University challenged me to think about over the course of my studies. As the program progressed, I noticed our collective answers as a class, were evolving. Everyone who either does mathematics or teaches mathematics has a sense of how they would answer those fundamental questions, but few of us would say exactly the same things. Those questions never occurred to me during my undergraduate mathematics courses, but I think if they had, I would have approached my mathematics courses differently. In fact, if they had, I'm sure my experience in those undergraduate courses would have been very different.

I enjoyed mathematics, but sometimes I felt like I was on the outside looking in. I had gone back to university at the age of 25; it had been 7 years since my last high school mathematics course. Looking back, my experiences on the front lines as a first-year university student were fraught with stress and exhaustion. I never quite felt like I knew what I was supposed to. I had finished high school with respectable marks, but had never really learned how to learn on my own, or study in a way that was effective in my first year of university. I think I passed my first-year courses in spite of my study habits, not because of them. My experience with mathematics courses were triage-based: I put my study efforts into facilitating survival, not success. My assignments were often completed at the last minute, since I routinely underestimated how much time would be needed to complete them, and to study for my other courses. Since it had been so long since I had done high school mathematics, I did not have the background

knowledge that was expected, and had to overcome this by working with a study group for almost all of my assignments. As I finished a variety of mathematics courses, I found myself attracted to the underlying logic behind everything, but was acutely aware that my high school courses – at least my own engagement with them, had not provided me with the tools required to understand the underlying structures and concepts needed to effectively engage with university-level topics; consequently, I was always one-step behind, yet doing three times the work of me peers. Or so it seemed. In spite of this, I really enjoyed studying mathematics, and I finished a BSc. in Mathematics and Chemistry, and found work at an environmental testing lab. As often happens (so I was told), people who go into sciences find themselves feeling unfulfilled, and crave something more altruistic like research or teaching. As intriguing as research can be, I found myself drawn more to teaching. My experience with scientific research so far had me feeling like more of a cog in a machine than a valued contributor of ideas, so I completed the PDP program at Simon Fraser University in 2006, and have been a teacher ever since. My undergraduate degree is a general degree in mathematics and chemistry, but over the years, my imagination has been captured more by teaching mathematics, than by teaching chemistry. This has been due in no small part to the Master in Mathematics Education program and courses at SFU, and in particular it's combination of depth and breadth in a variety of topics in mathematics education.

One by-product of the master's program for me was a new interest in research. I had come to appreciate how much can be learned from really simple questions. The right question provides insights into the thoughts, values, and attitudes of the research participant (in this case the student). When interviewing my first research participant, I asked "what would you do to convert an hour and 35 minutes to a decimal number using decimals?" and was intrigued when they used proportional reasoning strategies instead of the decimal conversion that I had been expecting. This student had critical thinking skills that I hadn't been giving him credit for – which both concerned and inspired me.

Another really significant by-product of the master's program was a new appreciation of the power of group-work in high school, and in particular, random group assignments. Each member of the group had something to gain from the insights of the others, creating an enriched learning environment, and allowing the group to reach conclusions that they may not have been able to as individuals. This had not been my classroom experience in high school at all. But I did feel that based on my own

experiences in university, as well as how I saw it effecting my own students, it was an important component of a high-school student's learning environment.

My understanding of what it means to do mathematics was changing as I participated in the master's program. It had come to resemble more of an intricate web of ideas and contexts building in three dimensions, from the students outward, rather than as a static template from which to select and apply method and calculation. The living, breathing concept of mathematics had changed my perception of what was important.

I wanted to provide my own students with the best possible chance of success in university. If there was a way I could adapt my practice to better prepare my students for university, so they wouldn't be as mystified by the experience as I had been, I wanted to find out what it was. I was learning about a variety of resources and practices in my master's program that I could use to help my students be more engaged, and this seemed like a critical aspect of student success.

Based on what I saw in a lot of curriculum documents for high school, there was an intention for high school teachers to be developing critical-thinking skills and application-skills in their students. There was an intention for students to work collaboratively, in a way that afforded them opportunities to share and apply creative strategies to solve problems. I wasn't convinced I was doing enough in this regard, and changed my classroom tactics almost immediately to include group-work using whiteboards, and more open ended problems. Still, I had heard, as most teachers do, that students weren't as prepared for university mathematics as they should be. I wondered if there was more I could be doing to help bridge the gap between high school and university mathematics classes for them.

How students learn seemed like only one aspect of the issue. What they learned seemed something that needed to be considered as well. At first, I thought the underlying issue might be the nature of the high school course itself. In the master's courses, the subtle connections between different topics and approaches added value and dimension to my own understanding. When comparing that learning experience to what happens in high school, it seemed like the Precalculus 11 and Precalculus 12

courses provided a survey of different topics relevant to the study of calculus, but with no inherent narrative relating topics to each other.

Perhaps students didn't know how to apply concepts from one topic to the next because this had not been an important part of their understanding of what mathematics is, and how to think mathematically. Perhaps they were forgetting certain skills because they weren't using them and building on them from topic to topic.

It also seemed that students might be struggling because they lacked certain knowledge or understanding that could be afforded by emphasizing concepts in the high school curriculum. What is a function? What information can be gained from a graph? What are the different ways to simplify an expression? Fluency with algebraic manipulations and fluency in reading graphs and diagrams took practice.

This led me to the naive intention of influencing curricular changes with the results of my thesis investigation. I thought perhaps a better alignment between the Precalculus 12 course, and Calculus 1 as it is taught at the university level, might be achievable if I could identify the root causes of the apparent gaps.

Student engagement with homework problems and lecture materials also became a curiosity. What did they do with their lecture notes? Were they expected to read ahead? Were study groups effective? Which of these things should I be encouraging, and which of them should I be enforcing? What did a modern university lecture look like now? I knew from experience that the pace was much faster than a high school student would be used to. I wondered how the professors themselves felt about the calculus course they were teaching; if they felt they had to rush through the content, and in what ways they tried to engage students.

The image of a university lecture, where the professor-monologue reverberated off the walls of the lecture theatre might not be a fair and accurate stereotype. In order that my students be ready for calculus in university, it would be necessary to find out what university calculus was aiming to be, where the professors saw themselves in this process, and how they saw the students. I was also really curious about how students saw themselves. The juxtaposition of student and professor perspectives intrigued me, and seemed like something that I could pass along to my own students.

Because I wasn't sure what the underlying issue actually was, I wanted to ask a variety of questions to find out if I was on the right track, or if other things were going on. I wanted to find out if there were incongruities between high school and university teaching and learning, and if so, what they were. The research question that guided my study was: What are the differences between high school mathematics, and first-year calculus as perceived by students and lecturers?

In this thesis, I will explore and compare perspectives on the experiences of first year calculus students in general terms, and in terms of their own sense of readiness for first year calculus, and also obtain the perspective of university lecturers on the matter of student readiness. In Chapter 2, I review literature that examines first-year student experiences, as well as research into calculus learning itself. This is followed by Chapter 3 describing the methodology I used in my research, including descriptions of the research participants, data collected, analysis of results, and immediate themes that emerged. Chapter 4 contains the results on a question by question basis from the student participants, with comparative and contrasting results from the lecturers. In Chapter 5, I look at a cross-analysis of student and lecturer responses as they speak to more global and abstract themes. Finally, in Chapter 6, I address the research questions, conclusions, and implications for high school teachers.

Chapter 2. Related Literature Regarding Calculus Readiness in University

I am not the first to have this concern about readiness and students' ability to apply their understanding. Work has been done on this topic from a number of different perspectives. The 'Calculus Reform' movement in North America in the 1980's marks a significant shift in pedagogical approach on the part of the universities. The relationships we now take for granted between equations and graphs, is just one example of how universities shifted their focus from abstract concepts to physical representations in order to help students better apply their learning to real situations, which is after all, the end goal of many university programs. It was hoped this approach would help students become more fluent with different representations of a variety of classes of functions, and become the flexible thinkers their future instructors and professors were expecting (Hallett, 2006). A recent MAA report has compiled data from a variety of institutions, analyzing the demographics and intentions of students taking first year calculus courses, as well as institutional practices that contribute to the retention of STEM students (Bressoud, Mesa, & Rasmussen, 2015). They recognized that in spite of the 'Calculus Reform' movement, there were still notable deficiencies in student performance.

The 'Calculus Reform' movement introduced a set of initiatives that were intended to improve student performance in calculus by making university calculus more "human-friendly". The Williamstown conference in 1982 resulted in Ralston and Young publishing a series of articles which explored the necessity as well as the feasibility of changing students' experiences in first year mathematics courses. They recognized that calculus was no longer serving a common goal, and discussed the merits of discrete mathematics as a complement to calculus, and the use of computers in providing students with a visual reference using dynamic-geometry software. Being able to visualize classes of functions would help students organize new knowledge into a more complete narrative, and help them see the connections between different representations of mathematical objects. Ralston and Young also recognized that incoming students did not have a common background of knowledge and skills, and that even though they wanted students to finish their calculus courses with a specific set of skills and understandings, students were not entering their first year with a uniform skill set (Ralston & Young, 1983). Ralston and Young recognized that it would be necessary

to overcome the difficulties of implementing curriculum revision on a large scale so that university offerings remained consistent across the United States.

Following the Williamston conference, “Toward a Lean and Lively Calculus”, (Douglas, 1986) and “Priming the Calculus Pump: Innovations and Resources” (Tucker, 1990) provided universities with working examples of modified curriculum, delivery, and assessment from real examples that were piloted in a variety of institutions. In addition, these provided reviews of a variety of the projects that had been funded to implement innovations in the spirit of calculus reform. The purpose was to improve the experiences of students, and in turn, improve retention rates.

Twenty-five years later, student readiness is still a concern, however a more holistic approach is now being taken to address student readiness for university life and expectations; not just for the rigours of calculus. The 2015 MAA study by Bressoud, Mesa, and Rasmussen compiled information about the demographics of students who enroll in calculus, the impacts of classroom practices that are believed to influence student success, and the characteristics of successful programs such as they are defined by the MAA. Their hope was to influence calculus instruction on a national level. (Bressoud, Mesa, & Rasmussen, 2015)

In the sections which follow I look at the practices that permeate high school classes and university lecture halls, with a specific focus on recent curricular developments at each level. This is followed by a look at some of the research that has been done around articulating the ways that students are not ready for university calculus as well as efforts to facilitate student success.

2.1. High School Calculus

2.1.1. The position of the NCTM on calculus courses in high school

Should all high school students planning to study mathematics or the applied sciences at the post-secondary level just take calculus in grade 12? Many high school administrators and districts would say yes, since they offer the course, even though Precalculus 12 is the prerequisite for university calculus. However, the NCTM has taken the position that calculus in high school should not be a requirement for students entering the sciences in university.

> Although calculus can play an important role in secondary school, the ultimate goal of the K–12 mathematics curriculum should not be to get students into and through a course in calculus by twelfth grade but to have established the mathematical foundation that will enable students to pursue whatever course of study interests them when they get to college. (National Council of Teachers of Mathematics, 2012)

Whether or not high school administrators are aware of this position is unknown. They do recognize that students aren't as ready as they should be for calculus, and one way for a school to offer what seems like competitive programming is to offer a calculus 12 course, with much of the content being the same as university Calculus 1. Apparently, whether or not a high school calculus course serves as an advantage when students reach university, is debatable.

2.1.2. The arguments in favour of high school calculus courses

Ferrini-Mundy and Gaudard found that high school calculus classes enabled students to build their algebra skills. Students with a full year of calculus or an AP course tended to do better by a letter grade, than their peers with no prior calculus experience. But a brief introduction or survey style course was not actually useful, and algebraic proficiency seemed to be the only advantage. In addition, any advantage students had over their peers when entering their first-year calculus course, lasted only until their second year (Ferrini-Mundy & Gaudard, 1992).

> Our findings suggest that first-semester college calculus students who have studied a full year of secondary school calculus, whether it be an AP course or not, are more successful in the course than those who have had either no calculus in the secondary school, or only a brief introduction…. the major difference in performance between students who had no secondary school calculus and students who had a brief introduction seemed to be due to procedural proficiency… By the second semester, differences among the four groups [no prior calculus, an introduction to calculus, a full year of high school calculus, AP calculus] had essentially disappeared. (Ferrini-Mundy & Gaudard 1992)

2.1.3. The arguments against taking a calculus course in high school

Although it might seem that a survey course in calculus at the high school level would prime students for deeper exploration of topics at the university level, research into the efficacy of high school calculus has produced some good reasons why students should not take calculus in high school. The Mathematics Association of America's

Committee on the Undergraduate Program in Mathematics (CUPM) panel found that in fact, high school calculus courses often had detrimental effects on students' success in university calculus. This is because a survey course can give students a false sense of understanding, and it can also reduce the sense of wonder and excitement that comes from learning [something new].

> The lack of both high standards and emphasis on understanding dangerously misleads students into thinking they know more than they really do. In this case, not only is the excitement taken away, but an unfounded feeling of subject mastery is fostered that can lead to serious problems in college calculus courses. (CUPM Panel, 1987)

This sentiment is echoed by more recent research as well, where it has been found that the level of proficiency with precalculus topics that is expected, and the ability for students to analyze the information given in a question and select the most appropriate approach, is lacking in many first-year calculus students. Even students who complete an AP calculus course in high school may not necessarily have the expected level of understanding that will allow them to move beyond calculus topics to what comes next. The level of depth and rigour of an AP course is not consistent with a university course.

> The problem is that success in AP Calculus does not require this level of proficiency. It is and only claims to be evidence that one can solve the standard problems of calculus. It is not evidence that a student has acquired the preparation needed for future success. This is why MAA and NCTM issued their joint statement on Calculus in High School. In some sense, the worst preparation a student heading toward a career in science or engineering could receive is one that rushes toward accumulation of problem-solving abilities in calculus while short-changing the broader preparation needed for success beyond calculus. (Bressoud, Mesa, & Rasmussen, 2015)

Bressoud's five-year study in which "213 colleges and universities, 502 instructors, and more than 14,000 students" were surveyed "to learn who takes Calculus 1 in college, why they take it, their preparation for this class, and their experience in this class" found that AP calculus was only effective for students (meaning those students passed their first-year calculus course) who attained a grade of 4 or higher on the AP exam, and even then, that was no guarantee of success. They found that 18% of the students surveyed who had attained a 3 or higher on the AP exam still received a D or F in their first-year calculus course, and another 18% received a C. (Bressoud D. , 2015)

For high school students deciding whether or not to take calculus in grade 12, the results of the MAA study could mean that if a student earns less than a 3 on the AP exam, he or she will have likely wasted their time and effort, particularly if they took AP calculus, but opted out of writing the AP exam because they didn't expect to do well on it. High schools which offer calculus because they think it will give students an advantage for admissions to university might have a misguided notion of readiness. Brassoud's findings indicate that just offering calculus or even AP Calculus does not appear to be the answer. Students who entered calculus with less knowledge became discouraged and quit even if they started out with a high level of confidence and sense of readiness for their college calculus course.

As much pressure as high school administrators might feel that they need to offer Calculus in grade 12, Bressoud suggests that a high school calculus course is only as effective at preparing students for calculus as its' students and its' teachers. Students must be highly motivated and highly capable. Similarly, teachers must be qualified to help students build a dynamic and robust understanding of calculus concepts. There are many high school calculus teachers who do not have a deep background in post-secondary mathematics. Their students in turn will be less fluent and flexible in their thinking, and less aware of the nuances that are necessary to understand university or "fifth-year" level mathematics.

> It is important that this political pressure be resisted and that the choice of a fifth-year program be made by the mathematics faculty of the local school and be made on the basis of the interest and qualifications of the mathematics faculty and the quality and number of accelerated students". (Bressoud 2015)

Since the research to this point indicates there are no guarantees for student success based on their prior calculus background, and in some cases it may even be detrimental, proposing a high school calculus pre-requisite is no more the answer here in British Columbia than it is in the US. Addressing the calculus curriculum, and student ability to meet the needs of the related subject areas that calculus serves, has been an ongoing endeavor at the university level as well.

2.2. Curriculum for First-Year University Students

2.2.1. The pilot projects that have endured

"Toward a Lean and Lively Calculus", (Douglas 1986) and "Priming the Calculus Pump: Innovations and Resources", (Tucker 1990) provided universities with working example of modified curriculum, delivery, and assessment. These were real examples that were piloted in a variety of institutions, as well as reviews of a variety of the projects that were funded to implement innovations in the spirit of calculus reform. A variety of pilot projects around student involvement and engagement, as well as the use of technology and dynamic geometry software, and emphasis on applications to external disciplines have resulted from these pilot projects, and have permeated North American university classrooms.

Calculus reform in the US was focused on what was possible at the university level, and since our education system here in BC, especially for mathematics, is so closely tied to our counterparts south of the border, the issues concerning student readiness, and the changes taking effect in US universities have been mirrored here in Canada.

A modern working example of the scope of the paradigm shift was a program developed at Duke University in 1989 (Bookman, n.d.), which is still offered today called "Project CALC". The focus of this course is very much in line with the NCTM standards for high school mathematics.

> [Students engage in] real world problems, hands-on activities, discovery learning, writing and revision of writing, teamwork, and intelligent use of available tools. (Moore & Smith, 1992)

To help facilitate change, textbook content was also affected by the reform movement. Modern US university calculus courses and textbooks have adopted many of the methods and practices that were funded by the National Science Foundation (NSF).

> The most obvious measure of the impact of calculus reform is that the features initiated in the 1990s are now commonplace in "traditional" courses. Some of the new features have been transformed—in some cases their originators would say beyond recognition—but many have been adopted "as is" by standard textbooks. There is a wider variety of problems

> than before, and the "Rule of Three," which originated in calculus reform, has found its way into a large number of calculus texts. (Hallett, 2006)

The wider variety of problems and the use of multiple representations (graphical, numerical, analytical, and verbal) make it harder for students simply to memorize template problems—though still not impossible— and hence encourage conceptual understanding.

In spite of such ideas as the "Rule of Three", where the relationship between numerical, graphical, and symbolic representations are emphasized now in both high school mathematics classes, and university lectures, a common lament is that students are still struggling to transfer their understanding fluently from one representation to the next.

2.2.2. Offering calculus courses based on prior experience and career paths

A brief survey of many Canadian Institutions' websites reveals their expectation that students are perhaps still not as prepared as needed. Offering different types of calculus courses to meet the needs of students with different levels of exposure to calculus from high school was a common recommendation of panels and researchers who wanted to maximize student success (CUPM Panel, 1987; Burton 1989; Bressoud, Mesa, & Rasmussen, 2015). This practice has now become the norm at SFU, UBC, and many other Canadian institutions.

To accommodate the different students and their various levels of mathematics preparation and different planned career paths, universities and colleges offer a variety of pre-calculus courses based on a "calculus readiness test" or on student marks from high school. For example, the University of British Columbia offers Calculus 104, which is specifically designed for students with a low grade in Pre-Calculus 12 (University of British Columbia, 2016).

SFU offers an array of courses as well, such as Mathematics 100, which is intended for students with only Precalculus 11 or Foundations 11, and is designed to prepare them for first year calculus. Mathematics 150 is Calculus 1 with Review, and recognizes that even students planning to go into sciences and who did take Precalculus

12 in high school, might still need some extra time to review prerequisite skills along the way (Simon Fraser University, 2016).

In addition to offering courses based on student experience, many universities also offer calculus courses that are intended to serve the needs of different faculties, such as Biological Sciences, Engineering, and Economics.

2.3. University Initiatives to Investigate Student Readiness for Calculus 1 Courses

The research into first year student experiences appears to have been conducted to satisfy two main purposes: that which investigated the teaching and learning of calculus, especially the efficacy of calculus in high school, and that which was intended to inform universities about first year student behaviours beliefs, needs, and how best to support those students.

The first category of research could be used to assist universities in deciding what pre-requisites were most appropriate for first year calculus (CUPM Panel, 1987; Burton, 1989; Ferrini-Mundy & Gaudard, 1992; Hallett, 2006; Kraemer, 2011; Bressoud D. , 2015; the second, and that which I will discuss next, was done so that universities might better understand first year students' limitations and needs as students in a new environment, and develop appropriate interventions and support structures to improve retention. (Cliff, 2000; Pascarella, Edison, Serr Hagedorn, Nora, & Terenzini, 1996; Robert D. Reason, 2006; Brinkworth, McCann, Matthews, & Nordstrom, 2009; Ben-Avie, Kennedy, Unson, Richardi, & Mugno, 2012; Bressoud, Mesa, & Rasmussen, 2015)

2.3.1. Student needs and expectations for their new environment

In addition to curriculum knowledge, there are other variables which impact student success. In many cases, the successful attitudes and strategies that students had in high school are no longer effective. In order to be successful, students must recognize the differences in their new environment, and adapt their attitudes and behaviours accordingly. In some cases, they are able to identify these nuances in expectations and what the successful behaviours will be, but in other cases, they are not.

Universities have continued to recognize and research the difficulties that first year students face and the impact of those difficulties on student success. Brinkworth's 2009 two-year longitudinal study about student perceptions of their own needs and expectations revealed a desire for formative feedback, ready access to instructors, and the impact of extra-curricular activities on academics (Brinkworth, McCann, Matthews, & Nordstrom, 2009). Attendance habits, and study group habits were also examined; in short, the habits and expectations they would be bringing from high school.

Some disjunctions between the lecturer perceptions of the feedback they were providing, and the student-perceptions around timely feedback were identified, and one of their recommendations was to offer first year student orientations and academic interventions. A specific recommendation that separated this study from others was that post-secondary schools should collaborate with high school teachers in preparing checklists for students, but no details about the content of such a checklist were offered by the study. (Brinkworth, McCann, Matthews, & Nordstrom, 2009)

2.3.2. Student perspectives about pace

In Scheja's 2006 study, he noted that students identified extreme teaching paces as a problem because they didn't have time to practice using concepts between lectures, and during the lecture they didn't understand what was going on because it was going so quickly (Scheja, 2006). Calculus in university is delivered in less than half the time it is in high school in British Columbia (48 hours of lecture time for a typical 3 credit college course vs.120 hours of instructional time for a 4-credit course for British Columbia (Government of British Columbia, 2016). Many students in their position likely experienced what Cliff referred to as dissonance (Cliff, 2000) when they attempted to reflect on their learning. They could not identify what wasn't working for them, or used the same study and review tactics in spite of those methods proving unsuccessful. They also wouldn't have as much time to reflect on new information as they had been afforded in high school. This reflection time is critical in building understanding, and even more important is how they spend that time. Simply repeating question types is not a sufficient coping strategy for a fast-paced course.

The fact that these students appeared to expect understanding to develop from repeated attempts at solving problems and applying procedures, would seem to be something of a hit-and-miss approach. (Cliff, 2000)

2.3.3. Recognizing and correcting ineffective behaviours

Students' lack of ability to explain the reasons behind their own poor performance is common. In Cliffs study, they lacked awareness of their own study approaches, or continued to use approaches involving rote learning to improve understanding. This was the area that seemed the most dangerous for high school students going into university according to Cliff. In high school, there was often a teacher directing them on how to improve their habits, or how to improve their work, or to redirect their thinking when it was clear they were off-track. There was little opportunity for students to identify the approaches they were using that were ineffective – especially if it was their understanding, not their effort that had caused them trouble. (Cliff, 2000)

When students don't really understand the reasoning behind 'necessary' processes and procedures (Hewitt, 1999) they are unable to build on their understanding or correct their own misconceptions. They may become more confused if they try to overcome their difficulties by examining the connections between what they see as a procedure, and what is happening in terms of the mathematics involved.

Rather than studying the relationships, because they don't have any strategies to do this, they might just repeat different types of questions, hoping that they encounter enough of the different relationships to do well on an exam. Cliff identified that students who cannot identify their own shortcomings in university courses, would struggle to overcome those shortcomings.

Another pattern amongst students with dissonant orchestrations was their apparent inability to explain, either to themselves or to others, the reasons for any study difficulties, academic failures, or under-performance. Alternatively, they appeared to explain their difficulties in ways which were inappropriate or inadequate for the academic context in which they are operating. An example of this would be where the student knew they were expected to understand a relationship, but tried to accomplish this by repeating calculations. It was in these senses that their approaches appeared to be

dissonant with their perceptions. It could be argued that these students lacked sufficient metacognitive awareness to improve their situation. Biggs (1985), for example, argues metacognition is manifested as the capacity of a learner to demonstrate awareness of the approaches and strategies they use for studying.

Some students with dissonant orchestrations appeared to be aware of their strategies and approaches, but there appeared to be a lack of congruence between their approaches and perceptions. So, for example, they appeared to address needs to understand with strategies more suggestive of needs to reproduce content mechanically. Or they appeared aware that they were engaging in rote-learning approaches to problem-solving, but seemed to perceive that these approaches would be linked with examination success anyway. (Cliff, 2000)

2.3.4. The successful mindset

Research into the general perceptions and experiences of first-year students has been an increasingly popular research topic for universities in the United States, but it is not a new question. As early as 1996, studies have been examining the characteristics of successful students. Taking responsibility for their own education is important for a student's success. When students transition from high school to university, their support system is disrupted, and in some cases, it is dissolved completely. Whether they can succeed independently is one aspect of student success, regardless of their mathematics skills. Their "internal locus of attribution" (Pascarella, Edison, Serr Hagedorn, Nora, & Terenzini, 1996) or a sense that they themselves are responsible for their own destiny, is one characteristic of students who find success in their first-year courses. This research and that which was conducted by Cliff around successful study habits, suggests that the successful student will be one who is able to recognize when their approach to learning from high school is working, and is willing and able to adapt or change their strategies when it is not working.

2.3.5. The effect of a full course load and extra-curriculars

Since university course work requires so much more independence than high school courses, universities examined how the students perceived their new environment, and to what elements within it they attribute their success or failure. One

perception might be that university is extremely fast paced, and is not conducive to a balanced life-style; that students with full course loads will struggle to succeed, and that they must assume responsibility for their grades and homework, even if they do not have exceptional teachers.

In regards to teachers, Pascarella found the opposite to be true. Students who said they experienced what they would consider 'good teaching', were more likely to attribute their own success to their own hard work. Secondly, students with heavier workloads, and who engaged with more extra-curricular activities and campus based activities, actually had an increased internal locus of attribution of their success. (Pascarella, Edison, Serr Hagedorn, Nora, & Terenzini, 1996) This implied that the more involved students were, the more likely they were to assume responsibility for their own success.

There were many opportunities for student engagement and involvement in university communities. Clubs, sports teams, fraternities, sororities, social events, and musical events are just a few options at any given campus. But in order for these programs to become part of the student experience, effective student orientation was also found to be important. Ibid.

2.4. Where Unanswered Questions Remain

In many cases, the outcomes of the studies put forward by universities have resulted in recommendations and action items for universities only, and minimal recommendations for high schools and high school teachers. The only explicit mention of high schools was that there should be communication with them, but what that communication should be was not specified (Brinkworth et al. 2009). Even the more recent MAA study stops short of addressing high schools with their results.

> This report does not address how calculus is or should be taught in high school, yet it is relevant to high school and middle school teachers and administrators because it opens windows into the situation their STEM-intending students will encounter when they enter college…For high school teachers and administrators, this report may open their eyes to the obstacles and struggles our universities face and the very challenging environment students encounter as they make the transition to post-secondary education. (Bressoud, Mesa, & Rasmussen, 2015)

In fairness, the research was focused on questions that universities had a vested interest in answering, in order to make their own environment as functional and productive as possible given the nature of the modern calculus student. What was not addressed by the research is a perspective that will be informative to high school students and teachers. For example:

1. What kind of communication between high schools and colleges/ universities would benefit students?

2. What is the best way to prepare high school students for the level of understanding needed for Calculus 1 and beyond?

3. What is the best way to prepare high school math teachers to teach mathematics effectively?

4. What are the characteristics of high school math classes that produce successful first year calculus students?

5. What are the characteristics of the high school students themselves who do have a successful experience in their first-year calculus course?

6. How is success defined in a first-year calculus course?

Also absent from the literature is information that can be passed on to high school students about what university level mathematics courses are actually like, what students can expect, and more importantly, exactly what will be expected of them. I wanted to identify the disjunctions if they exist, between the student point of view and professor point of view, and provide insight into the nature and cause of student struggle.

2.4.1. The initial research questions

There were more unanswered question than could be addressed in one study, so I refined my research focus from “What are the differences between high school mathematics, and first-year calculus as perceived by students and lecturers?” to:

1. How ready do students feel for university mathematics based on their high school experiences, and in what areas do they struggle with their first-year calculus courses?
2. Do students and lecturers perceive readiness in the same way?
3. Do students and lecturers attribute struggle to the same reasons?
4. What are the differences between high school mathematics and university mathematics courses from the perspective of students?
5. What advice do students and lecturers have for incoming students?

Chapter 3. Methodology

Following the review of relevant literature, particularly that which pertained to locus of attribution and dissonant behaviours, I decided to use a two-pronged approach to look closely at students' perceptions of first year calculus, and lecturer's perceptions of students.

Data were collected via interview questions intended to invite anecdotal responses about student readiness. In this section, the setting, interview questions, participants, data collection, and analyses are detailed.

3.1. Setting

The student and lecturer interviews were conducted between January and July of 2015. Participants were recruited from Simon Fraser University in Burnaby, British Columbia, Canada, and the University of British Columbia, located in Vancouver British Columbia, Canada. These two institutions were chosen for the study based on their popularity as a choice for many of my students[1], their differences in size[2], and their reputations for excellence[3]. They were very popular choices for the students at the high school where I teach. Another important consideration was the location of both of these universities. Since I wanted to conduct face to face interviews, it was most convenient to find participants who could be accessed within a convenient driving distance from my home in Vancouver, BC.

The SFU Office of Research Ethics gave approval to conduct the research as reported here.

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[1] University of British Columbia was listed as the 1st choice of British Columbian students in 2015 (Ash, 2015).

[2] UBC had approximately 61,000 enrolled students in 2016 (Univerisity of British Columbia, 2016), and SFU had approximately 34,500 (Simon Fraser University, n.d.).

[3] UBC places among the top 20 universities in the world (University of British Columbia, 2017), and SFU was ranked #1 in 2015 as the best comprehensive university in Canada (Maclean's, 2015).

3.2. Lecturer Participants

3.2.1. Recruitment of lecturer participants

The first lecturer I was able to connect with came from a letter of introduction from my thesis supervisor. He was able to connect us via email. From there, the lecturer I had interviewed suggested some names of other people in the mathematics department, some male and some female, who might add variety to my interview results. This method of sampling is referred to as “Chain Referral Sampling”. It is a variation of non-probability sampling which makes use of referrals from one party to the next when seeking interview participants. It has typically been utilized when studying hidden populations, (Heckathorn, 2002) and is a method of sampling by convenience, whereby the researcher might be referred to a study participant by an acquaintance, and then the first participant refers the researcher to the next participant. “Chain referral sampling is suitable when members of the target population know one another and are densely interconnected” (Heckathorn, 2002). It was applicable in this case because many members of the mathematics departments at SFU and UBC know one another, if not personally, then at least by reputation.

Since the referrals are linked, it is important to choose an appropriate person to begin this process. Erickson (Erickson, 1979) warns “inferences about individuals must rely mainly on the initial sample, since additional individuals found by tracing chains are never found randomly or even with known biases.”

There is inherent bias in the selection of the first candidate, and the first candidate will influence who is to follow. It was important that my first candidate be representative of the university faculty teaching calculus, meaning they utilized teaching and assessment strategies that were typical of many others in the department, and likely represented the majority opinion regarding calculus instruction and student success at the university. This first referral was provided to me by my thesis supervisor.

To maintain anonymity of the next interview candidate, and to offset any biases introduced by my first candidate, I requested this first candidate recommend three potential colleagues who were seen as having differing opinions or teaching styles, or perhaps a different point of view. I would then select only one of them to interview next.

If the candidates wanted to talk amongst themselves that was up to them. I made it clear I was not looking to interview only like-minded individuals. My second participant was not only of the opposite gender, but did indeed articulate different teaching methodologies as well.

I carried on in this fashion until I had interviewed three SFU lecturers. As different as the SFU lecturers professed to be from one another, there were enough commonalities in their responses that I felt I was able to generalize after only a few interviews. I turned my attention in a similar vain to making contact with lecturers at UBC. Again, my thesis supervisor recommended a person to contact first who was well established in the mathematics department at UBC. This particular contact was also among the referrals suggested by other lecturers at SFU. From this contact, I continued to use the same referral method to seek another participant. In total, I was able to interview five university lecturers: Three from SFU, and two from UBC. Even with this small sample size, commonalities in their responses were evident. Their responses about student preparedness and where students struggle were very similar in nature.

3.2.2. Addressing gender-bias in lecturer responses

The lecturer sample set was also intended to include a combination of male and female lecturers, however one of the females who had agreed to an interview was subsequently unavailable. Another who I contacted indicated that she hadn't taught calculus in quite some time, and may not have recent or relevant data for my study. Convenience and availability forced the remainder of the interviewees to be male. The final demographic was 4 males and 1 female.

3.2.3. Anonymity of lecturers

Because the academic community is so small and well connected, it was necessary to ensure anonymity by the use of pseudonyms. The pseudonyms chosen were gender neutral in order to maintain anonymity of the lone female respondent. In addition, direct quotes from lecturers were used, but no reference to which university he or she was affiliated with was given in the results.

The 5 lecturers interviewed will be referred to as AJ, Bobbie, Casey, Dana, and Everest.

3.2.4. Teaching experience of lecturers

In regards to teaching first year calculus courses, experience ranged between 6 and 20 years, and included those with 10 years and 15 years of experience.

4 of the 5 lecturers held PhD's in mathematics; the fifth had an MSc. in mathematics. Only one of the lecturers held a formal degree in education.

3.3. Student Participants

3.3.1. Recruitment of student participants

When I first started thinking about who my ideal student participants would be, I imagined having a collection of responses from first year calculus students in what I believed to be the most 'main-stream' calculus courses[4]. I wanted to interview students who were experiencing varying degrees of success, and compare and contrast their results to a series of questions about workload and study habits.

When attempting to recruit students, I first tried to deliver a survey through one of the lecturers I had interviewed. He had granted me access to survey his class via the internal SFU email system. After crafting and distributing my survey, I received 30 responses – all from students who were willing to answer a survey, but not willing to be interviewed in person. I had hoped to be able to pick and choose the students I would interview based on two things; that they were in first year, and that they were representative of different levels of success: struggling, coping, and thriving.

Once I realized I wasn't going to be able to recruit any students this way, let alone specific ones, I decided to change tactics and sought them out in their natural environments: lecture theaters, and the campus Mathematics help centers. I contacted an instructor of the Calculus 1 course at SFU, and received permission to come to his

[4] A list of the different types of calculus courses is included later in this section.

class and introduce myself and my study[5]. From there, I was able to interview 3 students after class on the same day. I also set my sights on the SFU course workshop where first-year calculus students went to receive help, and the UBC Mathematics Learning Centre in hopes of reaching students from different programs. In each case, I contacted the TA in charge of the help centers at SFU and UBC on a given day, and was able to determine at what stage in the course students were, in order to increase the likelihood of encountering as many students as possible.

In addition to on-campus recruitment, 3 students were recommended to me by a colleague. These were students who had done very well in her AP Calculus class. Student interviews were now going to be based on opportunity sampling (McLeod, 2012).

I immediately recognized the bias I was introducing to my study results. These students were "successful examples" of first-year students, and included students who had needed to repeat their first-year calculus course. I decided to shift the intent of my surveys from looking for a variety of student success levels, to finding out what made a student successful. This seemed like an even better way to focus the results in a way that would help prospective students. Rather than focusing on the different ways students struggled, I might be able to articulate a framework for success.

When I went to SFU, students had just written their second mid-term, and were coming in to find out their grades. This proved fruitful, as I was able to gather data on their first and second mid-term marks. Additionally, they now had perspective on what was working for them in terms of study habits and managing their workloads. As I gathered more and more data, at first unable to find first year students, my own perception of the demographic I had captured began to change. I was collecting data from students who had already found a way to be successful in first year.

3.3.2. The value of changing my sampling strategy

As much as I had hoped to capture the raw reactions of first year students, I came to appreciate the value of the collection of students I had found. They had come from a variety of high school backgrounds, had needed to repeat their first year courses

[5] This particular lecturer was not a study participant.

in some cases, and had all found a way to be successful in Calculus 1. Even more valuable was that in spite of the diversity in their backgrounds, their responses were very similar in nature.

Some of the students who were interviewed had indeed failed their first-year calculus course, and had had to retake it. These students were able to articulate what had been the difference between failing and thriving for them. Regardless of their grades at the time of interview however, every single participant indicated that they had struggled in some way at first when transitioning from high school to university.

After only a few student interviews, some commonalities began to resonate among respondents. I interviewed more and more students, expecting there to be differences, but their responses just seemed more and more similar. The results and the other common themes could be quite beneficial to new students, and informative to lecturers.

3.3.3. Addressing gender bias in student responses

As with the demographics of the calculus lecturers who were interviewed, the representation of female respondents was not proportional to the number of males. There was in fact only one female who volunteered for an interview. She had been in a lecture that I had been watching, where I had put out a general request to students to participate in my study. Though her response to each question were the same as at least one of the other males' responses, it is unknown if other female respondents would have answered the same way she did.

3.3.4. Anonymity of student participants

All students who were interviewed indicated they were of legal age. To preserve anonymity, pseudonyms were used for each of the participants. In order to identify them as first, second or third year students however, each of their names begins with F, S, or T.

3.3.5. Summary of mathematics courses taken by the participants

During their interviews, students identified their pre-requisite courses, and calculus courses. The following table contains a list of these courses, and their descriptions as per each university's website.

Table 1 Summary of university first-year calculus courses

UBC Mathematics 100 – Calculus I: Differential Calculus with Applications to Physical Sciences & Engineering (Derivatives of elementary functions. Applications and modeling: graphing, optimization)
UBC Mathematics 101 – Calculus II Integral Calculus with Applications to Physical Sciences & Engineering (The definite integral, integration techniques, applications, modeling, infinite series)
SFU Mathematics 100 – Precalculus (Designed to prepare students for first year Calculus courses. Topics include language and notation of mathematics; problem solving; algebraic, exponential, and logarithmic and trigonometric functions and their graphs.)
SFU Mathematics 150 – Calculus I with Review (Designed for students specializing in mathematics, physics, chemistry, computing science and engineering. Topics as for Mathematics 151 with a more extensive review of functions, their properties and their graphs. Recommended for students with no previous knowledge of Calculus)
SFU Mathematics 151/152 – Calculus I/II (Designed for students specializing in mathematics, physics, chemistry, computing science and engineering.)
SFU Mathematics 154 – Calculus I (Designed for students specializing in Biological Sciences)

U of A Mathematics 100 – Calculus I (Calculus I Restricted to Engineering Students)
U of A Mathematics 101 – Calculus II (Calculus II Restricted to Engineering Students)

3.3.6. Introducing the students

Fiona was a first year student at SFU, and emailed me after I introduced myself to her SFU class. She had not taken Calculus in high school, and had received an A in Pre-Calculus 11. It was her second time taking Calculus 1, because she had received a D in Mathematics 154 (Calculus 1 for the Biological Sciences), and wanted to improve her grades. She was now taking Mathematics 151 (Calculus 1 for Science students).

Felix was a first-year General Sciences student at SFU. He consented to an interview at the SFU mathematics help center. In spite of having an A in Pre-Calculus 12 from high school, he had only achieved a C- in Mathematics 151 (Calculus 1 for Science students). He re-took a similar course, Mathematics 150 (Calculus 1 with Review), and achieved an A+. He was in the Mathematics 152 (Calculus II) summer school course because he recognized that he needed more time to understand course material. He indicated he liked the pace of the summer school course, and that he didn't have any other courses to contend with at the same time.

Farook was an SFU general sciences student in his first year. He was also recruited at the Mathematics help centre. He indicated that he had felt completely prepared for the university calculus course because he had taken the Pre-Calculus 12 course twice (first at school, and then online) in order to get a better mark. He felt the online course had been easier, and thought his algebra skills had benefited from his taking the course twice. His mark in Pre-Calculus 12 had been 84%, and he had achieved a grade of 25/40 on his first Calculus 1 midterm, and grade of 32/40 on the second one. He didn't think these were good marks, because in his words, he had 'made some silly mistakes'.

Fulton was a computer sciences student in his first year at SFU. He and Farook were friends, and I was able to interview him right away as well. Though he and Farook were friends, many of his responses were quite different in nature to those of Farook. He had only completed Foundations of Mathematics 11 in high school with a mark of 78%. He realized he would need to do upgrading courses at UBC, after which he took Mathematics 100 (Differential Calculus with Applications to Physical Sciences and Engineering), and received a grade of 83%. His marks in UBC Mathematics 101 were 32/40 on the first mid-term, and 34/40 on the second. He also was lamenting silly mistakes on his mid-term.

Frank was a first-year engineering student at SFU, and volunteered to be interviewed after I had visited his Calculus 1 class. He had taken Pre-Calculus 12 and Calculus in high school as a combined course. His high school mark had been 93%. In spite of this seemingly solid background, he reported that he had failed his first-year calculus course. He said that he had expected the calculus course to be easier because he had done the same content in high school. Unfortunately, he didn't do all of the university homework, and found the exam questions much harder than what he had experienced in high school. His reason for taking the summer school course did not come up during the interview.

Stefan was in his second year of the UBC engineering program. His name was suggested to me by a colleague from the West Vancouver school district. He had excelled in high school, achieving 93% in Pre-Calculus 12 and 5 out of 5 on the AP exam. He reported that his mark in Calculus 1 at UBC had been 92%. Though he didn't struggle in university, he made several comments alluding to the difference in workload. He had not used his AP status to accelerate his coursework.

Sam was also referred to me by the colleague from West Vancouver. He had achieved 99% in Pre-Calculus 12, as well as 5/5 on the AP Calculus Exam. He was in his second year of a program called 'Engineering Physics' at UBC. He too had not used his AP course as a pre-requisite. His first-year course had been Mathematics 100 at UBC.[6]

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[6] Though Sam and Stefan had had the same high school teacher, their perceptions of their high school classroom experience were completely different. Sam felt his high school experience had

Spencer was another second-year student referred by my colleague, but he was doing a software engineering co-op program at the University of Alberta. He had achieved low 90's in both Pre-Calculus 12 and AP Calculus, but had chosen not to write the AP exam. He had an IEP in high school, and had developed some strategies in order to achieve the level of comprehension he felt was necessary in order to achieve his goals.

Tahir had done AP Calculus in Saudi Arabia, and was now a third-year engineering student at UBC. (I recruited him at the UBC Mathematics help centre). Though it had been some time since his freshman year, he assured me that his first year had made an indelible impression on him, and he would be able to recall his transition from high school to university with no trouble at all. He had received a mark of 4 out of 5 on the AP Calculus exam, and his first-year calculus mark had been 60%.

Taj was another third-year student at UBC. He and Tahir were friends, but also brought their own perspectives to each of my questions. Taj had done the International Baccalaureate program in high school. He had achieved a grade of 6/7 in the Mathematics HL course, and reported that his first-year calculus mark had been about 65%.

been very notes based, while Stefan had used the phrase 'discovery oriented' to describe this particular high school teacher's lesson style.

3.3.7. Summary of students' information at the time of interview

Table 2 Student participant information

Student	High School Marks	Prior University Courses & Marks	Current University Course and Marks (if available)
Farook SFU 1st yr	Precalculus 12 84%		Mathematics 151 midterms 1 & 2 63% & 80%
Fulton SFU 1st yr	Foundations of Mathematics 11 78%	Mathematics 100 83%	Mathematics 151 midterms 1 & 2 82% & 85%
Felix SFU 1st yr	Precalculus 12 A	Mathematics 150 A+ Mathematics 151 C-	Retaking Mathematics 151 No grade reported
Frank SFU 1st yr	Precalculus 12 93%	Mathematics 151 F	Retaking Mathematics 151 No grade reported
Fiona SFU 1st yr	Precalculus 11 A	Mathematics 154 D	Mathematics 151 In progress No grade reported
Stefan UBC 2nd yr	AP exam 5/5	Mathematics 101 92%	N/A
Sam UBC 2nd yr	AP Exam 5/5	Mathematics 100 A Mathematics 101 C+	N/A
Spencer U of A 2nd yr	AP Calculus "Low 90's"	Mathematics 100 A	N/A
Tahir UBC 3rd yr	AP Exam 4/5	Mathematics 101 60%	N/A
Taj **UBC 3rd yr**	IB Mathematics HL 6/7	Mathematics 101 65%	N/A

3.4. Interview Questions

3.4.1. Lecturer interview questions

To establish comparisons or contrasts between high school and university mathematics classes, and the nature of mathematics instruction at each level, I asked specific questions about course delivery, content, pre-requisite skills and the materials available to students from the lecturers. I also asked about what they intended for students to do with their lecture materials and homework assignments. I asked open-ended questions and found commonalities in responses, such as homework expectations, that were consistent across the board. I also gained insight into the lecturers' perceived relationship with the students as individuals and as a class.

The interview questions I created fell into three categories: Information about the lecturers themselves, the lecturers' thoughts about the first-year calculus course they taught, and their thoughts about students and student performance. I felt if I gained some perspective on these three areas of university calculus, I might be able to identify where students struggle. Is it because of a difference in teaching methods at the university level, the difficulty of the course, or perhaps a difference in what the instructors are expecting the students to do with their materials, and what the students are actually doing with them? I wanted to compare each of their first instincts when faced with an open-ended question about their impressions of first-year students. It also served to identify the perspective from which I could assume the rest of their responses were coming. From there I proceeded with my formal list. A complete list of lecturer questions is included below. It is separated into questions under the following headings:

Questions about the lecturer, Questions about the course, and Questions about the lecturer's perceived relationship with the students.

Since the interviews were meant to feel more like a conversation, where lecturers could include any information they thought was important from their perspective, not all of these questions were asked at each interview. In some cases, the conversation lead naturally to the lecturer discussing each of these topics, and in other cases, the questions were asked directly.

QUESTIONS ABOUT THE LECTURER

How long have you been teaching at SFU / UBC?

What courses do you teach?

How long have you been teaching first-year Calculus?

How would you describe your teaching style?

Which textbook are you using?

How is the textbook used in your course?

QUESTIONS ABOUT THE COURSE

Can you describe a lesson or sequence of lessons for your favourite topic in the course?

How do you feel about the first-year calculus course itself?

If needed: is there enough time to teach the content the way you would like to?

If needed: is the content itself what you would want students to get out of an introduction to calculus?

Do you feel the course is composed of stand-alone concepts, or is there a progression where old previous concepts are required for any subsequent work?

Do you distribute a course outline with a list of topics? I.e. if students want to read ahead can they?

How are the marks assigned in your course? (How many quizzes, midterms, final)

QUESTIONS ABOUT THE LECTURER'S PERCEIVED RELATIONSHIP WITH THE STUDENTS

What is your impression of first year students?

What assumptions do you have of first year students in terms of competencies and study habits?

Do you feel that your students are prepared to take on the course when they come out of high school?

If no: what skills are they lacking?

If needed: what is the role of homework in your class?

Are you familiar with the content and structure of the Pre-Calculus 12 course in high school?

If **yes:** Do you feel the pre-calculus 12 course prepares students for calculus? Why or why not.

If necessary: where do you feel the shortcomings are within the pre-calculus 12 course?

If **no**: What would you hope the pre-calculus course does to prepare students for calculus?

What are your expectations of students who struggle with the content or pace?

What are the students' sources of information and practice within your course? (i.e. are resources like lecture notes available, online practice or tutorials, videos of the lessons, a bank of applets etc?)

What is a successful mark on the first midterm in your opinion?

3.4.2. Student interview questions

In preparing student interview questions, I considered where I might be able to compare and contrast responses with the lecturer interview questions. At the heart of the motivation behind my study was a desire to bridge the gap between university-level expectations, and high-school preparation. I wanted to go beyond the surveys about student experiences in the literature, and not only get anecdotal responses about how students saw their own readiness, but also find out what attributes, attitudes and actions had enabled them to be successful, or what their struggles were if they didn't feel they were being successful. I was curious if the high school curriculum was sufficient preparation, and how the students engaged with their lecture materials. How did the classroom / lecture experience in university compare to high school, what pre-requisite skills were needed in order for them to be successful, and how did they define success for themselves? The ultimate goal was to have some key advice they might offer struggling students.

I asked students to compare their experiences in first-year calculus to their high school mathematics class. Specific questions, such as "what did you do if you had a question during the lecture" revealed some differences between the dynamics of a large

lecture hall and a high school classroom, from their point of view. From anecdotal responses, and responses to specific questions I found out if they went to every class, and what happened if they didn't. I also asked if they made use of study groups or if they were trying to survive on their own. I asked about how they spent their study and homework time, and used open ended questions to gain insight into their own sense of success or failure.

The following is the list of student-participant questions. They are separated into questions about the student, questions about the student's perceived relationship with their university lecturers, their perception of how their first-year calculus course compared to their high school math courses from a variety of perspectives, their definition of success, and advice they would give to other students.

QUESTIONS ABOUT THE STUDENT HIMSELF OR HERSELF

What courses and program are you taking this semester?

What was your final mark in Pre-Calculus 12?

Did you take calculus in any form in high school – if so what was your final mark?

ABOUT THE STUDENT'S PERCEIVED RELATIONSHIP TO THE INSTRUCTOR and THE COURSE

How would you characterize the first-year calculus course compared to your high school experience?

a) compare homework load

b) compare homework compared to tests

c) quizzes?

Stand-alone concepts vs. ideas carried through?

Did you receive course outlines? How did you use them?

Compare the teaching styles of calculus teacher to high school teacher

What did you do if you had questions during a calculus lesson in high school / university?

What did you do if you had questions about the calculus homework high school / university?

Do you feel you had time to do all of the assigned homework high school / university?

Did you have to miss any classes? If so, how did you make up for the missed class? High school / university

Which textbook did you use in the calculus course?

How was the textbook used in your course high school / university?

Were you expected to read ahead in high school / university? ….and did you?

Why or why not?

How were new topics introduced in high school compared to university?

Did the tests reflect the homework in high school and in university?

Did your concept of problem solving change when you entered university?

How were the marks assigned in your course? (How many quizzes, midterms, final)

How did you find the pace of the course high school / university?

Did the instructor give you an indication of what he or she thought would be required to do well in the course high school / university?

Did the instructor seem to expect or encourage students to work on problems together high school / university?

What skills if any did you feel you were lacking coming into this course?

If necessary prompt a response about mathematics skill, social skills,

and other soft skills

Do you feel your high school pre-calculus 12 course prepared you for calculus?

Why or why not?

What kind of access do you have to external support for the calculus course?

What are the students' sources of information and practice within your course (lecture notes, online practice or tutorials, videos of the lessons, a bank of applets etc)

How do you define success for yourself in high school / university this course?

What would you recommend to future students coming into that instructor's class?

3.4.3. Examples of related questions

Before asking lecturers direct questions about students, I started all of my interviews with the lecturers by asking "What is your impression of first year students"? When interviewing students, I started with something general like "What is your impression of first year calculus compared to high school?"

I felt this would mitigate the bias in their thinking that would come from answering the more specific questions that were to follow. At the start of each interview I wanted their raw unbiased and uninfluenced response when identifying their own impression of what the gap was between high school and university. From there I proceeded to ask related questions wherever possible. Not all of my questions for each group were related, but I sought to frame a question for each group in a way that would facilitate a comparison of lecturer and student responses. A short sample of some of these questions is included below.

Table 3 Examples of related student and lecturer interview questions

Lecturer Interview Question	Student Interview Question
What is your impression of first-year Calculus students?	How would you characterize the first-year Calculus course compared to your high school mathematics experience?
Do you feel that students are prepared to take on the course when they come out of high school? If no: where do you see them struggling?	Do you feel that you were prepared coming into this course?
Do you distribute a course outline with a list of topics? (Meaning if students want to read ahead, can they?)	Did you receive course outlines? How did you use them?
What are the students' sources of information and practice within your course (ie are resources like lecture notes available, online practice or tutorials, videos of the lessons, a bank of applets etc?)	What did you do if you had questions about the calculus homework in high school / university? What did you do if you missed a class?
What are the homework expectations in your course?	Do you feel you had time to do all of the assigned homework high school / university?
How is the textbook used in your course?	Did you read ahead?
What are your expectations of students who struggle with the content or pace?	How did you find the pace of the course high school / university?
Do you feel that your students are prepared to take on the course when they come out of high school? If no: what skills are they lacking?	What skills if any did you feel you were lacking coming into this course? -If necessary prompt a response about mathematics skill, social skills, other soft skills

What is a successful mark on the first midterm in your opinion?	How do you define success for yourself in high school / university?
Can you describe a lesson or sequence of lessons for your favourite topic in the course?	How was a new topic introduced in high school?
What advice do you give first year students, if any?	What advice would you give students coming into Calculus 1?

3.5. How the Analysis was Conducted

Even though my student-participants were not all first-year students, it was still my intention to glean information about their first-year experiences. Their tribulations had left an indelible impression that could be recalled with remarkable clarity; thus, I was able to use my original list of questions about their first-year experiences.

After each interview was transcribed, I summarized the responses to each question and recorded them on a chart under each participant's name. In this way, I was able to compare and contrast responses to each question, and look for themes within and between responses. In some cases, new information came to light that hadn't been part of the original list of questions. For example, my question about what advice lecturers give to struggling students led most of them to include information specifically about where students struggle. This had not been one of my interview questions, but they all included some reference to it in their responses. I then added a heading 'where lecturers see students struggle' to my data table, and recorded information if it was provided by other lecturer-participants.

As new topics got added to the response arrays for the students and the lecturers, some general themes were extracted. These were the result of combining codes which when considered as a collective, led to general unifying themes.

Responses which contributed to the unifying themes or provided specific insights to student and lecturer perspectives, are listed in tables under: Lecturer Interview Results, Student Interview Results, and Comparison of Answers to Related Questions.

Finally, it became apparent that even these general themes could be considered even more globally, and are discussed in a broader context as part of a cross-theme analysis in Chapter 5.

Chapter 4. Results and Analyses

In the sections that follow lecturer results and analyses are presented as a collection, and then the student results and analyses are presented.

<u>The themes which emerged from the lecturer interview results were:</u>

Time
Habits of mind as they relate to homework
How lecturers perceived understanding
Testing and university success
Advice to students who are struggling
Grading

<u>Student results were grouped according to the following emergent themes:</u>

Time
Students' perception of understanding
Comparing test preparation between high school and university
What wasn't said
Advice to other students

Given that the questions were based on a conversational style of interview, and the themes emerged from the data, it was not possible to establish a one to one correspondence between all student responses and lecturer responses, however a comparison of results for related questions is provided in section 4.3.

Each of the themes that did emerge are exemplified by representative excerpts from the interview data.

In Chapter 5, a cross theme analysis of student and lecturer responses is presented, and responses regarding time, understanding, advice to students, and their beliefs about mathematics are examined from holistic perspectives.

4.1. Lecturer Interview Results

Since many of the student interview questions had counterparts in the set of lecturer interview questions, it was informative to find out that much of what the students had considered important was not what lecturers first responses were about. In order to maintain parity in the presentation of each theme, I describe the themes for which the responses of students and lecturers were related, before identifying other themes in the lecturer responses. Before discussing the results related to theme, however, I address the issue of gender bias.

4.1.1. Addressing gender bias

When comparing the lecturers' responses to questions about their lecture styles and their expectations and observations about students, there were no notable gender biases: there was at least one male interviewee who responded the same way as the female had to each question; however it was not possible to determine if the female lecturer represented the views of other female lecturers.

4.1.2. Results of selected lecturer questions

Table 4 Results of selected lecturer questions

Responses that informed about "Where lecturers see students struggle"	• Analytical skills • Problem solving in unfamiliar situations • Starting homework right before it's due • Students might be unaware of what their real problem is • That they are expected to work for 8-12 hours per week outside of class • Not aware of the resources available to help them out • Knowing they can ask questions during office hours • Being away from home for the first time • Algebra skills • They see math as practicing the same calculation over and over • At some point their study habits stop working • They don't know where to begin a problem, read it, extract the relevant

	information from it, and connect their pre-requisite skills to solving it • Poor study habits • Gaps in their understanding • Problems are harder • Pace is faster • They are not independent problem solvers • They don't recognize problems for what they are
Responses to " what skills are they lacking"?	• Algebra • Analytical skills • They can't reason or generalize about classes of functions • Word problems • They have trouble reading the text • Translating verbal descriptions into math • They don't know how to start with a diagram and label the quantities

4.1.3. Time

Pace and pedagogy – 'doing more with less'

Though this wasn't actually first on their minds when it came to talking about students, every lecturer did acknowledge that they were in a constant battle against time from their own point of view. A couple of the lecturers used the phrase "trying to do more with less". Within 38 weeks of lecture, they were covering 35 chapters. On average this was three textbook sections per week.

In order to do more with less, as they said, a common practice was some kind discussion-based approach to the lectures. In some cases, the students responded to clicker questions, and engaged with the material that way, and in others, the students were skillfully guided through an interactive discussion about the topic. In this way, the lecturers felt they could address the big picture ideas necessary for students to navigate the minutia on their own. In one sample response, the lecturer described how they engaged the students in an interactive discussion, and asked for student approaches to

a problem they did not yet have the algebraic tools to solve. In doing so the lecturer deliberately gave authority to the students who then valued their own individual contributions, and could take ownership of their own learning. Less time for the course forces more onuses on the student to develop understanding through personal engagement. The following excerpt is included to provide insight into a specific lecturer's pedagogy.

> A typical class…what I've done in the past is I have tried to interact with the students as much as possible, within the lecture format. They are introduced to the material in class and I am generally not so concerned with covering every little detail. I may relegate some things to the readings, but I will typically cover at least 90% of what they need. I try to limit the number of examples I go through, in order to allow more time for class discussion and participation. So I don't like to race through a class. I usually will start with some sort of bridge, sometimes a problem as motivation, maybe it's a problem to catch their interest; a problem that they don't yet have the techniques to solve, because you know if you start by discussing all the theory and then get to the applications at the end, they've already fallen asleep. So I want to sort of pique their interest, so I'll start by posing a problem, and we'll realize okay, we don't yet have the techniques to solve this and then we'll go into develop some of the theory, but then revisit it at the end of the class. And throughout I will give a lot of problems to work on, but I don't like to just stand up and lecture to the student and solve problems in front of them while they sort of take notes while they're half asleep, so I try to get the students engaged…I try to get them all thinking about the problems and I like the lectures to be very dynamic… So you have to be encouraging with questions if you want students to participate, and to stay engaged. So while it is more of a lecture format, I don't have the students talking to each other a lot during class, but I do get them to participate as much as possible. - Casey

Though the lecturer did not state this explicitly, this approach would have also given them insight and eventually intuition about the tools, (such as algebraic methods, or graphing methods), their students were comfortable using when solving a problem, since those methods would be the ones a student would likely suggest first.

The lecturer had to balance their expectations of student competencies with the desire to only present main concepts and forego the detailed algebraic manipulations during the class period. The assumption was that many students would struggle with this. In some cases, the lecturers said they spent time on review of pre-calculus before the course. But not everyone said they did this. The notion of review during a lesson could be viewed through the lens of time in two ways. First, it takes time away from the

calculus content, and second, the assumed algebra skills are necessary for students to be progress on their homework problems in a timely fashion.

Class time and algebra skills

When lecturers were prompted to discuss “pre-requisite skills”, many referred to content knowledge such as polynomial factoring and logarithms.

Doing “more with less” as they put it, meant they were guiding the students through the major concepts of a topic, but leaving the detailed calculations up to the students to wrestle with (if needed) on their own time. Though the majority of lecturers interviewed from UBC and SFU conceded they were not familiar with the content of the Pre-calculus 12 course (4 out of 5 of them), they did recognize that many of the students who were attempting to transition from high school would have a very difficult time adjusting to the expectations of university calculus in terms of pace and the resulting homework load.

In the lecturers’ opinions, students who struggled with algebra, or who were weak or inflexible in their ability to work with algebraic expressions were at most risk of failing the course. One of the lecturers interviewed talked about a study that had been done by a graduate student in their department, to analyze the types of mistakes that Calculus 1 students were making on assignments and on exams. Gaps in students’ algebra skills appeared to be the most common cause for their errors. Often students knew that they needed to apply a process such as factoring, but did not know how to apply the process correctly. Students had been able to work around this gap in their understanding during homework exercises by using a computer algebra system such as Wolfram Alpha to carry out the process for them. This saved computational time and errors, but they had lost their skills in the process.

> Algebra was the big thing. When doing a calculation, there may be a place where they know they need to factor but don’t know how to do it, or know they need to complete the square and can’t seem to do it. Many students pull out Wolfram to do the factoring or complete the square for them… we say no calculators in first year, but it’s a shock for the students. They don’t necessarily have finger-tip skills. We know those skills transfer to other areas, so students can build a familiarity and can understand and can work with them. - Dana

The algebra skills are important not only so students can have a sense of what is possible when approaching a problem, but also so that they can become more efficient at making forward progress. Algebra skills are a means to an end in university, and are an assumed understanding when students enter their first-year calculus course. Lecturers save time in their lessons by avoiding showing step-by-step calculations during a lesson, and students who don't have these skills will spend more time trying to figure out their homework.

Lecturers' recommendations about time-management – 'do something every day'

In terms of homework time that should be allocated to calculus homework all of the lecturers indicated that their expectation was a ratio of 3:1 homework hours to lecture hours. The following quote was typical of each lecturer, and describes the content of the homework.

> One homework set per week covers three sections, for a total of twenty four prescribed problems per week, plus one or tw questions that weren't from a prescribed section, so they aren't referenced in the text – AJ

The lecturers were basing their advice about time spent on homework, on empirical data collated from the online homework platform. Students who routinely started their homework close to the due date did very poorly in the course. If they lacked the technical background necessary, then it would take them even longer to complete the homework and they would struggle to pass the course. If they waited until the day the homework was due, then there was a high probability they would fail the course.

> I do believe that from a lot of evidence, most of the students who fail are lacking the technical background, and they aren't engaged. With the online homework, we can see that some students are starting it 2 hours before it's due. In fact, 90% of the students who demonstrated this behaviour actually failed. [Online homework] is an interesting thing because you get to see behaviour. In particular, you get to see guessing behaviour." - Dana

The next 2 quotes articulate how two of the lecturers indicate to students what the homework expectations will be in their respective courses.

From Dana's point of view, homework time might be even longer for students because they are novices.

> I tell them they should do 5 hours per week minimum in addition to class time, and if they're a reasonable student, that should get them a 'B'. 'B' is where students are able to tackle most kinds of problems, and they still make certain algebraic mistakes. Students take 5 courses in a term here, and five eights are forty, so I told a couple of students, really, this is sort of like your first full time professional job and it might look like it's a 40-hour work week, but frankly it's a 60 hour work week because you're novices, and for many students that's probably true. – Dana

AJ tells students they should be striving for the 3 to 1 ratio of homework time to class time. AJ stresses the importance of good time management skills to students, and balancing their workloads.

> We use a 3:1 ratio, so for one of the classes which is 3 times per week, they should be spending 9 hours outside of the class time doing homework related activities. For the 4-hour course it's an additional 12 hours outside of class time. I stress that it's a rule of thumb; some will need less, some will need more. Personally, I needed more…also to structure that time throughout the week, and not to do it all in one day and then have 6 days to do your other activities. By the time it comes around and you go to do the next week's homework, you will have forgotten everything. So do an hour or two each day, and take one day off each week." - AJ

In order for students to manage their time well, they would need a sense of self motivation and initiative. They would also need to leave enough time to work through problems that were more involved.

4.1.4. Lecturer views on understanding – seeing the forest for the trees

"Guessing behaviour" was common when students hadn't allowed themselves enough time to develop understanding. What it meant to understand something came up in almost all of the responses, usually before I had asked about it as a direct question. To the lecturers interviewed, understanding centered on problem solving, and could be viewed from a very practical perspective, where students were striving for personal understanding. It also came from a more aesthetic one, where understanding mathematics could be seen as a process, and not necessarily as a result.

What follows are three direct quotations from different lecturers, to allow for nuances in their responses to be considered. They each refer to understanding in different ways.

The first quote presents a view of how to develop understanding by exchanging ideas using a discussion based approach during a lesson.

> There is some discussion that is content transfer, but I try to make the mathematics as real as possible. I want to shorten the distance between kind of let's call it research mathematics and first year calculus. It means looking at bigger problems than what they're used to, it means spending a longer time on problems, it means spending a lot of time just thinking and talking in the classroom, and having an exchange of ideas, it means interrogating answers that I get from them or I give them. It means revising and extending comments a little more than they're used to. The ideal I aim for is where the room feels very small, and we are really just having a kind of conversation in which we are just trying to solve a mathematics problem, which is after all, exactly what mathematicians do in their research. – Everest

The second quote describes a view of understanding where it must be developed beyond simple processes when students are working on homework assignments. Students need to apply flexible thinking and analytical thinking, have fluidity with basic skills, and know when and how to apply them.

> In high school, students can get away without knowing some really rudimentary things. I think they have the wrong view of math. They'll see math as answering questions; not problem solving. I think they see math as "here's a formula, now let's do ten examples of them", and that's that. There's no meaning attached to it. Understanding is understanding relationships, and the mathematics from different perspectives. Flexibility in thinking of quantities and their relationships, and also to understand the constructs themselves: what can I manipulate, what does it give me? – Bobbie

The third quote articulates the cognitive processes that must take place in order for understanding to develop. This lecturer had spent a considerable amount of time investigating student work, looking for insights into their difficulties.

> So, what are the differences between expert vs. novice behaviours? An expert has experience and will look at a problem and say what's the closest thing I know how to do that looks like this? Have I seen this before and can I remember the answer? So how do you take novices and provide them with experiences that move them toward becoming experts? They're terrified when I tell them they will eventually see problems that don't look like what they have done before. Eventually they have to be able to play

> around a little with it. So one piece is a mind-set piece. The other piece is what psychologists call reactive thinking or guessing vs. analytical thinking. An expert is someone whose reactive mode is just more accurate. They have enough experience that their first reactions are more accurate more often. Also, experts will engage in the assessment piece: here's my guess and is it right? Then what to do after if it's not? Mathematical literacy and 'fingertip skills' are the mark of proficiency. Students will be transitioning from novices to experts during their respective programs. Retention of skills, and knowing when they are best applied comes from experience, and solving a variety of problems; something which requires an investment of time, and some persistence. So how do we build those types of experiences for students? It's very easy in mathematics to think we are teaching problem solving. But in most cases, we are really teaching technical skills. We rarely have situations for the students that are new. – Dana

4.1.5. Habits of mind and homework – students need 'intellectual tenacity'

All of the lecturers pointed out that the homework problems in university take much longer to solve and are more involved than what the students worked with in high school. Unless students have experienced working on a problem for long periods of time, they might give up just because they don't realize how long they should have to spend solving it.

While the students recognized that homework in university required a much larger investment of time and effort, they didn't articulate it in terms of what they are actually doing that is different from high school. Some of the lecturers, on the other hand, were able to identify the behaviours and attitudes that were necessary when it came to succeeding with the homework.

Everest described these behaviours, in terms of tenacity and perseverance, as well as having the patience and time management skills to devote to more difficult problems.

> What they need are these basic [algebra] skills, and basic intellectual tenacity. Meaning they have to be able to work on a problem without giving up. Many problems the students receive are expected to take in excess of 45 minutes to solve. Students who have been exposed to problems that give way maybe only after half an hour, I think, are in a really good spot because they know this magic combination of time and pressure makes things happen. In many cases though, students lack time management skills. There is a new topic each week. – Everest

Students who do not allow enough time to effectively work through their homework assignments, either because of poor self-motivation or because they struggled with basic algebra, were identified as those who were at risk of failing.

The following quote from AJ describes a common issue with students who struggle, even if they had a positive mindset coming into their first-year calculus course.

> Students have a positive outlook on their pre-requisite skills, but university is deeper and more abstract. For instance, they can't reason or generalize about classes of functions. They struggle with word problems and verbal descriptions of situations to translate into mathematics. Simple things like starting with a diagram and labelling your quantities etcetera is lacking significantly. –AJ

4.1.6. About testing, grading, and university success – culture shock

From the responses collected from each lecturer about grades and testing, I found that testing in most first-year calculus courses was the same from institution to institution and lecturer to lecturer, to maintain some parity and comparability for students. Typically, the marks came from two mid-term exams, a final exam, and homework marks. Midterms made up about 35%-40% of a student's mark and was comprised of questions similar to what students would have experienced on a homework quiz. The final exams were worth at least 40%, but more often were worth 50%, and contained a few more involved questions. The remainder of a student's mark came from homework and quizzes.

I asked a few of the lecturers about re-writes, something that is common at the high school level. There are no re-writes or deferrals except in the most extreme of circumstance that are out of the student's control, and even then, there is a strict protocol and documentation that must be followed to receive permission for academic concessions.

One of the questions I had a personal interest in investigating was what lecturers deemed to be a good grade in first year calculus. What was the minimum threshold for success in their eyes, and how did they expect students to prepare for their midterms

and finals? The lecturers all indicated that they were aware that most students would be experiencing some culture shock when it came to their grades.

SFU, for their part, provided weekly quizzes, which resembled the style of question that would be on one page the midterms. In this way, students had a sense of what they were aiming for with their homework and study efforts.

Some lecturers did say that students would often ask them if the final exam covered the whole course. It was absurd to these lecturers that it might not. After all, the calculus courses they taught were a progression of ideas, and not an eclectic mix of concepts and methods to be studied in isolation. In order not to penalize students too heavily upon their transition from high school math to university math, if a student was ill-prepared for a midterm and did poorly on it, then their mark would be more heavily weighted based on the final exam.

Most lecturers considered a "B" to be satisfactory, meaning that with hard work, a student at this level was likely to be successful in the next course. Having said that, if a student intended to pursue mathematics as a discipline unto itself, then grades in the A range were necessary to demonstrate the requisite level of aptitude. Students needed to have at least 65% (C) in order to be considered "passing". Less than that, and the likelihood of them failing the midterm or final exam was high.

The following quotes illustrate each lecturer's perspective on the matter.

> B is for sure a reasonable grade…B is a place where students are able to tackle most kinds of problems, but they still make certain kinds of algebraic mistakes, or perhaps they still don't get certain concepts, but you know they are still solidly in the game. The averages in our courses are about a C+ B.
> - Dana

> A is excellent, B is good, C is satisfactory, and a C- or D is a marginal pass…If you think of it in terms of a course which is a pre-requisite for another math course, we would consider a student in the A range well prepared for the next course, a student with a B: it's conceivable they could do well in the next class with hard work, and a C student is in trouble in the next class. And if you give a student a grade of a C- it means they know just enough to pass the next course if they work very hard.– Casey

Since the majority of a student's grade was determined by the midterms and final exam, it might be difficult for a student to know if he or she is on track until it's too late. In order to address this, and to give credit for homework, without creating a system which

enabled copying, homework quizzes are often used. A homework quiz is a 10-minute quiz taken directly from the homework. One question is a short calculation questions, and the second is a longer question. With this system, students have an incentive not only to do their homework, but to understand it as well. There is a high correlation between quiz results and final exam grades. The length of these quizzes is meant to mimic 1 page of the midterm exam, and provide a sense of the level of difficulty to expect on their midterm.

> We decided that if we just assign homework but don't give any grades for it that will be a disaster. So I said ok, let's have them write an in-class homework quiz, and to give an incentive to make sure they understand the homework, we won't even change the questions in the slightest. We'll just take them verbatim from the homework…and they'll write a homework quiz, 2 questions, 10 minutes, once a week…Students notice very quickly that it's hard to reproduce a solution if it wasn't your own. Sometimes it feels like you're understanding by following an explanation, just sitting in the passenger seat, but when you go to solve the problem on your own, you understand very quickly that you don't understand it as well as you thought you did. And it turns out that there was a very high correlation between how they did on those homework quizzes, and their final exam. - Casey

Based on the correlation between student marks on the homework quizzes and their midterm and final grades, if it was evident that a student was at risk of failing based on their midterms, student services was alerted and could connect with the student to troubleshoot a solution. This was an example of a coordinated approach, similar to those discussed in the literature, to promote student success.

4.1.7. Advice to students who are struggling – 'don't wait until it's too late'

All of the lecturers were asked what advice they give to struggling students. Without exception, they all stated that most often students either aren't seeking them out for help, or are seeking help too late. Students might be unaware of what their real problem is. It might be basic skills, or the amount of time that is expected for the homework. Being self-aware and realistic was the first key. They may need to brush up on basic algebra skills and they may need to visit the workshops more often. They may need to set aside more time for homework, or they might be working with other distractions. Even knowing what to do in a study group can be hard and takes work to be effective.

In this quote, AJ discusses his perspective on student struggle. He identifies dissonant behaviours he sees in many students, and the practical advice and guidance he provided to a particular student.

> I'm just thinking about some conversations I've had [with students]. I'll ask 'do you have your computer open; is your cell phone next to you; are you text messaging; are you listening to music; do you have any distractions?' And yeah most of the time its 'I've got my Facebook open and I'm trying to multitask at the same time.' So for students who are struggling it's often not a matter of working harder, but working smarter.
>
> So my expectation is that all distractions are put away; that the very first thing they should be doing is working on problems. Pick a problem and start working on it. If you start to struggle then sift through the resources: textbook, pod casts, video lectures, whatever's available. Extract some more information and go back and work on the problem again. If you're still struggling with it use the teaching resources on campus: the tutorial center, the workshop, ask your instructor, post on the discussion board to get assistance. So it's about just working on the course content, working on problems. They are encouraged to make use of office hours.
>
> The other issue is with students who have been working on the course content and are still struggling. It's often pre-requisite knowledge. They don't know where to begin, they don't know how to read the problem, extract the relevant information, ignore the irrelevant information, and with the relevant information how to use it to solve the problem; how to connect that with their previous knowledge. And that's much more difficult to help them with this. - AJ

The expectation is that students, above all else, will seek help. This is necessary in the university setting, since the lecturers only have a relationship with the student, if students come to them. In high school, the student's teacher would likely initiate conversations about progress, and offer interventions for the student. Here, it is up to the student to self-advocate, which can be difficult for students not used to doing so.

In the following quote, Everest describes the reluctance that some students experience when faced with addressing their difficulties in first-year calculus.

> Denial is common, but they should seek help. Even if they went for help after failing the first mid-term they'd be in much better shape. Seek help because students don't often know what to do to get out of this rut, but a lecturer who has seen this happen a thousand times before has some small things to offer. - Everest

4.2. Student Interview Results

As I gathered more and more data, at first unable to find first year students, my own perception of the demographic I had captured began to change. I was collecting data from students who had already found a way to be successful in first year. As much as I had hoped to capture the raw reactions of first year students, I came to appreciate the value of the collection of students I had found. They were from different schools who had come from a variety of high school backgrounds; had had to repeat their first-year courses in some cases, and who had all found a way to be successful in first year university calculus. Even more valuable was that in spite of the diversity in their backgrounds, their responses were very similar in nature. After only a few student interviews, some commonalities began to resonate among respondents. In addition to time, students had much to say about the level of difficulty of their homework compared to high school, expectations around independent study, the nature of understanding and beliefs about mathematics, and their grades.

4.2.1. Results of selected student questions

As I interviewed more and more students, though some of their responses were different, most replied in very similar ways.

The table below shows responses to questions which provided insight in the differences between high school and university math courses, from the students' point of view. Though there were no questions which elicited a unified response, the collection of responses under each heading provides insight into the essence of the student perspective.

Table 5 Results of selected student questions

How is university mathematics different from high school mathematics?	• Pace is much faster • Learning happens outside of class • More depth of understanding is needed • Homework problems take way longer to solve • You have to figure things out • I spend much more time on homework

	• There is more history and beauty about math shown in university
How was high school mathematics different from university mathematics?	• You can learn by memorizing • Test questions were practiced in the homework • There were examples of every type of problem • You could spend more time on each topic
What skills were you lacking from high school?	• A sense of the big picture • Problem solving • Time management • Algebra
What is understanding to you now?	• Being able to figure things out without being told how • Understanding the steps and the significance of each one • Not just how to do something but what to do in different situations
What advice would you give to new students?	• Establish a routine • Learn to work with others • Don't fall behind • Do all the homework • Solve more questions • Review from the day before

4.2.2. Results Related to the Theme of Time

Time as it relates to the pace of the course – no flexibility

At the forefront of most students' minds when faced with the open-ended prompt "How would you characterize first year calculus compared to high school?" was a resounding and emphatic reference to time. They wanted to talk about the pace of university courses compared to high school, how much more time they were spending on homework, and time-management. Time was an important focus, even for those students who had found first-year calculus to be relatively easy.

At the time the study was conducted, Calculus 1 was delivered in 38 hours of lecture time. Without exception, all respondents indicated that a 50 minute lecture period was the norm at both SFU and UBC. By comparison, the high school calculus course, which includes all but one or two topics included in the university version, is delivered to students in 120 hours of class time (Government of British Columbia, 2016). The Advanced Placement (AP) Calculus course is also delivered in the same amount of instructional time, although in British Columbia, students write the AP exam in May.

On the other hand, students entering university from the International Baccalaureate (IB) Mathematics Standard Level curriculum received a comparable 40 hours of instructional time for the calculus component of their course (International Baccalaureate). The pace of the university course as a theme, found its way into the IB student's responses as well.

The most common descriptions were in terms of the speed with which a large quantity of information is delivered during a lecture, and the relentless advancement from one topic to the next. Many students remarked that one of the significant differences between high school and university was the absence of re-caps of the previous lesson. There was no flexibility in terms of time allotted for the delivery and exploration of a topic during a university lecture period.

The following excerpt from Spencer's interview captures the theme of time intensity.

> Here it's a straight 50 minutes of notes. The concepts build on one another and there isn't a whole lot of time between classes. They don't recap from one lesson to the next. There's very little ok last class we did this, here's a reminder how to do this operation…There's the whole concept of don't waste time. I feel that university takes more care with their time. – Spencer

Time as it relates to the lesson – no time for review

In high school, students had been able to request that topics be re-taught, that tests be postponed, and that extra class time or review days be added to the schedule when there was something that the whole class or an individual student was struggling to understand. Students reported that even their AP Calculus class in high school

afforded review days. The hours of instruction in a high school classroom are set, but how teachers use that time can be more flexible, since they have so much more time available to them.

This quote by Frank echoes the intensity of Spencer's thinking on the subject of time, and in particular show's how faster pace impacts his time for practice during a topic, time to review before a test, and time to revisit difficult concepts.

> Calculus in high school is not the same. Here the lecture pace is much, much faster. There are no lecture periods scheduled for practice, and if you don't practice you will fail really bad. Preparing for a midterm, you get less time because you are progressing really fast so you don't get a lot of time to study. In high school if you are struggling and ask for a recap, you can have that over and over again. Here it's impossible. – Frank

Students save time by limiting their questions during class time

Interestingly, an aspect of the university experience, completely within the control of the student, is also quite different. Students reported that they were conscious of the class time that might be spared if they refrained from asking questions. Of the 10 students interviewed, only 3 reported being willing to ask questions during a lecture. These three students proclaimed themselves to be among a slim percentage willing to ask questions at all during class, and admitted that even they were judicious with their questioning. Those who didn't ask question said it was because they didn't want to waste class time with questions that might be "silly", and would usually approach their friends or TA's with their questions instead. This is a different behaviour than what is often displayed by high school mathematics students. In high school, students are less cautious about asking questions that have already been addressed during a lesson, or that require prior knowledge. Spencer said he avoided asking these kinds of questions, and cited "wasted time" as part of his reasoning.

> You really have to think about what question you're going to ask before you ask it. Because the common consensus is if you ask a question and it's stupid, you're wasting 299 other peoples' time. If someone asks a question in university, you're losing class time. – Spencer

This result implies that if students had questions they intended on finding answers to, either they weren't ever going to figure it out, or they were planning to figure

it out on their own time. I wanted to get a sense of the strategies that students used to accomplish this. Most indicated that they did not make much use of instructors' office hours. Sometimes this was because the student didn't particularly like an instructor, but most of the time it was because the student was making use of his or her peers and the help center instead.

> Here you have to look for your classmates and go to the workshops and work on the course together. Lots of people do their homework in the workshops to get help as they go. - Frank

Time's relationship with homework-homework is a full-time job

There were three factors affecting how long it took students to complete their homework. One was the way in which homework questions were delivered, another was the total number of questions assigned, and the third was related to the complexity of the questions.

All of the students interviewed, including the one from the University of Alberta, reported that their homework was from WebWork, or something similar. WebWork is a platform on which students receive and submit homework problems electronically, and for which an algorithm submits a slightly different question to each student. TA's and lecturers can monitor trends in student responses and adapt questions and prompts accordingly. Without exception, all students reported that a minimum of a 3:1 ratio of homework time to class time in university was what was required of them to pass their first-year calculus courses. They said the WebWork assignments took, on average, 4 or 5 hours to complete.

The following two excerpts exemplify student responses when asked about their homework load in first-year calculus.

> The homework was WebWork. Here are 26 questions, now deal with it. You'd need a solid four maybe five hours to get through it. It takes so much time to type them in with brackets and exponents etcetera. - Tahir

> The quantity of it was so much different than in high school. In high school homework was optional, but now it's for marks again. Just the quantity of doing every little question…Like some of these questions, they take a long time to do…I remember math being like heavier than for the other courses. - Sam

A third factor contributing to the perceived differences between high school mathematics and first-year calculus goes beyond quantity, and identifies a difference in quality of problems as well. Some students reported that they often spent a considerable amount of time on what they considered more difficult or complex problems. These types of problems were often assigned separately from the WebWork homework, or offered as supplemental problems in the syllabus.

When asked about the differences between problem-solving in high school, and problem solving in university, students indicated that they spent a considerable amount of time figuring them out. They sometimes got "stuck" which contributed added to the time they spent on homework.

> They take longer because you might approach it the wrong way and have already done a whole page of work. I might have to solve it in more than one sitting. Discuss it with friends and work on it on and off. – Sam

Felix's approach to these types of problems was more solitary, but he was also willing to invest a considerable amount of time working on them.

> Well this would come across different for many people, but I will sit down and think about the problem for a couple of hours if it takes just to solve it. And those ones are not the homework problems, those are the ones I do that are extra. They are usually a lot more difficult. But generally, if you get stuck on a problem you need to refer back to the textbook and the examples and see if the professor has anything from the lectures that might help you, otherwise come here to the workshop. – Felix

Each of the students who were interviewed indicated that they made time to work on the problems which for them, could not be solved right away. All of the student participants said they had to struggle with their homework questions at least sometimes.

I wanted to get a physical sense of the homework expectations with the 3:1 ratio in mind. Since a 3-credit course is 2.5 hours of lecture time per week, the expected study time for each course would be 7.5 hours per week. Five 3-credit courses would then require 37.5 hours of study per week. To put this into perspective, if a student waits until 6:00 pm to start their homework, (as they might have done in high school, or if they participate in extra-curriculars after class) and takes one day off per week, they'll be working on homework until midnight every day just keeping up. This is in fact what many of the students reported they were doing. A really interesting and somewhat counter-intuitive result from the interview responses was that when students were

explicitly asked if they felt they had enough time to complete the homework to a satisfactory level, all but one of them replied “yes”. For me, this was a startling result, and would later become part of the paradigm shift from high school mathematics to university mathematics. This paradigm shift will be exposed in Chapter 5.

Time’s relationship to personal time-management- the importance of routine

Half of the student respondents reported that in their first year of university, one of the most important skills they were lacking wasn’t related to curriculum at all, it was time management. In high school, the workload and intensity ebbed and flowed. In university, even the AP students had found the pace in first year difficult to manage.

> Time management is the biggest skill I was lacking from high school. The jump from high school to university is huge, in high school you could get away with one maybe two hours of homework, and then in university you could be working all day. “ – Stefan

In addition to their calculus course, some were also enrolled in science and other courses which were also very demanding of their time. A few mentioned that first year physics – another course which has a heavy emphasis on calculations, was also quite demanding of their time.

Whether they had to balance their workload with other courses, or worked exclusively on their calculus courses during summer session, all of the participants stated that to be successful in calculus as well as their other courses, they had needed to do their homework and review concepts daily. Some students talked about this when asked for their advice to incoming students.

> I feel like a routine is really important, cause once you set a routine it’ll make your life really easier, and at the end of the year you’ll get a better mark. – Farjad

> Don’t leave things till the last minute. Review every day what you did in class. Don’t get behind. -Sam

Expectations around independent-study – much learning happens outside of class time

If students were leaving the lecture theater with questions before starting their homework, how might that affect their success with it? One reason why students were spending so much time on the "thinking questions" in their homework was that they were also having to "learn" the about the mathematics they were using as they went. When asked about the strategies they used in order to find the answers to their questions, most participants indicated that they did not make much use of lecturer office hours at all. Sometimes this was because they didn't particularly like a lecturer, but most of the time it was because they were making use of their peers and the Help Centre instead.

In high school, by comparison, much of student learning happened within the classroom walls, and it was there where students asked most of their questions. Independent study time was mostly practice based, whereas in university, students felt they were expected to figure more of it out for themselves. This was another contributor to the theme of "time" in their responses.

In the quote that follows, Fulton describes this difference in teaching approach, and the resultant expectation on the student at the university level. He describes his perception that his lecturer's role is to introduce the material, and his role is to learn the rest on his own. This requires time, and so this aspect of the student experience is included in this section.

> Prof's job is just to introduce you to the topic and you have to essentially learn the rest on your own. In high school, they'll hold your hand through the whole topic essentially. They'll do examples for every type of problem you'll expect on exams. In university, they just give you the bare bones, the most basic example they can give you, and they just let you figure the rest out on your own essentially – Fulton

Stefan also made reference to learning, and that it happened outside of class time. How students spend their time in and out of class, and what was happening during that time was one difference between high school and university.

> In [high school] there wasn't' much lecturing, but there was lots of hands on work with a worksheet, where each question was a little bit harder and the learning happened in the class, whereas in university the learning happened outside the class. - Stefan

As one student remarked, homework was no longer optional, even if it wasn't being graded. Since the content was delivered in class so briskly, students had to engage with the material in more depth on their own time if they were to do well on their exams, and be properly prepared for the next course. This was in contrast with their high school experience, where the expectation was that if something wasn't mentioned in class, then it wasn't fair game for a test.

Many stated that their level of success was directly related to the amount of time or work they invested outside of class time. They even reported that they had spent more time, not less, on topics they are struggling to understand. In high school they might have been apt to avoid or skip homework questions they didn't fully understand, or that they got stuck on. They all remarked that it was easy to be quite successful in high school without doing much work at all. They also realized they were being compared to one another, as well as being judged on their own merit. Often, what had been a sufficient amount of study and practice in high school was no longer sufficient in university, and they recognized they had to meet a new standard. Tahir, who had done the AP Calculus course overseas, articulated this key difference between high school and university.

> I expected to do well because I had done the course in high school, but then realized there are students who totally have it down and I have to keep up with them. The homework was very difficult and very time consuming. In high school you could get good marks without working hard." – Tahir

Felix, who had repeated Calculus 1, and was now taking Calculus II during the summer session, noted that his expectations around free time had had to change in order for him to be successful. So much so, that he had taken the summer school course in order to allow for the extra time he felt that was necessary for his own success.

> You'll have to do more questions than what is expected depending on your understanding. There's obviously not as much free time as in high school, so you must make some sacrifices - Felix

4.2.3. Students' perception of understanding evolved– accessing prior concepts and to solve unfamiliar problems

The previous section identified the ways in which time emerged as a theme from the student response data. One of the reasons students were spending more time on

their homework than they had in high school was because of the nature of their homework questions. All of the students had made comments indicating they recognized that they were expected to be able to navigate more difficult problems, and leaving time for this had become part of their homework landscape.

> Sometimes you can work on a problem and be off track and have to start over. Thinking questions take way longer – Sam

This relationship between time and understanding was sometimes articulated by students in their responses. For example, Farook and Fulton had just received back their second midterms on the day of their interviews. They disagreed about the "trickiness" of a particular question on the midterm. Farook, who thought a question was tricky, had said he had spent quite some time working on it on his exam. In contrast, Fulton had disagreed with Farook whole heartedly about the trickiness of that particular question, and had said he thought the same question was straight-forward because he had recognized how to do it.

On other occasions, understanding, as a topic, seemed to be separate for students. It wasn't just the amount of time needed to complete homework which differentiated the university experience from that of high school for many participants. These students had a different idea about what it was to understand mathematics, and their role in developing their own understanding. They described high school mathematics problems as word problem they had been shown how to do. They might appear on an assessment with modified numbers, or perhaps with some kind of recognizable variation. Those problems weren't novel situations requiring students to choose from a variety of solution approaches. The quote below captures how most students felt about problem-solving in their high school mathematics courses.

> In high school problem solving is kind of just memorization rather than actually problem solving. You just memorize a problem and know how to do it. - Farook

In a university mathematics course however, students had to learn how to troubleshoot and apply problem solving strategies in order to answer questions in a context that hadn't been discussed during their lectures or homework assignments. Not all test questions presented novel scenarios, but some of them did.

> Understanding is not just knowing how to do something but how to know what to do in different situations. - Stefan

Another way to see understanding was in terms of depth. Understanding to many participants meant they understood the inner workings of the mathematics they were using, not just how to apply formulas or multi-step algebraic manipulations.

> To understand the very basics of something would mean understanding to me. Like not just how to do a question or how to get through a question, but to get what each step is telling you and what it signifies, so like if you were given the question in another form you wouldn't get confused. [Understanding is when] you can see things for what they are, and be able to put them in the right place, and solve something." - Tahir

Being expected to recall and build on previous topics was a key delineator between university and high school in many conversations with participants. The students reported that for the most part, their high school curriculum had been presented as separate topics. Pre-calculus 12 topics jumped around from trigonometric functions to logarithms, then to combinatorics and probabilities, and finally there was composition of functions with domain and range. Though each of these topics are important pre-requisite knowledge for calculus, often they were studied and practiced in isolation of each other.

> In high school, once you are done a topic, you don't really see it again. - Stefan

The comparison between this relationship with the curriculum in high school, and the narrative that developed within a topic in university was another difference between high school mathematics and university level mathematics. Students' said their lecturer or instructor would introduce a topic with some kind of story or problem that couldn't be solved with the skills they had so far. Then the requisite skill naturally evolved and was explored as a way to solve this problem – a method or relationship that was not available to them up to this point. Students had to then retain those newly discovered relationships and skills as they moved from topic to topic. Understanding and mastery became the ability to navigate between topics rather than apply methods within just a specific topic. If a student struggled with a topic on a midterm, they would still have to improve their understanding of it to do well on the next exam. They would not be able to forget about it and move on.

> In university math, you have to refer back to what you've learned. Understanding would be if I were given a problem to solve with no way as to the how I should solve it, and I could find a way to solve it with all the stuff I've learned. I don't get explicitly told how to solve something. I'm just given a problem and asked 'here can you solve this yes or no?' – Felix

In one respect, the nature of calculus as a single branch of mathematics requires students to revisit concepts, expand on them, and continuously apply them throughout the course, whereas high school mathematics in Canada is a survey course of pre-calculus topics such as geometry and algebra, which are not taught as separate courses, so there is little sense of progression within a student's high school mathematics experience. Having said this, depth of understanding and problem solving skills within a topic such as quadratic functions or probabilities, can still be required at the high school level. But the students I interviewed reported that depth of their understanding within a topic in high school was not comparable to what was required of them in university.

> In university, I did more homework just because I really wanted to get the concepts down. Because I knew that they wouldn't be testing if I knew the concepts, they would be testing if I could apply the concepts. The tests reflected the homework sometimes, and others it meant you really had to understand the concept. In university, for a final exam, it wasn't a question that had been in the text with the numbers changed, it was a brand-new problem. In high school, they teach you to problem solve, and in university they need you to problem solve, but they don't necessarily teach you how to do it, but its required in order to do well in the course. - Stefan

4.2.4. Beliefs about mathematics – rules vs. understanding

The quote from Stefan in the previous section illustrates a shift in what it means to understand mathematics. This shift has more than practical value. One student responded passionately that he was bitter that he had been denied the opportunity for true discovery and understanding in high school. He said that he wished he had known what mathematics was really about in high school, because as it turned out, it was an area of study that he was truly interested in. He now felt deprived of experiences learning mathematics that he wished he had had.

> I believe it's become like a plague for a lot of students to not enjoy the subject even though there is actually a lot of importance and I'd say actually beauty behind the subject. But [this aspect of mathematics] is not shown in high school. In high school for a new topic, you are presented say

> something algebraic or just a handy tool but you're not ever told why you'd use it or how it came about. You don't get a chance to discover it on your own. You just get presented a formula and then you do some exercises – Felix

Other students' responses also revealed a shift in their beliefs about mathematics learning as a result of their first-year calculus course. Though they weren't all as passionate about it as Felix, their high school experiences were equally superficial.

> In high school, you just have to memorize rules, but in university it involves more understanding…there are more difficult and challenging questions – Frank

> You're expected to know everything in depth. – Tahir

4.2.5. Prerequisite skills – algebra is only a means to an end

Though they all indicated in one form or another that algebra skills were an important component of their homework, algebraic proficiency was not at the forefront of any of the conversations I had with students; time management was. This is not to say that students didn't see algebra skills as being important. On the contrary, all students acknowledged that algebra skills were important, and Farook and Fulton in particular made comments about their algebra skills being a limiting factor in their success. When reviewing the pre-requisite courses taken by all of the students who were interviewed, I noticed that 3 out of the 5 students who had Pre-calculus 11 or 12 as their pre-requisite, but no high school calculus course, seemed to have struggled more in their first-year courses. It feels ironic to me that when considering his own preparedness for university calculus, Farook felt that having to repeat Precalculus 12 meant he was prepared.

> I've taken Precalc 12 twice to get a better mark. I was properly prepared for all the algebra and all the basics. – Farook

The students who had done either the AP Calculus course or the IB HL course, had successfully completed their first-year calculus courses on the first try. Since the students' grades on the AP exam had been 4 or higher, and the IB HL student attained a 6 in that course (>90%), this small sampling is consistent with the findings in the literature regarding the efficacy of high school calculus courses.

Since proficiency with algebra skills was required for students to make forward progress on their homework, I expected it to come up when I asked students if they felt

there was enough time to complete the homework. If algebra skills were a factor which contributed to their struggles with time management, it didn't come up at all in their descriptions of what they found challenging about the homework. Of the student participants, all said there was enough time for the homework except interestingly, Taj, who had done the IB Mathematics HL program in high school.

Though these students were coping, they reported that many of their friends were not. They said their friends struggled most with basic algebra and procrastination. Consequently these friends were behind and struggling with their homework, and doing poorly on their midterms and their final exams. Frank had also fallen prey to poor homework habits in his first attempt at Mathematics 150, and had needed to retake the course. He said it was only with the hindsight that came from failing, even when he had taken high school calculus, that he was able to achieve an A on his first midterm the second time through. In the following excerpt from his interview, Frank describes how he assumed his high school experience would be equivalent to university calculus, only to find out that he was expected to apply his skills to solve problems, not just demonstrate them in isolation.

> Calculus in high school is not the same. The first time I took it I thought I knew it already from high school so didn't think I needed to do anything at all. But then in terms of actually doing the exam questions, they were much harder and I needed more practice. - Frank

4.2.6. Test preparation in high school vs. university – being proactive is the key

Since topics build on one another, the nature of testing was quite different compared to high school for these students, and so then was their concept of test preparation. There were no unit tests the way there often are in high school. Here the students wrote two midterms and a final only. This again required them to retain knowledge and understanding from one topic to the next in a way that had not been previously required of them. There were quizzes and hand-in assignments for different topics, but the midterms ensured that students would have to be able to retain information from the beginning of the course right up until the end of it. This required study strategies and attitudes that went beyond last minute review or cramming.

> There's more pressure and it's something you should not underestimate, because that's what students do; and you come out with averages of like 50% or something like that. A lot of the stuff that you learn throughout university in order to prepare for an exam, [requires that you] have knowledge of stuff from beforehand. If you don't have the stuff from beforehand it gets a lot more difficult to interpret and understand what you're studying. - Felix

Students not only reported doing all of their homework questions, but also reported employing a proactive approach to studying for exams.

To accomplish this, good time management skills were key. Spencer maintained a schedule where he endeavored to be 2 weeks ahead of the lecture schedule at all times; pre-teaching himself the material and then using the lecture time to clarify any issues or questions he had. This way when mid-terms came around, he was not just learning the material at the last minute as some of his other classmates might be, he was already in review mode.

> As much as I can be before mid-terms, I am usually two weeks ahead of the lecture all the time; so two weeks ahead of homework, lecture, review, then come mid-terms and I'm then caught up and coming along. - Spencer

Though Spencer seems like an extreme case, other students had a two week window in mind as well when preparing for cumulative exams. They said anything longer, and they would be needing to re-review. There was often new material coming during the lectures as late as one week before the exams. This seems like a long time, until we remember that the homework itself for all of these students' classes will take the better part of a week to complete.

> Finals come around in university and you're working all day. Lots of people I know, they studied a day or two before and that was it. For me it was generally one or two weeks because you don't want to start reviewing and then have stuff that's left over. – Stefan

4.2.7. What wasn't said and its' importance – locus of attribution

Before I identify what wasn't said, and its' importance, it is helpful to recall the other differences expressed so far between the students' high school experiences and their university experiences. When considering all of the responses together, it is evident that a successful student in a university calculus course requires a depth and breadth of understanding that simply isn't required in high school. The students must also be

willing to make a commitment to learning and concede that the demands on his or her time will be substantial. Learning in university is not only fast-paced but relentless and unforgiving of poor time management skills. University students also expect to see things on their tests that weren't explicitly covered in their lectures. In addition to their studies many of the students I interviewed also participated in sports and other extra-curricular activities. They were enrolled in engineering courses like chemistry and physics, each of which placed them under as much pressure as their mathematics courses. They were willing to put in multiple hours of work in order to understand first-year calculus concepts to the best of their abilities. These successful students, as busy as they were, fit the profile of a successful student presented by Pascarella et al.

As I was collating responses and grouping them into themes, *the absence* of two types of responses I had been expecting, never materialized.

The first thing that struck me about the responses of these students was that none of them assumed the attitude of complaint when talking about their workload in first-year calculus. This was in spite of a collective lament that high school had been poor preparation for university in terms of time management skills, even for those who had taken AP Calculus.

> You obviously don't have as much free time as you did in high school, so you have to make some sacrifices, that's not to say you have to dispose of your social life and not interact with others, there is always time to do that but that's what comes with going to university, you are becoming an adult you are becoming mature and that's what you have to manage. Are you going to live with your parents or are you going to go out there on the world on your own, … in a sense university [contributes] to you maturing to an adult age of self-responsibility and that kind of thing. – Felix

The second noteworthy absence from the data, was the feeling that they didn't have enough time for the homework. I had been expecting at least some of them to complain that the expectations were unfair, or unfeasible. The participants had described in detail the different demands on their time because of their calculus course (Web Work, going to the Help Centre, having to work through as many of the problems as possible, getting stuck and having to go back to their notes or the textbook), but each of them had been asked if they felt they had enough time to do the homework, and 9 of the 10 had said "yes". I considered this a very surprising result. Even when pushed beyond what they thought was possible, students still said they had time for their homework.

> When you think you can't take on any more, they give you more, and somehow you do it. There is very little wiggle room. - Spencer

Perhaps this is not surprising at all, when considered in context of their advice to other students. In the section that follows, student responses indicate an internal locus of attribution for their own success.

4.2.8. Advice to new students – do your homework every day

The resounding advice to new students was in regards to time management and especially not treating the homework as optional, since it was critical to building understanding.

Students made reference to establishing a routine, making a study schedule, reviewing class notes frequently, and not getting behind. The following list of responses is included since finding an answer to this particular question was one of my goals for this study.

Table 6 Advice to new students

Fiona	• Keep up to date with the lectures. Don't get behind. Understand all of the assignment questions. After the lecture, review your notes.
Felix	• You'll have to do more questions than what's expected depending on your understanding. • There's more pressure and it's something you should not underestimate, because that's what students do, and you come out with averages of like 50% or something like that. • For a lot of the stuff you learn throughout university in order to prepare for an exam, you need to have knowledge of stuff from before-hand. If you don't have the stuff from before-hand it gets a lot more difficult to interpret and understand what you're studying now, because what happens in high school is you could get away with memorization a lot; here you can't. • Generally if you get stuck on a problem you need to refer back to the textbook and the examples and see if the professor has anything from the lectures that might help you, otherwise come here to the workshop.

Farook	• Do all your homework so you can't complain after the test.
Fulton	• I feel like a routine is really important, cause once you set a routine it'll make your life really easier, and at the end of the year you'll get a better mark.
Frank	• Don't assume you already know the topics you saw in high school. • Don't get behind.
Stefan	• Don't study the day before. • Don't take more courses in the first year than you can handle. • Learn to time manage and make a schedule for studying. • Don't get behind because there's no time to recover. • Keep up with the material, and make sure after lecture you still review it; not just the day before the midterm.
Sam	• Don't leave things till the last minute. Review every day what you did in class. Don't get behind. Studying with friends reinforces your own understanding.
Spencer	• Stay ahead of the lectures if you can
Tahir	• none[7]
Taj	• Solve more questions. Don't underestimate how hard it's going to be. Expect 40% of you will not make it. • Learn how to work with others.

Taking calculus in high school facilitated a sense of familiarity with the concepts, but there was no equal in their experience in terms of the depth of understanding that was required, and the level of independence that was expected. The students held themselves responsible for their own success, and didn't complain that a certain professor was the cause of their struggles or failures.

[7] Tahir said he didn't know what advice he could offer that Taj had not already said.

Even if you took Calculus in high school don`t assume you already know it. Don't treat the homework as optional. You need to put in more effort because the questions are harder and it [the course] goes faster. Make an effort to try to understand the concepts from the lectures and actually do the practice. Try to spend more time on it. Its' going to get harder, and this is your only GPA booster because they [the courses] just get harder; so you will struggle if you don`t decide to work hard. – Frank

4.3. Comparison of Answers to Related Questions

The following table summarizes the results of the related questions presented in section 3.4.3.

Table 7 Results of related questions

Lecturer Interview Question	Student Interview Question
What is your impression of first-year Calculus students? • Students have a positive outlook on their pre-requisite skills, but university is deeper and more abstract • They struggle to remember basic skills • They can't reason or generalize about classes of functions • They have trouble translating written situations into mathematics • Certain gaps in understanding carry through from year to year • They are not used to having to answer questions on a test they haven't practiced in the homework • Students have the wrong view of math. They don't see it as asking questions and problem solving	How would you characterize your first-year calculus course compared to your high school experience? • Expected to do a lot of work yourself • Pace is different, it's hard to keep up • Expected to know everything in depth • There are more difficult and challenging questions • Test questions aren't from the textbook • The questions are asked in a different way • In high school problem solving was just memorization rather than actual problem solving • Learning happened outside of class • You don't get as much immediate help • Less scheduled practice • You don't get as much time to prepare for midterms • Homework isn't mandatory anymore • In high school you just had to memorize rules

Do you feel that students are prepared to take on the course when they come out of high school? • No (all respondents) • They think they are, but they aren't • The list of topics they actually "need" is exaggerated • Many students jump into Calculus without being prepared. • Taking a course you aren't prepared for isn't necessarily the fastest way to finish a program If no: where do you see them struggle? • Basic skills • Analytical skills • Don't know where to begin with a problem • Don't know how to read the problem • Don't know how to extract the relevant information and ignore the irrelevant • Don't know how to connect relevant info to pre-requisite knowledge • Problem solving in unfamiliar situations • Starting homework right before its due • Knowing they should be working 8-12 hours on homework • Knowing they can ask questions during office hours • Being away from home • Knowing what to do in a study group	Do you feel that you were prepared coming into this course? • No (8) • Yes (2) If no: What skills were you lacking? • Algebra • Problem solving • Time management

Do you distribute a course outline with a list of topics? (Meaning if students want to read ahead, can they?) • Yes (all respondents)	Did you receive course outlines? • Yes (either in print or online) How did you use the course outline? • Used it to work ahead • Used it for review • Didn't really use it
What are the students' sources of information and practice within your course (ie are resources like lecture notes available, online practice or tutorials, videos of the lessons, a bank of applets etc?) • Textbook • Lecture notes • Videos • Supplementary problems • Student peers • Tutorials • Online homework	What did you do if you had questions about the calculus homework in university? • Help center / ask TA • Ask a friend • Ask the lecturer What did you do if you missed a class? (none of the students reported missing more than one class) • Asked a friend or TA what was missed • Look at the notes, then the text

How is the textbook used in your course?
 	
• Students are expected to read ahead
• Extra problems only
	Did you read ahead?
	
	• Yes
	  - to read ahead
	  - to refine their understanding
	  - for extra problems
	• No
	  - their course notes were sufficient for most purposes)
	  - the text was too difficult to understand

What are the homework expectations in your course? • Expect students will spend 3-4 hours of homework time for every hour of lecture • 1 homework set per week covers 3 sections for a total of 24 prescribed problems per week + 1 or 2 that weren't from a prescribed section so they aren't referenced in the text • WebWork questions should take 1-2 hours and are due 1 hour before the next lecture • Written problems may take 1-2 days to complete and get handed in • Textbook assignments are optional and are useful as other examples • Homework quizzes represent one page of the mid-term	What was the homework load? • Obviously not as much free time as in high school so you must make some sacrifices • 3:1 ratio of homework to class time seems right (all respondents) • Homework was WebWork – here's 26 questions, now deal with it • Need a solid 4 maybe 5 hours to get through it • 16-25 problems per week • Homework isn't optional anymore • You can be working on a problem and be off track, and have to start over Do you feel you had time to do all of the assigned homework in university? • Yes, but it took a long time (9) • No, not with all the other classes (1)
What are your expectations of students who struggle with the content or pace? • Don't continue to keep doing the same thing • Come to office hours • Go to the workshop • Email the lecturer • Talk to friends • Read discussion boards • Improve algebra skills	How did you find the pace of the course high school / university? • Even high school AP Calc was better paced out • University Calculus is much faster than high school • About the same (1)

What is a successful mark on the first midterm in your opinion?

- 16/30
- C+ or B
- B
- 80%
- B+

Typical University letter grades (University of British Columbia, 2016)

Percentage	Letter Grade
90-100	A+
85-89	A
80-84	A-
76-79	B+
72-75	B
68-71	B-
64-67	C+

How do you define success for yourself in in first-year calculus?

- Good mark
- Being prepared for the next course
- 60-70%
- Being able to say you understand what's going on

Typical BC high school letter grades (Government of British Columbia, 2016)

Percentage	Letter Grade
86-100	A
73-85	B
67-72	C+
60-66	C

Can you describe a lesson or sequence of lessons for a typical in the course? • Illustrate a need for a new method or concept by presenting a problem students don't yet have the techniques to solve • Encourage them to wrestle with the problem • Encourage students to present their ideas • Make use of and expand on student ideas • Reduce the number of examples to provide time for discussion • An engagement that goes back and forth • As the term goes on, I show fewer and fewer of the steps	How was a new topic introduced in high school? • In high school for a new topic you are presented say something algebraic or just a handy tool but you're not ever told why you'd use it or how this was stemmed • You just get presented a formula and do some exercises • With a new topic they would go slowly, and try to show different types of examples • She would present a problem that we don't yet know how to solve, and then we would discover how to solve it
What advice do you give first year students, if any? • Be persistent • Be independent • Ask yourself "what kinds of questions can we ask here, how long should this take, and when should I give up?" • Take ownership of your work • Put your cell phone away • Do one or two hours of practice each day • Expect to spend some time working on each problem • Spend time in the workshops • Don't leave the homework to the weekend, you will have forgotten what you did earlier in the week • Use the textbook definitions and examples to create your own learning	What advice would you give students coming into Calculus 1? • Do all the homework • Establish a routine • You'll have to do more questions than what is expected depending on your understanding • Work with others • Don't study the day before • Don't take more courses than you can handle • Learn to time manage • Don't get behind • Don't assume you already know the topic

Chapter 5. Unifying Themes

5.1. Comparing the Language in Student and Lecturer Responses

The table below summarizes and groups the corresponding student and lecturer responses by theme, rather than by question. It is interesting to consider responses that seem like 'two sides of the same coin', where left side of the table contains the student perspective, and the right side reveals the lecturer perspective.

Table 8 Comparing language in student and lecturer responses

STUDENT RESPONSES	LECTURER RESPONSES
4.2.2 Time as it relates to the pace of the course • no flexibility	4.1.3 Time as it relates to pace and pedagogy • doing more with less
4.2.2 Time and its relationship to homework • homework is a full-time job 4.2.2 Time and its relationship to personal time management • the value of routine 4.2.8 Advice to other students • do your homework every day	4.1.3 Lecturer's recommendations about homework and time management • do something every day

4.2.2 Expectations around independent study • the prof's job is to introduce you to the topic, and you essentially have to learn the rest on your own • sometimes you have to start over on a question • do more questions	4.1.5 Habits of mind as they relate to homework • students need intellectual tenacity
4.2.3 How Students' perception of understanding evolved • in high school, problem solving is just kind of memorization • understanding is not just knowing how to do something but how to know what to do in different situations • you can see things for what they are, and be able to put them in the right place, and solve something 4.2.4 Beliefs about mathematics • in high school, you just have to memorize rules, but in university it involves more understanding • you are expected to know everything in depth	4.1.4 How lecturers viewed the development of understanding • seeing the forest for the trees • expert vs. novice behaviors • flexibility in thinking of quantities and relationships, and the constructs themselves
4.2.5 Pre-requisite skills • algebra is a means to an end • homework habits are skills that were lacking	4.1.3 The role of algebra skills with respect to class time • algebra is a means to an end • students who struggle with algebra are most at risk of failing the course

4.2.8 Advice to other students	Compilation of Results About Where Lecturers See Students struggle
• do your homework every day • routine's important • solve more questions	• expert vs. novice behaviour • having the "wrong" view of math • students are surprised to learn the final exam is cumulative • students don't know how to apply their knowledge to unfamiliar problems • They start their homework too late

5.2. What It Means to Do Mathematics

For most of the students I interviewed, it was evident that their concept of mathematics as a discipline had changed since high school. Specifically, their answers reflected a shift in their perceptions of understanding and problem solving.

5.2.1. Problem solving skills

> Problem solving is at the heart of any university level mathematics and should be at the heart of any mathematics class. – Casey

This sentiment by Casey, was echoed by the other lecturers, as well as on an institutional level by the mathematics departments at SFU and UBC. The mathematics department web pages for both universities use identical language around problem solving, critical thinking, abstraction and logic when describing their visions of mathematical training and curriculum.

> Mathematical training emphasizes powerful and versatile modes of thought including abstraction, critical thinking, logical analysis, and problem solving. – SFU Department of Mathematics (Simon Fraser University, n.d.)

> The curriculum emphasizes powerful and versatile modes of thought including abstraction, critical thinking and logical analysis. You will also study the art of problem solving using logic, precision, intuition, and imagination – (University of British Columbia)

When discussing where students struggle, Bobbie described the process of doing mathematics in terms of looking for relationships, and analyzing what information has been given, and how it can be used. The description of the process of doing mathematics contained therein was not the view that Bobbies students had of mathematics upon entering their first-year calculus course.

> …understanding relationships, and the mathematics from different perspectives. Flexibility in thinking of quantities and their relationships, and also to understand the constructs themselves: what can I manipulate, what does it give me? - Bobbie

The student interview results revealed a significant difference between students' high school mathematics experiences, and their Calculus 1 courses in precisely this regard. The students tended to describe their understanding of mathematics from high school as more superficial; often describing their experience in terms of practicing the same thing repeatedly and not in terms of having to think about mathematical objects such as functions from different points of view. They were shown procedures and asked to remember arbitrary processes and memorize relationships, rather than to consider the necessary results of those relationships in order to create meaning. They said their university mathematics questions required a deeper understanding of concepts than their high school questions had.

Farook used the phrase "next level" when comparing first year calculus to his high school pre-Calculus course, and had said that in high school, he had always been shown how to solve the questions that would be on his unit tests. He viewed the unfamiliar problems that he was facing in university as having a "twist".

> In high school, problem solving is kind of just memorization rather than actually problem solving. You just memorize a problem and know how to do it. Whereas in university, there will always be a twist to the problem you did in the textbook or did in the class.

Farook had needed to extend his understanding to unfamiliar situations, but sometimes thought these kinds of questions were tricky, and said he often struggled to recognize the relevant details within them. He was not the only one whose high school experience had fallen short of university expectations when it came to problem solving.

Student descriptions of their high school experiences were not in line with university level problem solving expectations at all.

From the lecturer perspective, doing mathematics involved looking for relationships and exploring the nuances within individual concepts. First-year calculus students struggled with this expectation.

5.2.2. Perceptions of high school level problem solving

The following table outlines the various responses that compared high school mathematics and expectations around problem solving to the expectations around problem solving in first-year calculus.

Table 9 Student responses when comparing university mathematics to high school mathematics

Farook	• Pre-calc is like simple algebra. This [university calculus] is next level • In high school, all of the test questions were from the textbook • In high school problem solving is just kind of memorization rather than actual problem solving
Fulton	• In high school, they'll hold your hand through the whole topic • In high school, there will be examples for every type of problem you'll expect on the exam
Felix	• In high school, you are presented something algebraic or just a handy tool, but you are never told why or how it was stemmed (referring to how came about) • In high school, you just get presented a formula and then do some exercises

Frank	• [University mathematics has] more difficult and challenging questions • In high school, you just have to memorize rules but in university it involves more understanding • [In university mathematics] the questions are asked in a different format • [In university mathematics] you have to understand definitions
Fiona	• [In university mathematics classes] understanding is more important than marks, because the skills build on each other • [Calculus 1] is not like Pre-calculus, the problems about limits and derivatives are totally different
Sam	• Questions in the [university calculus] homework require more problem solving. They are more abstract.
Stefan	• In high school, it was more discovery oriented[8]
Spencer	• In high school, it was the same as here - finding a way to solve or approach an unanswered question.
Taj & Tahir[9]	• [in high school] with a new topic they would go slowly and try to show different sorts of examples. • [In high school] they would show us what things might be on a test • [In high school] you could get good marks without doing much work

[8] Stefan's high school experience was very similar to what he described he experienced in his first-year university calculus course in terms of teaching approach and problem solving – except for the workload, which he said was the most difficult thing to get used to.

[9] Tahir said he agreed with Taj and had nothing new to add.

In the vast majority of cases, the participants' high school experiences did not explicitly develop the problem-solving skills, or the problem-solving mindset that the lecturers said they valued and expected of first-year students. In high school, most student participants were not expected to engage with mathematics in a way that valued solving problems they hadn't seen before. In fact, none of the participants talked about problem solving at all, when asked to characterize their high school experiences. These students did come to their first-year courses, if not with "the wrong view of math" as Bobbie had said, then at least with an incomplete view. Felix had spoken quite passionately and bitterly about this very absent component of his high school experience. He felt he had missed out on a great deal of wonder and discovery.

> In high school the way it is taught is very poorly presented to students. That is why you often get a lot of people who generally just don't enjoy the subject… I believe it's become like a plague for a lot of students to not enjoy the subject even though there is actually a lot of importance and I'd say actually beauty behind the subject. But it's not shown in high school. - Felix

5.2.3. Algebra skills are required for understanding

Mastering algebraic tools had been the end in itself in most participants' high school experiences, but in their first year calculus course, they needed not only to be familiar with those tools, but also understand how to make effective decisions in using them to approach unfamiliar problems. Calculations alone were not considered "doing math" at the university level.

The following quote from Sam describes a perception of problem solving more in line with university expectations. Sam was already in second year, and had survived the gauntlet of first year courses, so he had some perspective on the issue. He had indicated that university level problems were more abstract, and to understand how to solve them would mean he could do it without explicitly being told how.

> Understanding would be if I were given a problem to solve with no way as to the how I should solve it, I could find a way to solve it with all the stuff I've learned. I don't get explicitly told how to solve something; I'm just given a problem - here can you solve this? Yes or no?

This attitude toward problem solving was the ideal, but had to be developed over time for students. Everest had commented that students should be able to experience what it is to be a mathematician working on a problem that is genuine, and not just an exercise in a textbook.

> The ideal I aim for is where the room feels very small, and we are really just having a kind of conversation in which we are just trying to solve a mathematics problem, which is after all, exactly what mathematicians do in their research. – Everest

In order to facilitate this kind of exchange, some depth in students' understanding was necessary, as well as a solid grounding in the algebraic tools available to them. This was one way in which university mathematics lecturers facilitated the transformation of students from novices to experts.

5.2.4. How understanding was perceived by lecturers and students

In section 4.2.3, student and lecturer attitudes and definitions of understanding were discussed. In high school, "doing math" had involved regurgitating methods to answer questions students had usually seen before. In university, this was not considered "doing mathematics". When comparing the level of understanding that had been required in high school, with the understanding that would be necessary to engage with unfamiliar problems, students were able to articulate how their perception of understanding had changed. Tahir had advanced past a purely algorithmic approach, and indicated that he sought to understand the reasons behind what he was doing.

> In university, you're expected to know everything in depth…understanding is to understand each step in a process; to understand the significance of each step. - Tahir

Stefan had described understanding in terms of problem solving. He recognized the need to be able to choose the correct approach.

> Understanding is not just how to do something but how to know what to do in different situations. – Stefan

Engaging with mathematics in a meaningful way had caused these students to redefine what it means to understand it.

5.2.5. The role of time in what it means to do mathematics

In order to be successful in their first-year calculus courses, students had needed to be much more diligent with their time-management than they had needed to be in high school. This was a byproduct of the nature of how they were approaching their studies, and what it means to do mathematics, in two ways. First, to become proficient, students needed to work on a large quantity of problems, which took time, and was not happening during class time.

> You are expected to do a lot of work on your own. – Farook

Only by trying a variety of problems would they be able to learn how to make the decisions that would enable them to transform from novices into experts. Second, since some of the problems were more complex, students also needed to spend more time than they had been used to on individual problems. If they had poor time management skills, and were leaving things to the last minute, they would never reach the stage where the time they were putting into their studies was paying off. Conversely, those who did have good time management skills, and were able to invest the time needed to solve harder problems, ask questions, or get help, were the ones who had found a way to be successful.

> The learning happened outside of the class - Stefan

Abstraction, critical thinking, and logical analysis are not instant processes. Everest used the phrase “magic combination of time and pressure” to describe time’s relationship with doing mathematics.

> Many problems the students receive are expected to take in excess of 45 minutes to solve. Students who have been exposed to problems that give way maybe only after half an hour I think is in a really good spot, because they know this magic combination of time and pressure makes things happen. – Everest

5.3. Persistence and Perseverance

Persistence to meet short-term goals such as solving a problem, and perseverance to achieve long term goals, such as working one’s way through a program, or retaking a course to improve understanding, were also themes that permeated many of the results.

Spencer had been trying to stay two weeks ahead of the lecturers for his first-year calculus course, and had reflected that he had felt pushed to his limits. He had been able to persevere the very difficult schedule he had made for himself.

> When you think you can't take on any more, they give you more, and somehow you do it. There is very little wiggle room. - Spencer

The expectations around independent study would require students to be persistent in finding solutions to the problems they were struggling with, and if students were demonstrating this persistence within their assignments, then they would indeed need to spend the recommended time on their homework. All of the students I interviewed indicated that they made use of the help centre on a regular basis, and spent the recommended amount of time on their homework. Persistence was reflected in responses about independent study, and how much time it takes to finish the homework, and the mindset needed to re-start a problem that was proving difficult to solve.

> Sometimes you can work on a problem and be off track and have to start over. Thinking questions take way longer. – Sam

From Everest's perspective, persistence was eloquently described as 'intellectual tenacity'.

> The [prior] calculus knowledge that they have is over-rated, and not that relevant. What they need are these basic skills, and basic intellectual tenacity. To work on a problem without giving up. –Everest

Bobbie described persistence more in terms of student actions and attitudes:

> Students need to ask: What kinds of questions can we ask here? How long should it take to solve a problem like this? When should you give up? I encourage them to help each other in the workshops…It's an engagement that has to go back and forth to solve a problem. – Bobbie

The expectations described in the above interview excerpts describe mindset and how students should be spending their time, more than just how much time they should be spending.

In the following quote, AJ describes persistent behaviours in terms of what students should do to overcome their difficulties.

> For students who are struggling it's often not a matter of working harder, but working smarter. So my expectation is that all distractions are put away;

> that the very first thing they should be doing is working on problems. Pick a problem and start working on it. If you start to struggle then sift through the resources: textbook, pod casts, video lectures, whatever's available. Extract some more information and go back and work on the problem again. If you're still struggling with it use the teaching resources on campus: the tutorial center, the workshop, ask your instructor, post on the discussion board to get assistance. So it's about just working on the course content, working on problems. They are encouraged to make use of office hours. – AJ

Students had to persevere, even when they felt out of phase, as Scheja described, because there was no flexibility in the schedule of their lectures. Even though the students I found in the help centers for interviews were experiencing a relative level of success, their responses often implied a constant battle with time. Students were wary of getting behind and needed strategies to mitigate the effects of the inherent lag in understanding. Their strategies for perseverance were articulated in their advice to new students. Many responses could be seen as advice that encouraged students to maintain a continuous effort or have a long-term plan in the face of difficulties.

> Don't leave things till the last minute. Review every day what you did in class. Don't get behind - Sam

> You'll have to do more questions than what is expected depending on your understanding – Felix

> Routine is really important, cause once you set a routine it'll make your life really easier, and at the end of the year you'll get a better mark. - Fulton

> Start studying one or two weeks ahead of time. Don't take more courses in the first year than you can handle. Learn to time manage; make a schedule for studying. Don't get behind because there's no time to recover. – Stefan

While student responses described strategies for perseverance, the lecturers had been able to articulate why some students might be more out of phase than others, and would need to persevere if they wanted to succeed. If students were 'typical', and met the general profile the lecturers were expecting, perseverance would be required to overcome their array of deficiencies in regards to technical skills, study habits, and expectations around problem solving. In the following quote, AJ describes the types of deficiencies that need to be overcome.

> I expect that students will have difficulty with the transition. There will be a lot of gaps. Either they have forgotten it or were exposed to it but only learned it at the surface. I assume they don't have good study habits, and will have difficulty with the fast pace of the course…The problems are harder and require a deeper level of understanding of ideas; they realize that their approach to studying at some point no longer works…They need to become more independent problem solvers. Even if they have seen it before, they aren't prepared to answer the types of problems that are presented. – AJ

Many of the students I interviewed were exactly the type these lecturers were describing. They had had to recognize where their weaknesses were, and affect a plan to overcome them. If not, they found the course challenging to do well in. Tahir had been one of these students. It is not clear from his quote whether he recognized what the expectations of him had been in first year. Now in third year, he said he was getting much better grades.

> Questions which hadn't been shown as examples in class would show up on exams. University was forcefully made so that no one could score well….just the way we were tested on it – no one dare get above 60%. It seems ridiculous that the class average was 57%. You were just expected to know stuff. - Tahir

It must have been quite a battle for Tahir to persevere and maintain a passing grade if he felt this way about the course. He had scored 4 out of 5 on the AP Calculus exam, but had passed first-year calculus with a mark of 60%. Perseverance would also be required for students to acclimate to a self-identity that did not cast them as A students in their new environment.

5.4. Manifestations of Resilience

The students I interviewed demonstrated resilience in two different ways. One of the things I noticed when I was interviewing students who were at the help center to collect their midterms, was that many of them wanted to find out from their lecturer where they had gone wrong on their exam papers. They weren't simply there to find out their marks. I met Tahir in the help center doing exactly that. This willingness and desire for improvement after he had already received his marks struck me as an example of resilience. Some students were quite frustrated by what they called "silly mistakes", but still wanted to know where they had gone wrong, so they could do better next time.

The second and more general expression of resilience from the students I interviewed was that many of them had opted to repeat their Calculus 1 course rather than change to a program that didn't require success in Calculus 1.

Fiona had retaken Calculus 1 at SFU (Mathematics 151) after receiving a D in Mathematics 154 (calculus for biological sciences) because she wanted to improve her grades. She demonstrated resilience by being willing to face her challenges again, and find a way to overcome them. Similarly, Felix had received a C- after having attained an A in high school pre-calculus 12, but re-did his first-year course to attain a better understanding. He then opted for summer courses because he recognized he needed more time to understand course material; Fulton had only completed the high school Foundations 11 course, but since he was determined to pursue sciences, had needed to take some upgrading courses; Frank had failed his first-year calculus course, and was retaking it in summer school.

In some cases, the cause of their failure to meet their own expectations the first time through had been poor time management, underestimating the level of difficulty that would be on exams, and over-confidence after their high school experiences. Though they had not been successful, they had recognized that their success was largely dependent on their own actions and habits, and had pressed the reset button on their calculus courses and tried again. None of them had cited poor teaching as the cause of their difficulties. They had all recognized that their level of understanding was insufficient to move forward in their programs, and rather than give up, had identified their shortcomings and repeated the course.

Each of these students had first identified the source of 'dissonance' in their studies, and made adjustments to rectify it when they took the course again. This degree of responsibility for one's own actions indicates their 'internal locus of attribution' for success, and a high degree of resilience. When faced with failure, they were able to regroup, learn from their mistakes, and try again.

This wasn't always the case. Based on lecturer responses to questions about student struggle, students often didn't know how to overcome their difficulties. Everest described his experiences speaking with students who were failing, and didn't know how to recover. From the lecturer perspective, resilience started with seeking help.

Denial is common, but they should seek help. Even if they went for help after failing the first mid-term they'd be in much better shape. Seek help because students don't often know what to do to get out of this rut, but a professor who has seen this happen a thousand times before has some small things to offer. – Everest

Chapter 6. Conclusions

6.1. Addressing the Research Questions

After considering the results of my interviews, especially the unifying themes, I found that I was able to add a significant amount of data to answer questions not only about readiness and struggle, but also about the characteristics and habits of successful students. The nature of the study of mathematics was revealed to be quite different in university courses than it is in high school, and the habits students need for success included persistence, perseverance, and resilience. These underlying themes were omnipresent in a variety of student responses to the research questions:

1. How ready do students feel for university mathematics based on their high school experiences, and in what areas do they struggle?
2. Do students and lecturers perceive readiness in the same way?
3. Do students and lecturers attribute struggle to the same reasons?
4. What are the differences between high school mathematics and university mathematics courses from the perspective of students?
5. What advice do students and lecturers have for incoming students?

   Upon consideration of the data, information about the following questions was also obtained

6. What are the values of university lecturers, and how are those values manifested in their lectures and interactions with students?
7. What are the qualities and habits of successful students?

6.1.1. Are students ready? Where do they struggle?

The students that were interviewed all reported that the time they were spending on their mathematics homework in university was far and above the level of effort that had been necessary for them to be successful in high school. They had not entered university with appropriate study habits, and had not anticipated the workload. Even if they thought they were ready for first-year calculus at university, they had needed to make major adjustments to how they were spending their time in order to be successful.

The nature of responses between lecturers and students about the homework load, and what students were expected to do with the homework were similar in nature between the lecturer results and the student results. Successful students were doing the amount of work expected by the lecturers, and thinking about problem solving on a level they had not experienced in high school.

In order to build a complete understanding of the mathematics they were learning about, the students needed to build a deeper understanding than had been required of them in high school, and this took time. Patience and tenacity had been necessary, as had learning to make time for really learning mathematics, which meant going beyond just answering the cookie cutter type questions that had dominated their homework in high school.

6.1.2. How do students and lecturers perceive readiness?

There were mixed responses when students were asked if they felt they were ready for first-year calculus. Some indicated that they were based on their algebra skills, but not in terms of time management. Others said they had been so poorly equipped, they had needed to retake the course. The three students who had not taken a calculus course in high school had needed to repeat their Calculus 1 course. They had A's in their high school courses, and had assumed this meant they would be ready for university calculus. Of the students who did take a high school calculus course, one still needed to repeat his first-year calculus course, because he had assumed he understood

the content that would be on his university quizzes and midterms. He said by the time he figured out this strategy wasn't working, he was too far behind to catch up.

Lecturers saw readiness not only from the point of view of pre-requisite skills, but also from the point of view of problem solving skills and intellectual tenacity. They noted that many students entered their courses without these skills.

6.1.3. Do students and lecturers attribute struggle to the same reasons?

The technical skills necessary to make progress on a problem was something many students had cited as a struggle for them in their Calculus 1 course. Lecturers also noted they saw algebra skills as an area of grave concern. Students and lecturers also cited poor time management as a major hurdle that students had to overcome. This was due in part by the unrelenting pace of the course, as well as students not being aware of how much time they actually needed to be investing in their homework problems in order to effect the level of understanding that was required for success in a university level mathematics course.

6.1.4. What are the differences between high school and university mathematics courses?

What it means to do mathematics was a pervasive undercurrent to the interview results. In terms of cause and effect, it was the cause, and the other themes in the research results were actually the effect. The mathematics was definitely different if a student's high school experience was mimickery instead of mastery. Some of the students interviewed did report that their idea of homework from high school had been to reproduce what had already been demonstrated to them. Those who said the work itself was similar, just more intense, had experienced a more nuanced approach to their high school lessons. While procedural knowledge and skills are important tools for any student, they are merely a means to an end in university – not the end itself, as was the case in some of their high school experiences. Analysis of big picture concepts, thinking critically about the nuances and connections between different representations, and thinking creatively to formulate a strategy to solve unfamiliar problems, were not commonplace in their high school classroom experiences.

Another notable difference between high school mathematics and university mathematics was the expectation that students would retain and reuse their knowledge all the way to the end of the course. Since their marks in university depend on cumulative exams and quizzes, students were less inclined to ignore the concepts they struggled with, and actually spent more time trying to understand them. This is another difference that might come as a shock for students who were used to studying for a test, and then being able to forget about the previous topics with no consequence.

6.1.5. What advice was there for students?

The advice to other students were all variations of “do your homework everyday”, “do as many problems as you can”, and “don’t underestimate how demanding it’s going to be”. Beneath these headings were the strategies and skills that students would need, to be able to follow this advice. Making time to do problems every day, having strategies to overcome getting stuck, and being self reliant formed the details.

6.1.6. What values did lecturers display in their teaching and accessibility to students?

SFU and UBC mathematics lecturers expressed similar values when it came to teaching mathematics. They each expressed care and concern for students, and put quite a bit of time and energy into thinking about creating classroom experiences and homework expectations to help foster student learning. The lecturers I spoke to endeavored to create a context and a narrative through which they could invite students to think mathematically. Sometimes this was accomplished by taking student ideas and using them during the lecture, and other times it was accomplished by working on different kinds of problems during the lecture period. When students struggled, and were most likely to keep their struggles to themselves, the lecturers I interviewed all said they wanted students to come and talk to them during their office hours, even though they knew that most students were reluctant to do this.

6.1.7. What are the qualities and habits of successful students?

The students in second and third year had been able to attain a blend of soft skills (or mindset skills), content based skills, and higher level thinking skills. They were

transitioning from novices to experts. Even though the material they were learning was more difficult, first year had still been more difficult for them than their second and third year courses. They had established a routine for doing the majority of the homework questions, and had no illusions about the consequences if they didn't. They knew they had to invest more time where they struggled, not less, and had found ways to work their way through their difficulties, mostly by coming to the help centres, or looking for helpful videos online.

In some cases, students had to repeat the course before they were able to find success, overcoming dissonant behaviours and attitudes that had prevented them from being successful the first time. Unlike many high school students, they attributed their successes and failures to their own actions and attitudes, and had learned how to set goals for themselves and meet them. Their concept of what it was to do mathematics, had evolved from repetition of algebraic manipulations, to being able to solve problems without explicitly being told how. Critical thinking, reasoning, patience, persistence, and resilience were part of the process of doing mathematics. They didn't shy from struggle, but recognized that struggle was part of this process.

6.2. Implications for High School Teachers

The implications for high school teachers are two-fold from my own point of view. First, in order to build an appreciation for the nuances and connections that exist within, and between topics in mathematics, there needs to be as much emphasis on learning about mathematics as there is learning to "do" mathematics in primary, middle, and secondary school. So often, it's tempting for teachers to say things like "just do this" referring to an algorithm or rule-based procedure. The nature and beauty of mathematics is lost if this is how students are receiving their mathematics training. Moreover, this type of algorithmic approach is not preparing students for success in higher level mathematics courses. It seems, that to effectively prepare our students for university success, we need to help them experience and be successful solving unfamiliar problems; articulating connections between concepts, methods, and structures; and seeing the role of algebra skills in the context of problem solving. This holds true even for our weakest students. It is the big ideas they will remember; not the processes and procedures. We need to provide the expectation, as well as the

opportunity for deeper thinking; and opportunities for them to engage with problem solving tasks in a supportive environment.

Secondly, since “doing something every day” is such an integral component of learning, we need to develop classroom expectations and structures where students must “do something every day”. This is especially true in senior grades, when assignments aren’t always due the next day, and may or may not be checked. Perhaps posting daily practice, and/ or problems that are due the same day they are assigned would be an effective way to train students not to leave their homework work to the last minute. This may help students develop the time-management skills, as well as the “expert behaviours” that develop when a student is exposed to a variety of problem types, and is thinking mathematically on a more regular basis.

6.3. Limitations

Though I wasn’t able to keep my sample to first year students, I feel this deviation from my original study design was beneficial. I was able to not only find students who had struggled in the past, but also who had found a way to be successful. This is important, since advice for my own students was one of my original aims.

The limitation I do see here is that since many of my student participants were found at the help centres, their opinions about working with others are biased in favor of that strategy. Because the students might know each other from the help centers, they might have developed similar opinions and say similar things.

The small sample size of lecturers may have different views about students and their perceived role and relationships with students. The lecturers I interviewed seemed genuinely interested in student success. A larger sample size might have introduced results that are not represented here. Additionally, these data represent a snapshot at three similar institutions (SFU, UBC, and U of A). Widening the sampling pool may reveal connections to other elements of success, and areas where students struggle, such as lack of connection with peers etc that were not explored here. This collection of data represents only students who were already coping with their studies, or who had decided to retake Calculus 1, rather than drop out or switch to a different program.

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