Reading to Learn Mathematics: Textbooks, Student Notes and Classroom Communication

by

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Thesis Submitted in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

in the

Mathematics Education Program

Faculty of Education

SIMON FRASER UNIVERSITY

Fall 2018

Approval

Name:	**Sandra Hughes**
Degree:	**Master of Science**
Title:	**Reading to Learn Mathematics: Textbooks, Student Notes and Classroom Communication**
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Date Defended/Approved: December 17, 2018

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Abstract

‘Reading to learn mathematics’ has diverse interpretations: from reading to decoding text to reading mathematical literature. This blind study examined the impact of enhanced reading of the mathematics textbook in a Pre-Calculus 11 classroom. Students read and made personal notes on new content before there was any discussion or direct instruction. Their work was collected and examined for aspects and features of the mathematical text noted and whether work was directly copied or uniquely created. Prompts such as, ‘Create notes for a friend who missed class’ were used. The voice of their written work was compared to the voice of the textbook. Results indicated it was not the correctness of explanations or interpretations that mattered, rather the personal involvement with text that allowed for understanding. Further, students demonstrated increased ‘why’ questions, a broader use of mathematical register during class discussion, and changes to their personal connection to their learning.

Keywords: mathematics textbook; reading; addressivity of voice; personal student note-making; mathematical literacy; classroom communication

For Graham, Alexandra, Brandon and Garrett

and for Merlyn

Acknowledgements

Many thanks to Dr. David Pimm. I was inspired those thirty summers ago when you visited SFU as a guest lecturer from England, and I had the opportunity to hear you lecture on your book. Having built upon those words of wisdom in my career since, I am fortunate to have had the experience to finally meet you, to study under you, and to learn from you.

My thanks also to Dr. Peter Liljedahl. You are a force that everyone needs to experience. Thank you for convincing me to choose this Masters' program. Your support, encouragement and belief in all your students is motivating.

I would also like to thank my cohort. Even though we meet on rare occasions, I know that you are there to answer questions and connect anytime. It was wonderful to be part of a group that continually challenged each other, as well as challenged themselves, to better our practices and understanding of mathematics education.

To the Masters' of Mathematics Education program and all my professors, my thanks for all your inspiring words and better yet, your practical and insightful ideas towards ways of improving mathematics education for all students.

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Chapter 1. Introduction

"Why do I have to worry about literacy when I'm going to be a math teacher?"

(Roepke & Gallagher, 2015, p. 672)

Steve: So, what big plans do you have for the summer?
Me: I'm working on my thesis.
Steve: Oh, neat. What's it on?
Me: Reading to learn mathematics.
Steve: Oh, like how to do word problems.
Me: No. [*I hesitated. There were, after all, only minutes to mingle during lunch.*] That's one interpretation of reading in the mathematics classroom, but not …
Steve: So, like putting stories in the classroom.
Me: Not what I'm investigating, though I did try that last year with my Grade 9 summer school class. We're reading the textbook in my Pre-Calculus 11 class …
Steve: Oh, and everyone knows there's good stories in the textbook!

The above conversation took place with a fellow teacher during our school's farewell luncheon and was typical of other conversations I had had previously about my thesis topic. How do you explain reading to learn mathematics? I have found that the superficial definitions and interpretations of what it is to read and what it is to do mathematics keep the disciplines separate despite on-going, important movements such as 'Literacy across the Curriculum' (May & Wright, 2008). Predominantly, reading is associated with word comprehension and mathematics with numerical comprehension. Fundamentally, it is stories versus calculations and, often, literacy versus numeracy.

My desire to explore reading to learn mathematics was motivated by my personal educational and professional experiences, and fostered through my passion both for reading and for mathematics. As a child, I could not understand those who did not want to read, nor those who did not like mathematics; reading and mathematics were the tools I used to make sense of the world. While reading became my favourite hobby as I progressed through school, mathematics became my academic pursuit.

In Grade 13 in Ontario, I enrolled in three mathematics classes. There was a comfort in the familiarity of the environment and its routines. In each class, the desks were aligned in separate rows; there was no talking nor sharing of work or methods, though we were

normal teens and did try to talk and have fun. All three classes were also formatted in the same way. We would start by reviewing homework questions, before receiving a new lesson and then be given more homework. It was a chalk-and-talk style of teaching and this routine was only punctuated by quizzes or tests. I enjoyed my mathematics classes and I liked all the teachers for their singular personalities. It was an environment in which I flourished – and one where I succeeded.

The general pattern of my mathematics education did not change once I attended university; lecture and notes, homework then tests. Once I became a teacher, and began teaching high school mathematics, I taught my students as I had been (indirectly) indoctrinated. My mentors and fellow mathematics teachers at that time followed this same pattern. They were exceptional mathematics teachers and lovely people who also cared about their specialty. If there were any differences among us, it would have been the examples we used or our classroom policies. As far as my experiences indicated, no other style existed apart from this teacher-centered approach.

Yet, as a secondary mathematics teacher, there were many days that I experienced some disquiet and unease with my usual position at the front of class, explaining mathematics. It was not due to my students, nor my subject. I have always known that mathematics is part of my core and that teaching is part of my being. I enjoyed bantering, joking, working with and teaching my high school students. They, in turn, enjoyed my teaching as expressed through the many smiles, conversations, gifts and letters I received from them and their parents over the years. However, while I was definitely engaged in my teaching, it was clear that not all my students were engaged with mathematics. Mathematics was a requirement of graduation and they were ‘doing the math’. I always questioned how I could teach mathematics better or present the beauty of mathematical discovery to my students. I did attempt small forays into other ways of presenting material, but the school mathematics environment at that time did not support this and I was floundering on my own.

At this point in my career, my growing family demands and the purchase of a small business removed me temporarily from the classroom. However, I missed teaching more than I imagined I would and began tutoring. This, in particular, allowed me the opportunity and time to work with and listen closely to the sense-making of my students. What I witnessed through tutoring is that there can be different ways of making meaning

and interpreting the same piece of text. Different students notice different things. Different students bring different backgrounds and experience to bear on the text.

As a tutor, I could have my students explain what they were reading before I assisted with their understanding. I found that it was not the correctness of their explanation that mattered most. Rather, it was the degree of engagement and personal involvement with the text that allowed for further development of understanding. I did wonder then if it were possible to allow for personal meaning-making in the secondary mathematics classroom. My desire to explore the relationship of and between reading and mathematics was strongly influenced by my tutoring experiences.

While working at the family business by day, tutoring at night and raising a family, I made the fortunate decision to enroll in the Master of Mathematics Education program at Simon Fraser University (SFU). It was both an eye-opener and a game-changer for me. Not only did I further my own mathematical understanding, knowledge and rigour, I met educators and colleagues who were also looking to engage their students better. Through my coursework and the myriad articles and books I read, I gained new perspectives on mathematics teaching and mathematics education. I 'met' many powerful figures who were working at the world level and who inspired me to read further. One such text was titled 'Math wars' (Schoenfeld, 2004). Until that moment I had not realized that there were factions, traditionalists versus progressives, fighting for control in the world of mathematics education. For me, this 'war' contrasted the direct instruction background and education that formed my mathematical experiences against the beauty and creativity of mathematics that I enjoyed and hoped for my high school students to experience. Other authors, such as Jo Boaler (2002), provided research on project-based or inquiry-based approaches to mathematics education.

I now had names, ideas and voices to attach to my previous disquiet. Where previously I could not recognize why I occasionally felt uncomfortable with my traditional teaching of senior mathematics, I could now recognize the source of my consternation. Before teaching secondary mathematics, I taught elementary school. I began my teaching career as an intermediate elementary school teacher, teaching Grades 4–7. I did not equate then my elementary background and teaching experience to high school. It was as if I were two different teachers. As an elementary teacher, I would have been termed 'progressive' in my interdisciplinary approach to education and my focus on mathematical thinking, its applications and reading across disciplines. *Teaching for*

Thinking (Raths et al., 1986) for example, was common reading and an application in my elementary classrooms. However, six years later in, there were no progressive role models in high school mathematics, nor, if I ventured from traditional mathematics teaching, was there any support. At that time, I did not recognize how to apply my teaching philosophy to the secondary level.

Through my Master's program at SFU, I also recognized that my personal ability to survive and thrive in a traditional mathematics classroom was not the norm for the majority of students. As a high school student, I was not happy with only the answer; I wanted to understand why. I questioned why certain procedures worked, for example, and often worked to understand on my own. I also relied on my mathematics textbooks, both in high school and in university. In university, however, there was a greater dependence on them. I learned very quickly that lecturing professors were not normally able to cover the whole syllabus. Textbooks were required because professors opted for ones that would support their lectures and, for example, fill in the holes or further arguments that they did not have time to cover.

The opportunity for self-reflection afforded me through my coursework and readings at SFU connected directly to observations I had professionally. For a research paper for one of my Master's courses, I began by considering what interested me, and returned to what I felt were the pillars of school education; reading and mathematics. That research paper introduced me to Borasi and Siegel (2000) and their book *Reading Counts: Expanding the Role of Reading in Mathematics Classrooms.* Their research on reading in the mathematics classroom proved a significant academic influence on me and their ideas inspired me to consider exploring the possibility of significant reading in the mathematics classroom. I connected their work to what I had observed and reflected on over the past two decades of working with mathematics students. While I did not have access to such specialty classrooms where their book was situated, I was interested in exploring the possibility and worth of more extensive reading in the course of regular high school mathematics classrooms. Their work was instrumental in fuelling my imagination and inspiring me to read further.

I had been wanting to return to classroom teaching for some time, and now that my family was established, this happened to be an opportune time. While it was a demanding and hectic period of change, because I was still completing my Master's coursework, helping at our family business and tutoring, this was the year I happily

returned to mathematics teaching as a Teacher On Call (TOC) in a Greater Vancouver school district.

For background reference, being a TOC often meant being in a different school daily and, for me, this provided a door into other mathematics classrooms across this large teaching district. I was able to glimpse practices and protocols employed by other mathematics teachers and I compared them with my recent Master's degree education at SFU. As a TOC, I taught in a variety of mathematics classrooms. Some *felt* like mathematics classrooms with textbooks, geometric shapes and posters, for example, all promoting mathematics. Others were more austere in appearance and contained nothing to indicate which type of classroom I was in. Some had desks singularly and separately spaced, while others organized desks in groups. From talking with other teachers at different schools, it was also clear that there were different emphases in different schools. While there was a shifting dynamic toward a problem-solving approach in this district, most mathematics departments were still organized traditionally.

Very quickly I obtained a half-time, temporary position at a semestered[1] school. I was still a TOC, but this meant that I would be teaching at the same school every afternoon for the duration of semester two. The school at which I obtained a temporary contract was more traditional in its approach to mathematics, and there were several indicators of this. For example, the mathematics classrooms were teacher centered, where teachers delivered daily lessons from the front of the classroom, while students were to take daily notes and complete daily seatwork. The teachers also followed the organization of the textbook from Chapter 1 to Chapter 9 and used this approach to delineate their course outlines. I found that, by Pre-Calculus 11, my students were ingrained in this direct approach to mathematics instruction and it was difficult to change their entrenched routines. Even when I adjusted small protocols, like not following the textbook order of curriculum, my students expressed concern. While I felt the tension of change to move toward a more inquiry-based approach to my teaching, I was only there for a short time and it felt like I was swimming upstream even to implement such minor changes.

These observations and circumstances proved important in my decision to teach these mathematics classes traditionally. As I mentioned earlier, this was the teaching style of

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[1] In a *semestered* high school, the entire academic year is split into two parts. Semester one runs from September to January and students write final exams in January. Students then start Semester two in February with new courses and write those finals in June.

my past – and this familiarity with it would provide adjustment time for me while I returned to classroom teaching and accustomed myself to this school and its community. Also, these students were used to experiencing this teaching approach and it would be easier to continue this path rather than expend energy trying to change their mathematics culture. In addition, as a TOC, I was teaching in different schools in the morning, and then racing to teach at this school each afternoon. I did not have much time to consider what I felt were better teaching practices when practicality and survival were dominating my days. Even when I felt the tension of change.

However, it bothered me that I could integrate reading and mathematics in elementary school, but high school students did not seem to read in the mathematics classroom. They copied notes from the teacher and when it came time to complete homework assignments, they took pictures of the questions and answer keys from their textbook. Many students did not carry their textbook home regularly, and most times they did not bring their textbooks to class.

The summer before I taught this Pre-Calculus 11 class, I taught a Grade 9 summer school class. It was a class where the purpose was to preview and work through the course material *before* the regular school year. These students were eager to get their textbooks, as the latter "were their window of advantage to next year" (Hughes, 2016, p. 22). We completed many textbook readings in this class and these students wrote journals about their mathematics and what they had read and thought. The words of two of those students haunted me as I taught traditionally in this Pre-Calculus 11 class. Early on, one summer school student wrote, "When I look at the textbook, I look most at the questions as they are what is needed to be completed". Another student spelled it out more clearly, "The thing I look at the least is the Example because before the teacher gives us any homework she gives us a lesson by telling us how to do the chapter and solve problems so when it's time to do the homework I know so much that I go straight to do the homework" (Hughes, 2016, pp. 22-23).

I was curious to know what my students were reading when they looked at their textbooks. For over two decades of classroom teaching and private tutoring, I have watched my students stare, tilt their heads and puzzle over mathematics texts. What is it they are reading? What are they looking at? What are they noticing? What sense or meaning are they making of the text of words, symbols, notations, diagrams, illustrations and so forth, and often in combination, those things that are required to make meaning

of the page in front of them? These were some of the questions I hoped to explore in my thesis.

Chapter 2. Literature Review

The real purpose of books is to trap the mind into doing its own thinking.

(Christopher Morley, circa 1900)

In what follows, I will first examine literature on what it means to read so that this process can be understood in the context of mathematics. Following that, I will detail what it means to read mathematics and what it means to read to learn mathematics. As the textbook is a familiar component of the mathematics classroom, I then examine why it should be read. Finally, reading is a component of literacy, and its importance in relation to mathematics is discussed.

2.1 What does it mean to read?

During my career as an elementary teacher, I witnessed polarized arguments over 'Whole language' versus 'Back to the basics' in Language Arts education. This 'Reading war', as defined by Stephen Krashen (2002), pitted the 'Skill-building hypothesis' faction (phonics) against the 'Comprehension hypothesis' faction (whole language). To understand the arguments concerning reading education, three prevailing reading theories are briefly outlined next. In addition, it is also important to understand the process of reading and have a clear specification of what it means to read, in order to effect appropriate teaching strategies if reading is to be considered in the mathematics classroom.

The Traditional theory, or Bottom-up processing, focuses on the printed form of the text. It was influenced by the behaviourist psychology of the 1950s and built upon conditioning processes: that is, like Pavlov who studied the salivating response of dogs to the ringing of a bell (and not the arrival of food itself), this theory of reading instruction forms reading habits through repetition, drilling and error correction.

More commonly, this theory is known as *phonics* by educators. "In other words," according to Pardede (2010, p. 4), "language is viewed as a code and the reader's main task is to identify graphemes and convert them into phonemes. Consequently, readers are regarded as passive recipients of information in the text. Meaning resides in the text and the reader has to reproduce it." The phonics approach to reading has been widely

criticized, as it focuses solely on text features and structure and ignores what the reader brings to the text.

The Cognitive theory, or Top-down processing, was influenced by cognitive scientists in the late 1960s and considers the “mind’s innate capacity for learning” (p. 5). In comparison with phonics, where there is rule memorization and meaning resides in the text, this theory takes into account the reader’s background knowledge and their assimilation of text meaning to personal understanding and existing knowledge.

Though the cognitive theory of reading came into prevalence in the 1960s, the phonics approach is still powerful today and the Reading ‘wars’ a perpetual battlefield. In the late 1990s, for example, the ‘Whole language’ movement re-emerged to counter phonics. Whole language is basically a holistic approach that immerses students in reading and writing and does not emphasize mistake correction or solely phonetical rules. Its supporters can be viewed as the polar opposite of phonics supporters. (Lemann, 1997) Whole language was attributed to the ideas of Noam Chomsky in the late 1950s, and emerged as a philosophy and movement in the 1980s. (Shafer, 1998)

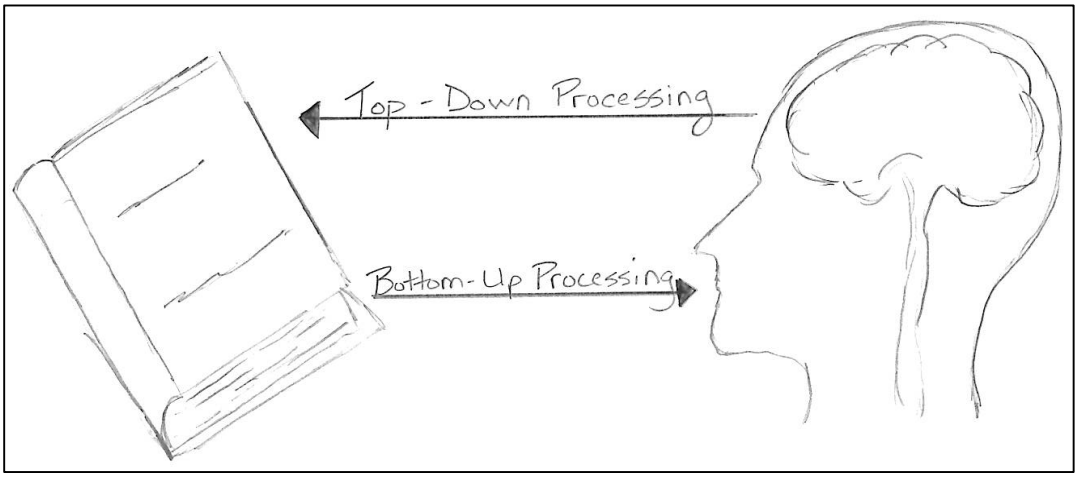


Figure 2.1 Comparing Bottom-Up with Top-Down Processing (adapted from Peglar, 2003, p. 1)

The third model, the Interactive Reading model, recognizes and incorporates both theories: reading is considered as an active process. There is a triad of the reader and their prior knowledge, the text, and the reading situation. Louise Rosenblatt’s (1978) transactional theory of reading is the one most credited for change. Other authors and reading educators, such as Constance Weaver (2002), have developed exercises and

strategies that help explain and educate about the generative process of meaning-making in reading. In her book *Reading Process and Practice*, Weaver defines reading as a sociopsycholinguistic process. The three root words refer to social factors, the mind and language respectively. Reading as a sociopsycholinguistic process refers both to the mind and to the text interacting in continually overlapping cycles within the social context of the reading of the text. Weaver listed that the background knowledge, cultural orientation, personal reading history, beliefs and feelings all affect how the mind interprets the reading.

My concern with reading in the mathematics classroom was not with teaching decoding skills so that students could better solve word problems. That is a limited view of the reading process as well as a limited view of reading mathematics. I was interested in the view of reading described by her as a sociopsycholinguistic process where there is an on-going exchange and interaction between the reader and the text. An instance given in Borasi and Siegel's (2000) book clarifies this. They cite the classic example of "The notes were sour because the seams split" (p. 27). The researchers of this study noted that, while students could recognize all the words, they could not make sense or meaning of the sentence until the context of 'bagpipes' was introduced. Their conclusion was that students have to do more than decode. In addition, without background knowledge of bagpipes, different students would still have different interpretations.

Rosenblatt's transactional theory of reading, as explained in Borasi and Siegel, supports the idea that "… from the perspective of transactional reading theory, texts are open, variation in interpretations within and across readers is expected, and meaning-making is a generative activity that is best characterized as duplicating the activities of an author rather than duplicating the text". To this extent, and discussed below, transactional reading strategies were supported in the classrooms of Borasi and Siegel's (p. 29) research. This, too, fueled my interest in supporting readers to interpret text.

The other consideration I had was that context was important. As I was thinking on this, I read an earlier article by Weaver (1996) entitled 'Teaching of grammar in the context of writing'. Despite years of grammar courses that she took for her degree and career, "decades of grammar studies tell us: that in general, the teaching of grammar does not serve any practical purpose for most students" (Hillocks & Smith, quoted in Weaver, 1996, p. 15). She argued that, in the context of writing, grammar does. For me, this

agreed with my interdisciplinary side of teaching; reading skills in the context of mathematics was purposeful, rather than teaching reading as isolated skills in mathematics.

2.2 What does it mean to read mathematics?

'Protocol', according to the on-line Merriam-Webster dictionary, is "a system of rules that explain the correct conduct and procedures to be followed in formal situations". Reading has established protocols that are implicit in the type of genre you are reading and can be taught explicitly in formal education. For example, reading fiction and reading mathematics would involve different protocols. While my mathematics education did not explicitly detail reading protocols, my reading education did. For example, in university I learned how to approach and interpret a poem compared with a short story. While mathematics does have reading protocols, many have argued they are complex, difficult, complicated and challenging for students.

Several articles and texts I read, such as Bossé and Faulconer (2010) and Barton and Heidema (2002) all contained variations of the following:

> Mathematics is the most difficult content area material to read because there are more concepts per word, per sentence, and per paragraph than any other subject; the mixture of words, numerals, letters, symbols, and graphics requires the reader to shift from one type of vocabulary to another. (Durham District School Board, 2006, p. 11)

Added to this, mathematical concepts are often abstract, so it is difficult for readers to visualize their meaning. What also makes reading mathematics both singular and demanding can be the independent interpretation or interplay of the numbers, graphs, algebraic examples, the mathematical register and theorems, for example, that students are expected both to follow and to understand. I have also found that I can easily read literature yet not know every word to understand the meaning of the passage, though I need to interpret every single symbol on a page of mathematics to garner its full meaning.

In their article on how to read mathematics, Simonson and Gouvea (2010) give advice on reading mathematics. For example, they advise readers that, "Understanding the text requires cross references, scanning, pausing and revisiting" (p. 5). That is, merely reading the page through is not part of the mathematical reading protocol. Often, reading

mathematics involves reading across the page and possibly right to left, up the page, even diagonally, in order to connect visuals to text. Connecting terminology to examples may happen across two different pages. Essentially, linear reading, or reading straight through, is not often possible for these reasons, in combination with the condensed meaning inherent in mathematical text and its symbols. Mathematical notation, for example, can have more than one word to explain its meaning.

I also appreciated their words of “A three-line proof of a subtle theorem is the distillation of years of activity. Reading mathematics […] involves a return to the thinking that went into the writing.” (p. 5) How do we consider the source in mathematics or teach our students to consider them? This is advice not often heard in mathematics classrooms. To further this idea, “In reading mathematics text, readers need to analyze and expand meaning rather than condense ideas.” (Adams et al., 2015, p. 500). How do we, for example, describe the power and evolution of the two simple horizontal lines that represent the equal sign? Again, what can appear as a simple item or two of notation, depending on its placement, can have impactful meaning.

The mathematics register, though composed of written text that seems to relate to vocabulary in English, does not always. Reuben Hersh (1997), a famous mathematician, described mathematics as a lingo, a dialect, rather than a language, because “it is a semi-dialect of English (or some other natural language), not a complete language” (p. 48). It is not a complete language because there are some things you cannot say using mathematics. He detailed many examples of the difficulty students have with the mathematical register. For example, learning that ‘=’ means equal or equivalent, and then applying it as such:

> But what does *equal* mean? When we say 1/2 = 2/4, we don't mean 1/2 is indistinguishable from 2/4. They have different numerators. They have different denominators. We regard them as *equivalent* for good and sufficient reasons. All this may be explained in an advanced course, on the rare occasion when a detailed construction of the rationals is carried out. But already in the fourth grade the = relation is an equivalence relation between fractions, not an identity. No one ever explains this, so there's no way for the student to understand except in terms of models like slices of apple pie.
>
> This non-understanding was manifested frighteningly when a calculus student was asked,
>
> “What is the minimum of the function

$y = x^2 + 2x + 5$?"

and answered "correctly"

"$x^2 + 2x + 5 = 2x + 2 = -1 = 4$ *minimum*"

> Maybe this is the outcome of years in high school spent factoring, multiplying, and dividing expressions that always remained equal. (p. 50)

This careless use of the = symbol has resulted in loss of meaning and process. While the answer to the question of 'What is the minimum of the function?' is 4 and is correct, there are clear indications that the last line has little truth. For example, -1 does not equal nor is equivalent to 4. Also, the original function of the question is a quadratic function, which quickly became equivalent to a linear function, and it is not clear why or where this expression came from. Most troublesome is that the one string of statements is not true. For example, the original quadratic function is not equal to its first derivative, which is what the 2x + 2 represents. One cannot be substituted for the other. In addition, to derive the value of -1, this student had to set 2x + 2 equal to 0, knowing that the slope of a horizontal line tangent to the quadratic function is zero and would be tangent at the vertex of the quadratic function. While the first derivative is a way to find this value of *x,* the 2x + 2 does not equal nor is equivalent to -1; -1 is the result of 2x + 2 = 0. Ultimately, to calculate the value of the minimum of 4, the student had to substitute the -1 into the original quadratic function. While the answer is correct, the work as written is not correct nor true, and does not make sense. It is difficult to 'read' this line of equivalences; the paragraph, or demonstration of understanding, is missing. It is comparable to a paragraph long, run-on sentence in English; it may be necessary to read and re-read the paragraph several times in order to make sense. Structures and conventions are inherent in all disciplines and exist to facilitate understanding of text.

My favourite quotation from Simonson and Gouvea's article was, "Know Thyself. Texts are written with a specific audience in mind. Make sure that you are the intended audience, or be willing to do what it takes to become the intended audience." (p. 7) How the textbook addresses students will be discussed further in Chapter 6 under the notion of addressivity. This quotation, however, does suggest that mathematics can be a tough read. How do we enable our students to read mathematical text? How do we develop the mathematical habit of mind of persistence, for example, needed to work not only through tough mathematical reading, but also challenging problems? "In addition, it appears that unproductive mathematical habits of mind are developing among students. "… many students expect that all mathematical problems can be solved in 10 minutes or

less." (Hughes, 2015, p. 26) Is reading mathematics text, and in particular the textbook, a means of developing mathematical habits of mind like persistence and a way to counter developing unproductive habits?

Often as mathematics educators we tend to read the text and interpret it for our students. Many teachers do rely on the textbook for examples. While some teachers will use the classroom textbook directly for examples, others will source out and use the examples from equivalent textbooks. Several authors, such as Draper (1997, 2002), are concerned that mathematics teachers avoid reading instruction and thus reading in the mathematics classroom because students are weak readers, and teaching them to read or using literacy and reading strategies in the mathematics classroom detracts from the time spent doing and learning mathematics. Thus, the textbook is not read for ideas, but used for practice. Draper also stated that:

> Researchers have consistently reported that content-area teachers believe that a) it is someone else's responsibility to teach reading and writing, b) they lack the ability and/or training to teach reading and writing, and/or c) they do not have the time to provide literacy instruction along with their full content curriculum. (2002, pp. 357-358)

I also believe that reading should be *part* of mathematics, not taught as a separate discipline within the mathematics classroom. If some of the reading to learn mathematics approaches, described below, were to be applied, reading would become its own entity in mathematics, not part of mathematics.

What I do not agree with is the separation of reading, or any component of literacy, in the mathematics classroom. The subject is mathematics: thus, the focus is mathematical content. For this reason, I do not agree with practicing reading strategies, nor do I want to create mathematical activities merely to focus on reading or writing. In such a case, "the movement of literacy into mathematics may result in the replication of rather artificial activities, rather than foster qualitatively better mathematics learning" (Bruce & Davidson, 1996, p. 284). In addition, while there are sources and resources to help with mathematical reading, how would a mathematics teacher know how to sort through them and know what to choose, or how to apply the resource or fit it in their schedule, for example?

2.3 What does it mean to read to learn mathematics?

The literature on 'reading to learn mathematics' can be diverse in its interpretation. A predominant view that exists is, "Mathematics is about problem solving and reading is an important component, especially for word problems" (Heidema & Mitchell, 2005, p. 3). This quote is supported by those that feel, "The students know how to do the math, they just don't understand what the question is asking" (Metsisto, in Kenney, 2005, p. 9). One mathematics teacher, concerned with his students reading the textbook to learn from it, stated, "I then devised a way to teach the reading skills that students need when reading mathematics textbooks" (Ostler, 1997, p. 37). He did find, however, that his students used these strategies as blueprints for decoding similar types of problems.

These ideas and approaches are not new. Reading in mathematics to decode, find information or to understand what the question is asking, have been a concern for some time. For example, in 'Reading in the teaching of mathematics', Orleans (1939, p. 349) states, "Whatever are our opinions concerning this phenomenon, the fact remains that, under the present circumstances, we must accept the pupils that come to us, deficiencies and all, and we must do with them what we can". While this may not be accepted as a politically correct statement today, it does reflect the continual concern of educators that students cannot read or interpret their textbooks or mathematical text and, thus, are unable to read to help themselves learn. Aside from the extremely traditional mathematics teachers that believe that the transmission of information is how mathematics should be taught and that reading should be taught by reading teachers, all authors do agree that students need to be able to read to learn mathematics. They differ on what this means and thus what or how this needs to be accomplished. However, as discussed above, mathematics does not consist of words alone. Reading strategies in isolation do not help students look at or interpret mathematics as *mathematics*.

Before engaging in their research, Borasi and Siegel (2000) found that the majority of literature they reviewed that connected reading to learn mathematics focused predominately on strategies to improve the extraction of information from mathematical text. They felt that this 'mining' metaphor as developed by Pimm (in Borasi & Siegel, p. 1) was a limiting and narrow view of the importance of reading in math classrooms. This is comparable to a phonics approach to reading. As Pardede (2010) further explains phonics, it is changing graphemes, which are letters or combinations of letters, into their

phonemes, or sounds. This requires repetitive practice to increase fluency, and thus, comprehension. If this is a practical and viable approach to reading mathematics, then there needs to be the consideration that students need to speak mathematically in order to have the opportunity to create the sounds of mathematical terms and symbols. Ultimately, the larger problem that exists in teaching decoding skills toward reading to learn in the mathematics classroom is that reading can be seen as the obstacle to its learning.

What Borasi and Siegel did find was that reading non-traditional forms of mathematical text engaged the students in their study and did support the learning of mathematics. Their study, and their initial focus on reading to learn mathematics, was to involve transactional reading strategies on mathematical text such as historical readings. While this may sound as a limited view on reading to learn mathematics, it did expand their thoughts and their research and lead them to document the many ways that reading occurred in the mathematics classroom. It engaged their students and, as example, revealed the humanistic side of mathematics.

Reading non-traditional forms of mathematics may be viewed as a utopian way of reading in a mathematics classroom. Borasi and Siegel were fortunate to be researching in a special mathematics program, different from traditional mathematics classrooms. In considering the typical mathematics classrooms in Canada where there are textbooks, there are students sitting at desks, and the teacher works their best to deliver the content, how can reading to learn mathematics be introduced? One difficulty is that the mandate given to mathematics teachers includes curricular content that demands time, time of the students and time of the teacher. In our Canadian classrooms, what does or what can reading to learn mathematics look like?

If we consider reading as a transaction where there is an interaction between the reader and the text, then reading in the mathematics classroom would definitely involve more than understanding written text through phonics. It would also have to be an interaction with all textual features on the page or on the board. My personal experiences with reading to understand mathematics has taught me that there are many ways to make sense of what I am reading. As a graduate student, I did not read the questions or the mathematics as my classmates did; we all had different backgrounds and experiences

and interpreted what we were reading differently. There may be one answer, but not one path. This would, and should, be the same for our students.

There are parallels to be considered between reading and mathematics that help explain the broader view of reading to learn mathematics. For example, mathematics is precise. In Pre-Calculus 11 one outcome is understanding exact values. Compared with Chemistry or Physics 11, where rounding and approximate values are needed, precision is mathematics. Yet mathematics is full of ambiguity. Ambiguity is the search for understanding and that is inherent in both mathematics and reading. Mathematics and reading are also both personal; we make sense based on our background and experiences, as well as the context we're reading in. As will be shown in Chapter 7, my students preferred different ways to understand mathematical ideas. In addition, both subjects require some active and generative effort.

Through his research and investigations into reading in the mathematics classroom, Magnus Österholm's (2003, 2006) work agreed with other research claims that reading mathematics is different than reading in other content areas. In addition he found that, "There is no common type of reading comprehension for *mathematical* texts in general, but one seems to need several types of skills for different types of mathematics texts with symbols" (2006, p. 340). With this in mind, how should reading to learn mathematics be approached in the mathematics classroom?

2.4 Why should there be reading in mathematics?

> Mathematics is fundamentally concerned with texts - not textbooks necessarily, but the written in general (Pimm, Personal Communication)

According to Borasi and Siegel (2000), educators have tended to underestimate the contribution reading can make to the learning of mathematics. They combined their specialties in mathematics and reading education, respectively, and undertook an interdisciplinary investigation that expanded even their view of how reading could be woven into a mathematics classroom. From why students might read in mathematics classrooms to what they might read and how they might read them, Borasi and Siegel challenged the accepted place of reading in mathematics class demonstrating that there is more to learn from text than facts or procedures.

Their book, in particular, launched my imagination, providing a lens into classrooms where reading is a conduit to learning mathematics. The difficulty for me, however, was that the classrooms featured in the book were not typical and, as mentioned above, many teachers are not yet in inquiry classrooms. I was interested in exploring the avenues that reading to learn mathematics could be fostered in any mathematics classroom, traditional or otherwise.

I was also not concerned with developing or promoting a particular teaching method. I was concerned with enhancing the role of reading in the mathematics classroom, a concern that has evolved through both my personal and professional experiences working with students and helping them try to make sense of mathematical text. Like Borasi and Siegel, I believe that students should have the opportunity to interpret the text rather than have the teacher continually translate it for them. In their article 'Reading, writing and mathematics: Rethinking the 'basics' and their relationships', Borasi and Siegel (1994) write, "Finally, the idea that knowledge and learning are acts of construction situated in a community of practice implies that instruction can no longer be defined as the efficient transmission of information from teacher to student" (p. 37). In the authors view, the inclusion of reading in the mathematics classroom is another teaching strategy to support the construction of mathematical knowledge.

In considering other literature and ideas about why reading should be included in the high school mathematics class, there are a range of ideas from performance on international tests to being able to read textbooks in university. Briefly, reading affects mathematical performance. From the Trends in International Mathematics and Science Study (TIMSS), it was found that best readers perform better on all demand levels. For example:

> On average across countries, the mathematics achievement difference between poor and good readers was larger on the high reading demand items than on the low reading demand items. The average mathematics achievement of the best readers did not vary much by level of reading demands, whereas the average mathematics achievement of the least proficient readers was higher on the items with low reading demands than on the items with medium and high reading demands. (Mullis et al., 2013, p. 15)

Paula Maida (1995), in her article 'Reading and note-taking prior to instruction', about helping students achieve mathematical literacy, focused on note-taking and reading the

text prior to classroom instruction. She discussed the National Council of Teachers of Mathematics (NCTM) goals of increasing students' mathematical communication as this will assist in developing mathematical comprehension. She writes, "Mathematical communication also forces students to be active learners as opposed to passive learners who simply accept and memorize procedures" (p. 470). She feels that if we focus on reading in mathematics class and, in her article, it is reading the textbook she is writing about, then the students will believe that the textbook contains useful information.

In 'Reading as a learning strategy for mathematics' (Else, 2008), another mathematics teacher also focused his research on the reading of the textbook as it accompanies students outside of class and thus can be used as a resource. His main research goal was motivation: to motivate the students to read through rewards, to motivate them through discussions on the importance of reading and to include reading of the textbooks during class time. He felt that if he motivated his students to read, he could create a more independent classroom.

While the title of this article implies 'reading to learn mathematics', there are substantial differences from my focus on reading the textbook. Else based his research ideas on Barton and Heidema's (2000) book *Teaching Reading in Mathematics* and the literacy skills they apply to reading in mathematics class. His summary of their reasons why students do not read their textbooks stresses the content-specific skills that are required to read mathematics. This is a narrower focus and one more on the difficulties of reading mathematics. Else also worked to motivate students to appreciate reading itself. To do this he used a reading guide to focus the students' readings, and it was worth marks. Students were required to complete the same reading guide for each mathematics lesson. The reading guide was very structured in comparison to the transactional strategies Borasi and Siegel (2000) used and describe in their work. Transactional reading strategies are open-ended, and I describe one from their book called 'Clone an Author' in Chapter 5 of this thesis.

In their article 'Using literacy strategies to teach precalculus and calculus', Tena Roepke and Debra Gallagher (2015) adamantly state:

> Mathematics is not a spectator sport! Students need to be actively engaged in the process of learning mathematics, not just passive bystanders.

> Students need to be fully engaged in mathematics through reading and writing to enhance their understanding of mathematical concepts. (p. 677)

Roepke, who also teaches mathematics pre-service teachers, could not explain to these students why they had to take content-area reading courses as part of their training when they wanted to be mathematics teachers. For this reason, she enrolled in her students' reading course and subsequently applied the literacy strategies she had learned to the university calculus course she taught, which she described in this article. Both authors felt that borrowing strategies from literacy instruction was a way to engage their students in math. More importantly, they also found that it was a way to inform their teaching as they could hear and read what their students were thinking and they felt they better understood what their students were understanding.

Still another article set in the university mathematics classroom is by Janet St. Clair (2007) titled 'Using a new synthesis of reading to promote a community of mathematicians'. St. Clair is concerned with the development of mathematically literate individuals. She defines them as those able to engage in 'inquiry math' talk which, as examples, involves raising questions, making and testing hypotheses, generalizing and debating. Using the three transactional strategies that Borasi and Siegel used, St. Clair videotaped and recorded her university students engaged in activities such as reading and discussing articles about René Descartes and co-ordinate geometry.

This problem has become evident through my research: most of the research on reading does not involve senior secondary mathematics classrooms. There is a plethora of articles and research on reading and mathematics at elementary school, some at the middle school and university levels, but it is difficult to find research at senior high school.

So why should reading be included in the senior mathematics classroom? Aside from creating independent thinkers and considerers, reading is important to the future citizens whom teachers help shape. For those students choosing to pursue post-secondary education:

> Many students do well in high school mathematics courses without reading their texts. At the college level you are expected to read the book. Better yet, read ahead. If you read a section before listening to a lecture on it, the lecture will be more comprehensible, and if there is something in the text you don't understand, you will be able to listen more actively and ask questions. (Hubbard, n.d.)

In consideration of the above, and in consideration that reading can contribute more to the mathematics classroom, it is noteworthy that reading to learn mathematics is uncommon in most mathematics classrooms.

2.5 What to read in mathematics: mathematics textbooks

Across my lifetime as both a student and a teacher of mathematics, I have encountered, read, and worked with and through numerous mathematics textbooks. Some I have liked and continue to use and refer to over the years, while others held a more temporary position. Regardless, I have always been excited to receive a new textbook and explore the mysteries it holds. I have found my mathematics students are the same and value the concrete textbook and the advantages they feel they give them (Hughes, 2016). To consider what it might be that students are reading in their mathematics textbook, it is important to examine the structure and organization of mathematics textbooks in general, their influence on the teaching and learning of mathematics, and what it may be that textbooks offer both teachers and students.

"A textbook can be described as a book designed to provide an officially sanctioned and authorized pedagogic version of human knowledge" (Gracin & Matić, 2016, p. 31). In a content rich course such as mathematics, a textbook is a useful reference source for mathematical content and explanations at every grade level. While publishers today will boast of updates and new features, how different are current mathematics textbooks from their predecessors? According to Howson, Keitel, and Kilpatrick:

> Teachers in the USA depend heavily on textbooks [...] As a result, textbooks are marketed much like automobiles: publishers' representatives find out what the consumer wants and then the publishers compete to offer it to him in the most attractive package possible. Novelty is at a premium; yet the changes offered are frequently cosmetic and rarely radical. (in Love and Pimm, 1996, p. 395)

The above quotation indicates that there has been little real change to mathematics textbooks despite cosmetic additions, such as updated photos or internet connections that further explorations or readings. Mathematics textbooks are generally organized as other discipline textbooks; chapters focus on a topic, and the chapters are further divided into sections. Each section in a mathematics textbook was found to be similarly formatted in the Exposition – Example – Exercise structure described in Love and Pimm (1996). The first part, Exposition, explains an idea, then the Examples portion contains

worked examples, and the final section holds the practice exercises. The only difference between textbooks are the titles they give these sections.

There is no lack of literature that details the prevalence of mathematics textbooks in mathematics classrooms, from around the world and across the grades. With this prevalence comes influence. For example, the following concerns declining achievement scores on the TIMSS of fourth-grade Dutch students and how this could be related to the textbooks the students used: “Although several factors influence what mathematics teachers teach children, there is much evidence that the curriculum and the textbooks are important determinants of what children are taught and what they learn.” (Kolovou et al., 2009, p. 31) According to Love and Pimm (1996, p. 397), “In many schools (published) materials were the principal or sole determinant of the children’s programme of work.” Haggarty and Pepin (2002) state, “For example, although we made the point in the previous paragraph that textbooks were a major resource for teachers, teachers themselves make decisions about how to use them” (p. 128).

Further, Bishop (1985) states: “In the USA, and to some extent elsewhere, one development put more effort into the production of the ‘ideal textbook’. (p. 25) Why the ideal textbook? Gracin & Matić (2016, p. 52) explain, “The results of the present study show that teachers design their instructions using textbooks as the main source for classroom activities. Other sources, such as other textbooks, web and e-material, worksheets, teachers’ guides, are used, but not as a main resource for teaching. These results correspond to the findings of the past study in Croatia, as well as to research results from other countries.” Keith Devlin, a famous American mathematician, phrased it, “Of course, teaching mathematics in the progressive way requires teachers with more mathematical knowledge than does the traditional approach (where a teacher with a weaker background can simply follow the textbook – which incidentally is why American mathematics textbooks are so thick)” (2010, p. 6).

In my professional experiences and practices as a senior high school teacher, I know that many of my colleagues decode the mathematics textbook for their students. As described by the above authors and researchers, the textbook explanations and examples are translated into lessons and lectures by the classroom teacher. I know some teachers that do this on occasion, while others do this as habit. In a study in 2009 by Brown (in Gracin and Matić, 2016, p. 53), “In their lesson preparation, teachers

decide what content, order, methods and pedagogical intentions to use and follow from the textbook. During the classroom observations, most of the participants used textbooks directly, following them as closely as possible, which can be seen as "offloading," i.e. using the textbook in a literal fashion." In my experience, this is common. For example, a colleague across the hall routinely skeletonized every explanatory section of the textbook for his class. That is, he retyped every example, without the worked solution, and every important term or phrase, without the definition, to create a lesson outline for his students, which he would photocopy and give to them at the beginning of each class. He then worked through every example and definition on his laptop as his class copied what he wrote. It was a literal recopying of the textbook without referring to the textbook.

If textbooks are prevalent and influential, then there is a need for students to be able to read and use them. According to Österholm (2003, p. 3), "At the upper secondary level, textbooks more often seem to be used as a collection of exercises, while at the university level, students are to a larger extent expected to read mathematical text on their own for learning". This is an interesting statement, and from my experiences, a truth. Students *will* have to read and know how to make sense of their textbooks at the post-secondary level.

As will be examined in detail in Chapter 4, textbooks have a voice. "Voice is a term used in many areas of linguistic research that study oral and written text" (Herbel-Eisenmann, 2007, p. 347). As Herbel-Eisenmann found, mathematics textbooks tend to have an authoritative voice and even authors who consciously tried to move away from this authoritative voice had difficultly doing so. Draper (1997) found that students find this authoritative voice intimidating. In addition, "A textbook has authority by virtue of its mere existence as a book [...]. Books are read differently from other forms of textual material - newspapers, magazines, advertising materials - with greater expectations of reliability and accuracy (Love and Pimm, 1996, p. 403). For these reasons, students should become familiar with reading mathematics textbooks. Perhaps if used for more than exercises and examined for meaning, mathematics textbooks could foster positive attributes in students such as questioning and thinking skills.

While there are other sources of mathematical text, such as stories at the elementary level, and historical text across post-secondary grades, textbooks are the primary source of information for mathematics students at the senior level.

2.6 Literacy across the curriculum or literacy in the mathematics classroom?

> Although there have been numerous successful educational reform efforts centered on increasing reading and writing across the curriculum, most have insufficiently defined whether the purpose for such in mathematics is to increase student literacy or to increase mathematical knowledge. With this ambiguity in place, many have failed to distinguish reading and writing **about** mathematics from reading and writing **in** mathematics. Therefore, reading and writing biographies of famous mathematicians has upstaged reading and writing regarding the nature and uses of the discriminant of a quadratic polynomial. (Bossé & Faulconer, 2010, p.10)

In a world where illiteracy is inexcusable and demands address while innumeracy is socially accepted (Fite, 2002, p. 7), reading across the curriculum, and in this instance the mathematics classroom, requires attention. However, as the above quotation highlights, the end goal or goals are not clear. I undertook this exploration two years before British Columbia introduced its new mathematics curriculum. It is now a good fit with the new curriculum and its Core Competencies of communication, thinking, and personal and social responsibility.

Bruce and Davidson (1996, p. 283) argue, “The assumption is that once students have acquired adequate reading and writing skills they can move those skills into the discipline areas, particularly in middle school and high school and, later on, in college”. This does not happen as learning is situational. Other authors (Adams et al., 2015) argue that reading teachers do not have the background or experience to teach reading mathematical content. Despite mathematics teachers not feeling trained to teach reading in their mathematics classrooms, they are more skilled and familiar with reading mathematical texts and textbooks.

However, mathematics teachers teaching reading in mathematics classrooms isolates this skill and, as argued previously, reading should be in the mathematics classroom to foster mathematical understanding and learning. Thus, promoting reading of the mathematics textbook and allowing students to write notes may have the desired effect of integrating literacy into the mathematics classroom.

2.7 Research Questions

There are many issues examined above, not the least being that mathematics teachers use and depend on the mathematics textbook for the information, structure, progression

and content that they use to teach their students. In addition, there is a lack of research and exploration into reading the mathematics textbook in the senior secondary classroom. Yet being able to read and being able to read and consider content text is an important educational goal. Reading is also an important literacy goal, but rather than teach reading strategies in the mathematics classroom, why not allow students the opportunity to read mathematics in the mathematics classroom? In this way, they are given the responsibility to decode and interpret, for example, rather than relying on the teacher to do this. Along with the experience and possible struggle of considering notation and varying forms of information, they may also exercise mathematical habits of mind. For these reasons, my research questions are:

In the context of a senior pre-Calculus mathematics classroom,

1) What are my students judging as important from what they are reading in their mathematics textbooks?

2) How do they express and explain the above in their self-directed written notes?

3) What are some of the impacts of the above on student activity within a mathematics classroom?

Chapter 3. Methods

> The Principle of Reflected Blindness restated as: You start to fool yourself that anyone listening has the same comprehension. (Easdown, 2006, p. 2)

There are two broad strategies for collecting data on reading: student interviews and student writing. For this qualitative study, I collected written notes that my Pre-Calculus 11 students created from assigned textbook readings. In addition, I also made my own notes after each class about what I observed. In what follows, I will provide: the context for my decision to work with this group and why I chose to collect their notes; a description of the course and its participants; a brief account of my classroom structure and organization; how data was assigned, collected, analyzed and how the results will be organized.

3.1 Setting

Both mathematics courses that I was hired to teach were at the Grade 11 level. The high school I was teaching at offered all three of the BC mathematics 11 courses available at that time; Pre-Calculus 11 for students considering post-secondary science-based futures; Foundations of Mathematics 11 for students considering post-secondary education (but not science-based); Apprenticeship and Workplace Mathematics 11 (AW 11) for students considering futures in the trades. Students in BC needed a Grade 11 mathematics course for graduation and any of these three courses would fulfill these graduation requirements. The Foundations 11 and the AW 11 courses were more limiting in their post-secondary options. For example, the Foundations 11 course provided options for university studies in business and the arts, but not in disciplines such as engineering that required theoretical calculus, while the Pre-Calculus 11 course accessed all university disciplines.

My half-time posting saw me teaching a Pre-Calculus 11 class and an AW 11 class. I could not consider the AW 11 class for this exploration, as there was no textbook for that course. This meant that my exploration had to take place in the Pre-Calculus 11 class.

The participants for this study were between 16 and 18 years of age. It was initially a class of 21 students, but this changed after the first month. Over the first few weeks of

the second month, student schedules were altered and two students moved, and the class roster eventually levelled at 16 students just before the break at the end of March. I taught this group of students every afternoon from the beginning of February to the end of the semester in June. Overall, they were a quiet group and only two students would voluntarily answer questions or contribute ideas when I tried to engage them in class discussions. Only one student listed math as a favourite academic subject on her student profile.

Three students concerned me during the first few weeks of semester, as their mathematical abilities were below average. Two were international students who I found over time did not want to learn English, nor complete any mathematics assignments. One did quit school early March.

The third student continually self-described herself as “bad at math”. I checked with her counsellor and learned that, like the remaining international student, she had achieved 50% in Foundations of Mathematics and Pre-Calculus 10 only by taking the course a second time in summer school. However, as with other students in my Pre-Calculus 11 class, she wanted to ‘keep her options open’. This was a phrase often used by students taking Pre-Calculus 11 who were not sure of their hoped-for career paths. As Pre-Calculus 11 provided more options for post- secondary studies, even students weak in mathematics would opt for it or were recommended to take it by teachers or counselors.

Pseudonyms are used for the students who participated in this study. The pseudonym and accompanying pronoun agreed with the gender of the student, though the nationality or other distinguishing features of the original student were not tied to the new name I provided.

Beginning classroom structure

I was teaching my afternoon mathematics classes in someone else’s classroom. The physical space had been arranged by an English teacher who had been teaching at this school for eight years and who used the classroom in the mornings. There was nothing to indicate that it was either an English classroom or a mathematics classroom. The desks were paired in rows and faced the front of the class. The teacher’s desk was also at the front. There were no projectors, neither overhead nor tablet, so the only medium for giving notes was the front whiteboard.

My decision to teach traditionally fitted in well with this layout. Typically, I began each class with a welcome from the front and then would complete a homework check or collect homework assignments. Next, I would go over any questions concerning the previous day's work. Following this I would spend approximately 30 – 50 minutes on a new lesson and then assign practice questions. Time left after a lecture was to work on exercises, and I would circulate to answer any questions and to make sure they used their time wisely.

This was the format of my typical class. I did work hard to engage all the students and, for example, would call on everyone for their ideas and encouraged all to take chances. If a student raised a hand to answer a question, I would usually say something such as, "Before you tell us the answer, explain to us how you got there" or "Explain why you thought that". At home, when I planned, I would create and then re-create lessons to try and engage everyone in the class, Nevertheless, it was still a traditionally organized classroom where I controlled the content, context and the information flow. And the forms of communication.

This was not the setting that I had envisioned as the venue for my reading to learn mathematics exploration. For one, I had wanted to be in my own classroom where I could organize the physical environment, rather than work within someone else's structure. Second, I had just completed my Master's coursework and had a wealth of ideas and strategies I wanted to establish before I began my exploration. Third, being called last minute as a TOC had left little time for pre-planning my exploration. These two mathematics courses were last minute additions to those offered that semester and I was only notified the morning I started there what I would actually be teaching. Reading was not part of a normal mathematics classroom routine and I had wanted to be ready and prepared. As I had found, small changes at this school were difficult.

3.2 The Exploration

That was the pattern of my teaching from the beginning of February through to the end of March. As will be detailed next chapter, I began this exploration on March 29th, nearly half-way through the semester. The only protocol I changed when classes resumed after the two-week break at the end of March was for my students to read the textbook and make notes before I discussed the content with them. Reading and note-taking did not

occur every day as, for example, there were times when we investigated an idea further or there were test days or other school activities and interruptions.

Classroom policies and procedures did not change for this exploration. As students already took notes from the board, the only thing that changed on the surface was that they were to read the textbook first and make their own notes before we discussed and/or summarized notes as a class. In essence, they made notes on what they thought or judged was important. There were prompts to these readings that followed the formats such as “Your friend was sick and missed class today. They asked you to e-mail them notes. Create notes that explain the important ideas, so that they can complete any missed work” or simply “Read page 273 and create the notes you think I would give you”. There were no restrictions on what or how they could write (though as the above instances indicate, there were sometimes suggestions about the form or purpose) and I did not direct them to focus on particular elements of that section; I merely let them note what they thought was key or important. Again, we did not do this every class and sometimes I assigned textbook readings and note-taking for homework.

In the dual roles of teacher and researcher, I continued to assign seatwork and homework as part of my normal classroom routine, with reading the textbook as part of it. Most often my students read and wrote independently, but there were no restrictions on talking about their readings with a classmate. Following reading activities, we had general class discussions and sharing of what was considered important and why. In the manner of my teaching, there were times when I engaged the class in further tasks that served to highlight the difference between reading literature and reading mathematics or I presented reading strategies to help them look at mathematical text differently.

For example, one strategy that I presented was Clone an Author, taken from Borasi & Siegel’s (2000) book. It is a transactional reading strategy; there is no set structure. It was improvised for my class in that, essentially, my students read a textbook passage independently and then for each important idea they noted, they wrote the idea on one stickie note. Following this they could rearrange their stickie notes in a way that showed or created meaning for them. This strategy is similar to mind maps, though the ideas are not tied to each other. It is also easier to move stickie notes or cards physically and thus to reorganize the information in a way that makes sense to the student (as an author). In

this way, if taken further, the organization can be discussed and compared with classmates and revised or further reorganized.

If I collected these writing samples as part of my routine homework checks, I did not consider them as data for analysis. For my student analysis, I wanted to consider what each student was reading without direct intervention. As I will discuss in § 3.4[2], only independent notes of textbook readings were examined for this study.

One personal concern I had for this exploration was that of time; I was not sure how long it would take for my students to read and consider explanatory text. With the ever-present pressure to cover the curriculum, I was worried about completing all the lessons and work I had planned. However, I did not change my outline, schedule of topics, nor list of homework assignments, but instead tried to muster my 'monitor and adjust' persona of previous years. I did set an internal restriction on myself not to panic if classwork went a bit slower.

The textbook

The McGraw-Hill Ryerson (MHR) *Pre-Calculus 11* (2011) textbook was the prescribed mathematics text for our district. Every section in each chapter in this textbook was organized around a 'three-part lesson', as the MHR textbook termed them. The parts to these lessons were always ordered, and titled 'Investigate,' 'Link the Ideas', and 'Check Your Understanding'. The 'Investigate' sections contained opening tasks; the 'Check Your Understanding' sections contained practice questions. (These were not the pages the students were assigned to read independently, but could be assigned for classroom or homework practice.) The middle part, 'Link the Ideas,' contained vocabulary, explanations and examples, and the authors of this textbook used phrases like 'help you think through' and 'help you understand the work' to describe this portion of each section.

As my exploration focused on the portions of the textbook that explained something to the student, my students read the 'Link the Ideas' parts. This portion of each chapter section was always the longest because it contained not only explanations, but also

[2] The symbol ' § ' is used to denote the sections of my thesis, while the word 'section' is used to indicate a section in the textbook.

worked examples and a ‘Key Ideas’ summary. For this reason, and particularly at the start of this investigation, I did not have them read the complete ‘Link the Ideas’ sections, because the amount of text was overwhelming. Appendix A, §1, provides an example of what my students read from the textbook. These are four pages from section 5.1 on ‘Working with Radicals’. The ‘Link the Ideas’ part starts halfway through page 273, seen in the first image, and continues to page 278 in the textbook. On Day 1 of my investigation, I had my students read only the second half of page 273 over to the next page, 274 rather than all six pages of this portion.

The Link the Ideas sections often provided different methods to solve the same problem, as illustrated on page 275. According to the textbook authors, “One method may make more sense to you than the others” (p. viii). They also had Your Turn questions after the worked examples. The “Your Turn part allows you to explore your understanding of the skills covered in the Example.” (p. viii). However, there are no worked solutions for the Your Turn questions, nor answers for them in the Answers section at the back of the textbook.

The majority of the Investigate portions that started every section of every chapter, for example from pages 272–273, involved a task and my students did not read or make notes on these pages as they contained no explanatory text. They were activities that most often contained directions and instructions. However, as further reading of the textbook uncovered, there were a couple of the Investigate sections that did work to explain and demonstrate mathematical ideas. For example, pages 74-75 contained explanations of angles and rotations, and then compared what is meant by ‘angles in standard position’ to those not in standard position through diagrams, not written text. This could be described as a visual explanation, though no concrete answers to the final visual are provided to the students. Thus, teacher support and explanation, for example, are required. As I will discuss in Chapter 5, part of this noticing of textbook explanations evolved through the teacher reading of the textbook and her professional development. Reading directions and instructions were not considered reading explanations, though I did want my students to look at and use more of their textbooks. The next chapter provides a detailed look and analysis of the MHR textbook that my students read for this exploration. It examines the structure, language and voice of the textbook.

3.3 Collecting Data

For this exploration, data was collected through written student notes of their textbook readings, and through field notes that I wrote daily.

Student written notes

As this exploration focused on reading a mathematics textbook, I was concerned with the novelty of the task. Only collecting one or two samples per student would not be representative of this exploration, as reading the textbook and creating reading notes once would be likely be both unfamiliar and peculiar. Would students change or stay the same if they had the opportunity to read more of the textbook? Thus, I decided to collect many more than one sample per student.

I also wanted to ensure that the data from my students was not biased by my interest: that is, I wanted to collect raw data; what were my students reading and/or able to consider on any given day of the study? If I interviewed them or told them I was collecting reading samples for my thesis, they would know I was interested in their reading responses. In addition, if I chose the interview path, my students would have to read and then debrief with me each time. As I did not know what types of results I would find, I could not anticipate which students to interview. This seemed a lengthy process as it would have to be repeated each time we read the textbook in order to be able to document continuous change, and could well prove a possible intrusion into class time. For these reasons I chose to collect written notes and keep it a blind study.

Homework is a by-product of student mathematical activity in my classroom. During a routine week, homework is either simply checked for completion or I collect homework to see how my students are explaining their mathematics. Reading homework was checked and collected in the same manner. Thus, my data is the notes that my students wrote and I collected this data occasionally as I would normally collect homework. Again, my students did not know until the end of this exploration that I was examining their notes for my own purposes. For this reason, I made photocopies of their work and returned the originals to them (occasionally with comments or other forms of written feedback). Most of the feedback was oral, during class discussions. There were times when I had them read and make notes, but I did not always collect their work for data. This was due to a multiplicity of factors, such as days of large student absence or merely wanting reading

and note-taking to be a consistent class routine. Over the course of this investigation there averaged eleven different written activities collected per student. As mentioned, with frequent student absences, the range across students was between 7 – 15 sets of notes.

Field notes

To keep track of my thoughts, experiences, reflections and decision-making with regard to this study, I kept a written account of daily classroom occurrences. I always jotted notes. I also noted parts of conversations I heard my students having or interesting questions they asked. Some days there were minutes where I could sit at my desk and record my immediate observations and comments. Other days, there was only time to scribble phrases on the piece of paper that I usually had in hand. Regardless, every day at home after school, my first task was to transcribe any notes and sit quietly for at least half an hour to reflect on and record further anything that struck me as interesting, frustrating or unusual, for example, or just to record the occurrences of that day. My field notes journal is labelled from Day 1 onward, after my first entry that detailed my thoughts just before the initial exploration occurred.

Over the ten weeks of this investigation I took daily field notes of my observations of classroom activities. Whether it was a day that reading textbooks occurred or a day of more traditional activities, I recorded my observations on student interactions with each other, with their textbooks and mathematics, their comments if they were noteworthy, and overall classroom environment. I also recorded my thought processes as I worked my way through this novel exploration.

While the thesis research questions address what is it that the *students* are reading, and what meaning *they* are making of the mathematics in their textbooks as judged by their writing, the exploration also involved my reading of the textbook and my development as a teacher of reading mathematics. I had many questions and concerns as I began this exploration. I was stepping away from my comfort level of organized control and off into the unknown. Would there be a revolt? Would they refuse to read the textbook? This question haunted me over the two-week break at the end of March because I knew my students were accustomed to being told what to know. What about time? Would this take

extra time and would I still be able to cover the curriculum? What complaints would I be dealing with? As Monday drew closer I slept less and worried more.

However, as worried as I was, I did believe that reading had a place in the mathematics classroom. In this section I will present some of my internal dialogue, anecdotes on situations and the class environment, student comments, and the process by which I added reading to my Pre-Calculus 11 class. I will note here what struck me as novel at the time, or what situations, conversations, assignments and class discussions added to my learning about my students' reading the Pre-Calculus 11 textbook.

3.4 Analysis of Data

The MHR textbook was the focus of my readings as well as my students. As will be detailed further in Chapter 4, I had to read the textbook first to decide what was accessible and informative reading for my students. Then, during class, my students read the textbook and created personal notes. In addition, I kept a written record of my classroom observations and my thoughts and impressions as we progressed through this study. My data for this exploration comprised photocopies of the written notes my students made of their textbook readings and my own field notes. I read and considered all the data that I collected.

Ultimately, there is no simple nor easy way to analyze reading. The focus of this exploration concerned my students reading the textbook and then scribing their thoughts on what they'd read. This was not research on writing; it was on reading. As mentioned above, evaluating reading in and of itself is a difficult task, if possible. Analyzing written thoughts of readings allows for greater data collection than interviews. Is evaluating written transcripts of a reading better than an interview? As Österholm (2006) wrote:

> One reason for letting the participants write their answers to questions is to make it possible to include a larger number of participants. One could argue that oral answers would be more valid since the answers in such a situation could be of a more direct and spontaneous kind, thus more directly reflecting an existing mental representation and decreasing the risk of receiving answers based on mental representations that are being constructed when the question is asked. However, no method can in a direct manner examine a person's mental representations, and it is not evident how great the benefits are when answers are given orally. Also, written answers can allow a type of answer that oral answers cannot,

> namely the use of symbols that a person does not know how to pronounce. (p. 337)

Reading and writing *are* interrelated and influence each other, and it is difficult to assess which contributed to what in this exploration. For example, there are researchers who have found that writing improves with increased reading, and, "As students write more frequently, their reading skills improve" (Pierce, 2018, p. 2) However, it is not within the scope of this thesis to try to delineate between these two processes. While I am reading and analyzing the written work of my students, it is with the purpose of trying to explore what they noticed and considered important to record after they read their mathematics textbooks. The specific and particular task for this exploration focused on reading the mathematics textbook. It was not an exploration of what my students would write with or without direction about their mathematics. Writing was merely the vehicle I used to generate the data about what they read.

The MHR textbook

In order to consider what it is my students were reading, I examined three aspects of the mathematical text: the vocabulary, the examples and the syntax. I was interested in seeing what my students were noticing and whether they primarily seemed to be looking at the words, the numerical examples, the diagrams, the structure and organization of the mathematics or at the work as a whole and trying to connect these features. I was also interested both in whether they were copying, modifying, or interpreting the text, as well as in *how* they modified the textbook text. (see below)

Table 3.1 Criteria for Evaluating Student Notes Against the Textbook Text

	Copied	Described/Modified	Interpreted
Vocabulary			
Examples			
Syntax			

For every textbook page that was assigned, I read and re-read that page and noted the above features. Then I created a personal summary sheet for that reading. On this separate sheet of paper, I first listed any relevant vocabulary. By this, I am not only referring to what the textbook refers to as 'Key Terms', which the textbook highlights and provides definitions for in the margin. I also noted terms key to my students' understanding of the mathematics for this particular chapter or section. For example,

from the first few readings from pages 273 – 278, there are no ‘Key Terms’ assigned by the textbook authors. However, I know that ‘index’ and ‘radicand’, for example, are words that are used repeatedly in this chapter and mentioned in other chapters as well. For that reason, I considered them relevant vocabulary for the first reading. As will be shown later, they were terms that my students also indicated as important for and relevant to understanding this chapter.

I next recorded the number of numerical examples from these pages. From page 273, for example, I noted five examples. Only one of the five examples was a worked example while the others were numerical, primarily used for defining and comparing terms. However, I considered them to be of equal weight in counting examples because each one required its own interpretation or even noticing from any student. I would count the textbook examples and compare this to the number of examples that my students recorded or generated.

Syntax is the form of a language or its surface structure. The MHR textbook was the structure and form of mathematical language that my students were reading. As will be discussed below, my students either copied the syntax of the textbook, or tried to create their own. In this way, they either retained or lost the mathematical meaning, or semantics, of the passage.

Student Notes: Copying versus creating

I was examining my students’ written notes, but they were writing as a result of reading the textbook. When I collected and photocopied their notes, I created at least two copies so that I could critically highlight or make marks on one of the copies. I was looking for specific things as I examined each page and would note, for example, what was missing or extra information that had been added. Every piece of student work was compared to its related mathematics textbook page and the summary sheet that I had written.

Meshed against this I considered the sense or meaning that my students were making of the mathematical text. I was considering a spectrum from copying to creating. To delineate this aspect, I considered Dan Kurland’s (2000) ideas:

What a text says	restatement
What a text does	description
What a text means	interpretation

As I analyzed the works of each student, I used different coloured highlighters to mark every page of notes they submitted: pink to indicate direct copying of text; yellow to indicate change; and green to indicate the student's own work and thinking. Below is an example:

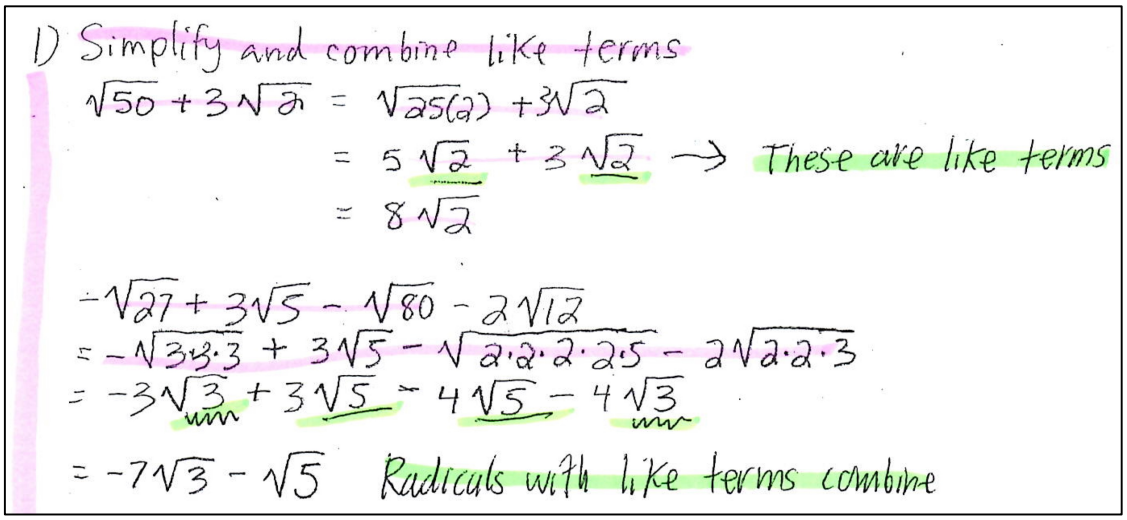


Figure 3.1 Example of highlighted student notes for analysis

At the end of this study, I considered the collective notes of each student independently, like a portfolio of their reading notes across the ten weeks of this study. For each student, I looked for what had stayed the same and/or what had changed (and, if so, in what way). For example, if they had copied the complete page from the first day of this study, did they continue to copy fully through to the end? Or did they only notice written text on day 1 and then subsequently change focus to numerical examples?

Syntax was mainly evident when my students made mistakes. Generally, I would look at the amount of information presented to my students on each page and consider the percentage they noted. When they made notes and the mathematics was out of context, so that it was nonsensical, or if they cut and pasted words that resulted in a meaningless statement, I would count that as a syntactical error. In other words, once the meaning of the mathematics was lost, there was a loss of syntax.

Choosing Student Notes for Analysis

The work of five of my students were chosen as exemplars. The analysis of the written representations of their readings is presented in greater detail in Chapter 6 compared with the analysis of the work of the rest of the class. The exemplar work was chosen as it

indicated several aspects of classroom potentials: range of academic change, range of personal change, range of classroom presence, as well as range of both mathematical and social communication. For example, the marks of several students improved, but one exemplar illustrates dramatic growth. Another exemplar did not have as dramatic an increase in their marks, but represented a radical shift in classroom presence; from quiet and uncertain to vocal and engaged in class discussions.

The remaining eleven classroom members did affect change; it was either on a lesser scale or within the range of change of the five representatives. A brief synopsis of their classroom presence and their work is included to provide both a reference to the exploration and how it impacted the classroom community, as well as to indicate how for some students, mathematical change and growth was not mathematical.

3.5 An Outline of the Remainder of this Thesis

The following chapter, Chapter 4, comprises a detailed look and analysis of the MHR textbook that my students read for this exploration. It examines the structure, language and voice of the textbook. This section also provides reasons for the readings that were chosen.

Chapter 5 is from my perspective. It begins with an outline of the content of the exploration through examples of the reading prompts that I created and gave to my students. This is followed by observations taken from my Field Notes and looks at three time frames of the study: the beginning, the middle and the end which coincide roughly with the unit on Radicals, Trigonometry, and Inequalities. It serves to provide the reader with the context of the activities that occurred and as a supplement to the student samples. They chronicled this exploration and also provided a cohesiveness and overview that would not necessarily be evident from student notes alone as they aid in understanding the culture, the social situations, behaviours and events of this exploration.

Following my Field Notes, Chapter 6 discusses the results of this exploration. It contains analyses on the written reading notes of all my students and lengthier analyses on the notes of five of my students. These five represent the spectrum of change for this class over the course of this exploration.

Chapter 7 considers a cross-analysis of the student data and is addressed as emerging themes. The final chapter looks to conclusions of this exploration and considers future avenues of exploration.

Chapter 4. Aspects of the McGraw-Hill Ryerson (MHR) Pre-Calculus 11 Textbook

> Textbooks form the backbone as well as the Achilles heel of the school experiences in mathematics. (Kajander & Lovric, 2009, p. 173)

In this chapter, I do not include any interactions with students. Rather, this chapter provides a summary of the structure, organization and certain linguistic features of the MHR textbook that my students and I read for this exploration. In particular, I examine the textbook to see what and how the textbook 'says' (voice) and to whom it 'says' it (addressivity). For example, does this textbook acknowledge the presence of a teacher, of student readers or students in a classroom? What experiences of mathematics does the textbook provide that could have the potential to influence a student's view of mathematics? This chapter also details how I chose the specific readings for this study.

As curriculum in this province is dictated by the Ministry of Education, the two distinct, competitive mathematics textbooks of the same year would present the same information, but in differing formats. When I first began working with the MHR textbook, I wondered why there were so few sections in each of its chapters. Compared with the eight and nine sections per chapter of its predecessor and competitor (respectively), this textbook averaged just three sections per chapter. My students were optimistic that there would not be as much work and their parents were worried there would not be enough. These were both interesting statements, in that they viewed the role of the textbook as a source of practice.

As I became more familiar with the textbook, I did not find the reduced number of sections to be beneficial. It was more difficult to find information: for example, five sections that previously explained and discussed graphing of the vertex form of a quadratic function were now all in just one in the MHR textbook. While my predilection is not generally for the delineation of a topic, this compression of sections could mean denser content and more time spent on any one section.

4.1 Format of the MHR textbook

This textbook is rigidly formatted and consistently structured: the four longer chapters are organized at the beginning of the textbook and are each approximately sixty-five pages in length. The five shorter chapters that follow average forty pages. There are also pages devoted to unit projects, cumulative reviews and unit tests: these features generally occur at the end of every second chapter. For this reading exploration, which occurred during the second half of the second semester, we read and explored five chapters:

Radical Expressions and Equations

Rational Expressions and Equations

Absolute Value and Reciprocal Functions

Trigonometry

Linear and Quadratic Inequalities

I was not following the textbook's table of contents, which is why the trigonometry unit, Chapter 2 in the textbook, was studied near the end of the semester. Of the five chapters listed above, the trigonometry chapter was the longest at sixty pages, and the other four were approximately forty pages each.

To understand the organization of this textbook, the authors created a six-page 'Tour of Your Textbook' section. These pages appear to be addressed directly to the student, as it is termed 'Your' textbook. Sentences such as "The first page of the Unit Opener introduces what you will learn in the unit" (p. vi) also appear to address the student, as the teacher should have already learned and understood this work. The language is more informal and direct in this section. For example, following the title, the personal pronouns 'you' or 'your' are used eleven times in the approximately 150 words on the first page. Of these 150 words, sixteen others are bolded and used repetitively. For example, 'project' is repeated ten times on this page and seven of them are part of the bolded words. Words such as 'project' refer to subsequent headings the student will encounter in the textbook, as in 'Project Corner' or 'Unit Project.'

The following introductory pages of the textbook explain the way the chapters are organized and what students are expected to accomplish. These pages detail how every

section in every chapter is divided into a ‘Three-Part Lesson’. It is known by students that lessons are what teachers do, and this could indicate that this explanation is for both student and teacher. However, the first page of explanation of the Three-Part Lesson, page viii, contains fourteen ‘you’ or ‘yours.’ This is the third page in a row where the pronoun ‘you’ is prevalent. There are no ‘we’ pronouns. It seems that the students may be on their own to work through these lessons as, for example, “It is designed to help you build your own understanding of the new concept” (p. viii). Students at the Pre-Calculus 11 level could have mixed reactions to this depending on their academic stance and background.

The above statement, and others on this page such as, “The **Reflect and Respond** questions help you to analyse and communicate what you are learning and draw conclusions”, raise questions. For example, are senior secondary students accustomed to reading and building their own understanding of new concepts? Are they or will they analyze what they are learning to draw conclusions? Are they able, used to, or familiar with communicating mathematics? Are these regular occurrences in high school mathematics classrooms? As I read further I wondered if the textbook was speaking rhetoric, or can students use this textbook to build their own understanding and/or reflect and draw conclusions?

The MHR does have a defined, rigid repetition that is the structure of this textbook This repetitious structure is pervasive and feels like a fractal; it has a self-similar repeating pattern. Each chapter does follow the organization outlined in the beginning of the textbook; similarly, each section follows the prescribed Three-Part lesson structure, and even within each section the categories and wording can be structured identically. As an example, the Investigate section on page 143 has three parts to it. In each part there are three ‘questions.’ Each question has the same number of components, and each component is phrased identically. There is a parallelism with the formatting of the five imperative directions (that are the questions) given within each part. For example, each part starts out with ‘Graph the following functions on the same set of coordinate axes, with or without technology.’ This imperative is next followed by the same directive, ‘Describe how the graph of each function compares to the graph of $f(x) = x^2$” in each of the three parts. This continues, and the remaining directions are nearly identical over a page and a half. (see Appendix A, §2)

What is also interesting is that *every* 'Check Your Understanding' section breaks the exercise questions into four sections titled 'Practice', 'Apply', 'Extend,' and 'Create Connections.' There are a similar number of questions for each part of the Create Connections practice work through the textbook. All this embodies routinization.

Table 4.1 Comparison of number of types of exercise questions for Chapter 3

Type of question	Section 3.1	Section 3.2	Section 3.3	Section 5.1	Section 5.2	Section 5.3
Practice	9	9	8	10	12	10
Apply	12	11	16	10	9	8
Extend	2	5	4	2	6	5
Create Connections	3	2	3	3	5	3

There is comfort and certainty with routine, and the repeated template for the format of these sections may provide familiarity. However, there is also the underlying theme that the orderly structure of this textbook reflects the orderly structure and certainty of mathematics, which lends to the notion that mathematics is orderly and structured, predictable and boring.

4.2 The Structure of the Three-Part Lesson

As discussed in Chapter 2, the MHR textbook chapter sections follow the 'Exposition – Examples - Exercises' format, but here are titled 'Investigate', 'Link the Ideas' and 'Check Your Understanding.' These three components are consistent, not only through their structure, but also through their layout and language.

The middle portion of each section, 'Link the Ideas', was the mathematical text for the reading assignments for this exploration. These sections were most often the longest, usually equaling the sum of the other two parts. The textbook sections averaged 12 pages in length. The Link the Ideas portion usually comprised 40 – 50% of this amount. It always included some explanation or background information, which was followed by several pages of worked examples. These examples could also have questions or statements typed in green font next to the examples. They were meant to prompt student thinking.

The Investigate and Check Your Understanding components, the first and the third portions of each section, were not considered for the reading tasks for this exploration. However, as they represented approximately half the textbook, their format and language were important to consider, particularly the practice section. For example, and as discussed below, student familiarity with and understanding of mathematics textbooks is largely a result from reading and completing practice questions for homework.

4.3 The Language of the Three-Part Lesson

Student familiarity with the language and structure of mathematics textbooks would come primarily from the practice questions. As detailed in Chapter 2, it is common practice for mathematics teachers to decode the textbook explanations for their students and then assign the corresponding practice questions. The experience of the majority of secondary mathematics students with reading mathematics is through reading textbook homework assignments. For this exploration, it is important to examine and understand how the textbook addresses the students in all sections of the textbook: the Investigate sections as they are the welcoming to every section and set the tone and voice; the Link the Ideas sections as they are the focus of this exploration; and the Check Your Understanding sections as they are the sections that students are traditionally most accustomed to reading.

Investigate – Language

As noted in my section 3.2, this part of each textbook chapter section was an activity. While the textbook authors describe the Investigate activities as '…designed to help you build you own understanding of the new concept," they were very structured activities. The students were directed on how to complete each activity, and how to consider each activity. There were no answer keys for the questions asked in these sections, which meant that the activities could be independent if they were completed within a classroom and with a final confirmation of the results within this environment.

Of note in this part of every chapter was that the language was predominately imperative. For example, textbook section 5.2 on multiplying and dividing radicals used exclusive imperatives such as, use, determine, verify and express, at least two times each. It did include inclusive imperatives such as suppose, consider, and generalize. However, of the 20 'questions' asked in this section, 16 were imperative directions. Only

4 had the form of questions “What is…”, What would…”, and “Why.” This means that in an independent exploratory section of the textbook, the student is being directed 80% of the time.

Link the Ideas – Language

Upon examining the Link the Ideas pages, the only pronoun found was ‘you’, and it was rare to find this pronoun. As an example, in section 5.1 on Simplifying Radicals, the pronoun you was only used twice over five pages of text, not counting the ‘Your Turn’ titles. Generally, these sections were written with imperative and declarative statements and no pronouns. The imperatives generally outweighed the declarative statements by three times. For example, on page 274 there were 9 imperative statements to 3 declarative, and on page 275 there were 11 imperative statements to 4 declarative. In addition, the imperatives were most often exclusive. That is, they were of the ‘scribbler’ variety where the student is told to complete some task. Write, Multiply, Express and Convert were common imperatives in the first section of the Radicals chapter.

Interestingly, while the green thinking prompts could be questions, they could also be imperative statements. For example, on page 315 of the Rational Functions chapter, one green prompt states, “Multiply the numerator and denominator by -1.”

Check Your Understanding – Language

As noted in table 4.1 above, this portion of every chapter section had its own internal organization. The homework question section was divided into four parts. The first three parts, Practise, Apply and Extend, were titled using exclusive imperatives that ordered students to complete practice questions, application questions, and then more challenging questions. The fourth and shortest part, Create Connections, was the ‘thinker’ or inclusive imperative title where, according to the textbook, students were to think about the Key ideas of the section. There could also be questions that extended mathematics to other subject areas. In approximately every one of three Create Connections sections, there were Mini-Labs. Of the four sections, teachers most often assigned homework from the Practise and Apply parts. This observation comes from years of teaching and tutoring experience. As well, a quick google search of seven different Pre-Calculus 11 websites verifies this observation. At most, if Extend questions are assigned, they are starred as challenge questions and are optional for students.

The title, ‘Check Your Understanding’ of the MHR textbook also goes by titles such as ‘Try This’, or ‘Exercises’ in textbooks of the same ilk. In other words, ‘Check Your Understanding’ is a semantic that means practice questions to both students and teachers. What types of questions were students asked in these parts of the MHR textbook? The following is a tally of the language used for the questions asked in the Practise components of the Check Your Understanding sections of 5.1 and 5.2 in the Radical’s chapter.

Table 4.2 Tally of words used to ask questions in Practice Sections 5.1 and 5.2

5.1	5.2
Simplify - 3	Simplify - 6
Express - 2	Express - 3
Copy - 2	Expand - 2
Complete - 2	Multiply - 2
Write - 1	Divide - 1
Order - 1	Rationalize - 2
Verify - 1	Identify - 2
	Determine - 1
	Write - 1
	State - 1
	Use - 1

While there were multiple parts for most questions in these sections, such as a), b), and c), there were no questions asked. Rather, as can be seen from the table, each question was an exclusive, or scribbler, imperative direction. Beth Herbel-Eisenmann (2007) noted that students need scribbler imperatives before thinker. However, there are no questions here.

The Apply questions follow the Practice, and the textbook authors describe them as “questions that apply what you’ve learned”. The questions asked in this textbook were composed of 50% imperative directions and 50% questions in section 5.1. For example, there were six “What is the…” type of question, but they were often connected to and followed by an imperative direction on how to answer the question. Section 5.2 had ten imperative instructions to six questions.

What appeared to be consistent was that this textbook posed questions predominantly by giving instructions on how to do something. Even the Create Connections questions contained more imperatives than questions. There were the occasional thinker imperatives, such as “Describe the similarities and differences…” (p. 292), but as Herbel-

Eisenmann posited, the consideration is to whom is the student addressing through these thinker imperatives?

Upon looking at the language above, it seems that the voice of the textbook is authoritative and exclusive. It does not share secrets, it dictates ideas.

4.4 Readability of the MHR textbook

Being a reading exploration, it was also essential to look at the textbook from a readability perspective. What students noticed could depend on the visual appeal. Readability describes how your eyes flow through the text and on the page. White space is visually important. For example, it provides brief reading breaks for the eyes from the text and work. Condensed information is overwhelming to the reader and in the case of mathematics, can make a challenging subject even more challenging to work through.

The 2011 MHR textbook has a variety of margins. There can also be print in these margins and this has many effects on the readability of the text. For example, page 145 (see Appendix A, §3) has four highlighted terms within the text and the highlighted terms and corresponding definitions are also included in the margin. The eye, however, is pulled around the page because the highlighted terms all demand attention. The text is concentrated to the top of the page while white space and visual rest are at the bottom. As the definitions are reprinted in the glossary and do not have to be reprinted and defined in this space, there is a question as to why they have to be included on this page at all. The glossary, like the answer key, could be used to avoid linearity in this textbook rather than create many focal points of attention on one page.

Another example of how the set structure of the textbook affects the page use and readability occurs on page 182 (see Appendix A, §4). In this example, the left 2-inch margin of the page is used to list materials, and this list contains one item. As a result, the investigation on completing the square is compressed to the remaining two-thirds of that page. The useful white space is trapped outside of the bordered text rather than distributed throughout the investigation. The text appears denser than it would if the text occupied the full space. When the text is dense it can present a barrier to information. For some students it supports the idea that mathematics can be a tough read. In addition, dense text can impart the feeling of urgency as one idea is connected to the

other and the next and you must keep going. This becomes an exhaustive cycle in non-fiction writing if there are no visual breaks in information.

4.5 Aspects of addressivity of the MHR textbook

The consideration of how mathematics textbooks speak to students on behalf of mathematics bears consideration. There exists a variety of approaches. I had to search hard to find examples of how this particular textbook addressed the student, the teacher, the classroom and/or the lesson. These elements of address were implicit and scattered through the chapters. In what follows, I will detail what I noticed by reading this textbook and consider what and how this might relate to people in the classroom.

Addressing the student

If the MHR textbook addressed anyone, it was the student, and this was mainly read through the pronoun 'you.' No other pronouns could be found in this textbook. As mentioned earlier, however, this pronoun was rare, particularly in comparison to the imperatives that dominated the written text. 'Copy, 'Write', and 'Use', for example, were common words the textbook used and these terms told the student what to do with the mathematics. This in of and by itself created the mathematical text as an authority. Essentially, students were directed or ordered what to do and how to approach their mathematics through these imperatives. For some students, this could be perceived or unconsciously understood to imply that mathematics was too hard to consider on its own or without support.

The textbook is a Pre-Calculus 11 textbook and the opening section titled 'A Tour of Your Textbook', could also be taken as addressing the student. The teacher would be familiar with the set-up and structure of mathematics textbooks, even the most current. As stated in Chapter 2, only the presentation and packaging of textbooks is different. The Tour of Your Textbook section also introduces the Three-Part Lesson. While these 'lessons' are the three parts that compose each section in every chapter, it could be considered that this also addresses the teacher. The language here, however, implies the student. For example, "It is designed to help you build your own understanding of the new concept." implies the student. The mathematics teacher should understand the new concept, although there are always new methods to be learned.

It could also be considered that the textbook is addressing the student through the 'Focus on' statements at the beginning of each section of every chapter. These sentences are written in the present tense, implying that the student is there, reading this currently. It is again an imperative direction to inform the student that as they begin the section, they should 'focus on' these big ideas to ensure success.

The Investigate pages in each chapter tended to address the student most directly as they had the greatest concentration of the pronoun 'you'. However, there are no answers for these tasks and students would not attempt them without a teacher present both for support, guidance and to ensure they are on the right track. For example, on page 101, #5a and b state, "Relate the two equations from step 4 to eliminate *h* to form one equation. Divide both sides by *ac*." This is not a task that most Pre-Calculus students would be comfortable attempting, let alone understand. To get to this point in the question, students had to follow instructions to copy and label an oblique triangle and understand terms such as 'altitude' which is not familiar to most. Without a teacher present, most students would not be successful with this task.

What is interesting about the Investigate sections is that, while they address students with interesting investigations, they dictate the steps, the process, and ultimately the reliance on a teacher for confirmation of understanding or explanation.

Images and diagrams are also read, and this textbook is full of photographs. It is rare to find a few pages in a row without photographs. Herbel-Eisenmann and Wagner (2007, p. 11) write:

> Pictures alongside verbal text impact the reader's experience of the text, just as text colours the reader's interpretation of pictures. For instance, we can compare 'generic' drawings and 'particular' photographs. To illustrate, a drawing of a boy could represent any boy whereas a photograph of a boy is a representation of only the one boy in the photograph.

While photographs can date a text, the unit projects date the book as well. For example, page 45 gives 2007 and 2008 forestry statistics that students can consider for their unit project. What is noticeable is that there are not many photographs of students in this textbook. There are many of adults in careers that use mathematics, and many of objects or structures, for example, but few of students. The opening paragraph for section 4.2, for example, refers to sports that involve quadratic equations, and there is a

photograph of students in a gym playing volleyball. The facing page has a photograph of a men's team playing volleyball. A quick scan of the textbook only revealed 6 other photographs of students. In a textbook full of colourful photographs, it is strange to find so few that place this textbook in the supposed realm of high school students.

Addressing the teacher

The MHR textbook has implied addressivity with respect to the teacher. There is no direct mention of a mathematics classroom teacher, or a 'they' or a 'we'. The textbook is, however, formatted for the teacher to use. Referring to all the points made above, the teacher can use the textbook as a resource and, in this respect, it addresses the teacher. The 'Focus on' directives, for example, while appearing to address the student, are restated Prescribed Learning Outcomes (PLOs) from the 2008 BC mathematics curriculum. For example:

PLO: "Describe the relationship between arithmetic sequences and linear functions."

Focus on: "describing the relationship between an arithmetic sequence and a linear function."

The Three-Part Lesson can also implicitly address the teacher as teachers do, for example, use the worked examples to give as notes to their students because students are not accustomed to referring to them, as discussed in my Section 2.5. In addition, the different headings for the Check Your Understanding would only be read and interpreted by the teacher. That is, if students are told by the teacher to complete questions from a certain page, they are not concerned with whether the questions are 'Practice' or 'Extend' questions. Students only notice what questions the teacher assigns them for homework, not the category they are under.

What is also addressed implicitly to the teacher is the useful structure of the textbook. Knowing that the textbook has covered all the PLO's, the classroom teacher can feel confident following the textbook organization. Indeed, many teacher course outlines and homework assignments are delineated so that they can cover the content of the textbook.

Addressing other students and the mathematics classroom

As with addressing the teacher, the textbook addressed other students and a classroom were implied by the types of directives given in the written text of the textbook. There were a few mandates, such as, “Work with a Partner”, that implied the presence of another student. Page 75, for example, also had the directive ‘Explain’ three times. Thinker imperatives such as explain imply that the student and their partner are explaining their thinking to someone, which implies the presence of someone who understands the activity. This could imply the classroom and a teacher.

The other activities that imply a mathematics classroom are the unit projects. These projects do not directly state that a student has to present their project to the class in a classroom. However, sentences like, “At the end of your project you will encourage potential investors to participate in the development of your resource.” (p. 3) imply a classroom. Again, there are no direct references to the classroom or teacher, but these types of statements are not logical to be considered real. For example, it would be quite difficult for a student, or group of students, to develop a petroleum, minerals, or forestry resource and approach investors.

There are also ambiguous ideas in commands such as, “You will then explore the proposed site of your natural resource” that are throughout the textbook. They often contain a high conviction modal word (will) combined with the pronoun you, the determiner ‘your’ and the directive to explore. While it is not clear that the student will not physically visit the proposed site, it is not clear that they are not expected to.

4.6 Choosing Textbook Readings

I knew from previous teaching and tutoring experiences that I would have to choose appropriate text for my students to read. Not every idea in every mathematics textbook is explained, or explained well and at the level of the particular class you are teaching. This exploration focused on reading sections of the textbook that explained, for example, a mathematical idea or process. As described earlier, certain sections of the textbook, such as the practice questions, do not contain explanations. I considered what pages of the textbook to read based on several factors.

As with every textbook, the MHR textbook has its strengths and weaknesses. I knew, for example, that the MHR contained the same information as competitive mathematics textbooks but with less sections per chapter. This required pacing as the amount of information per section was denser, and not always supported with sufficient explanation. Some sections had more written text than others. The combination of dense sections with minimal text did occasionally require outside readings from other textbooks. While shorter examples with minimal text are accessible to students, the lengthier examples without written support are more challenging.

For comparison, the first reading assignment on section 5.1 was one and a half pages in length. It contained several worked examples interspersed with and supported by full sentence text that explained these examples. I chose it because it had something for every student. That is, there were words, numbers, vocabulary, comparisons and algebra that would be accessible and interesting to different types and levels of readers and mathematics students. The examples themselves were only a few lines in length and progressed in difficulty. The overall length of this reading would challenge stronger students.

To contrast, section 5.3 on solving radical equations had four examples over four pages. Each example was a page in length. The four lines of written text before these examples only contained two reminders for students to identify restrictions on the variable and check for extraneous roots. The text here was not explanatory. There were also no explanations on what it meant to solve a radical equation and why or how this is different than other types of solving, which for many students meant that there was no context or meaning to what they were going to do. Even if the classroom teacher had followed the sequence of chapters so that students were exposed to solving quadratic equations and concepts such as extraneous roots in chapter 4, there is no certainty that students will transfer those general skills and learnings. Without reference, and perhaps even with, mathematics students are not prone to transfer skills and concepts from one chapter to the next. From my experiences, different textbook chapters mean new ideas and new aspects of mathematics to students. Also, solving radical equations requires a different process than solving quadratic equations and it would be difficult for many to understand the global idea of solving and apply this to the context of solving radicals.

Text like the above in section 5.3 *is* hard for students to read, particularly without indoctrination. Each example is lengthy and every page of this four-page part of the section involved different processes for each question; from the algebra of solving radicals and the algebra of setting restrictions, to the substitution involved in checking for extraneous roots. Another difficulty with the solving examples was that there was only one entry point: the question. As described above, section 5.1 had many access points.

The goal in choosing readings was to allow for different entry points of student learning through the different elements of mathematics such as written text, examples, and diagrams. It was to both challenge yet invite. I read to find readings that I hoped would trigger students to ask, 'What does this mean?' and "How can I make sense of this question?" for example. As my students became familiar with reading the textbook, and most became keen to read, I increased the demand.

Chapter 5. My Classroom Investigation: Prompts and Field Notes

> However, if the textbook is mainly used as a collection of given problems or tasks to solve, the reading is limited to the reading of the text of the given task (Österholm, 2006, p. 325)

This chapter serves to provide an overview of this exploration as well as detail and explain my development and evolution as a teacher of reading the mathematics textbook. The prompts included below are samples of those I created for my students. They range from Day 1 to the end of this exploration and provide a skeletal outline of the topics and content my class completed. Following this I recount and describe entries from my field notes from three of the six chapters covered during this exploration: Radical Expressions and Equations, Trigonometry, and Linear and Quadratic Inequalities. These chapters account roughly for the beginning, the middle and the end of the exploration. They also allow for different viewpoints of content, of student activity and of my responses to their work and our reading of the textbook. In addition, as this was a very disrupted time of the school year, my notes convey my thought processes and the activities they lead to even though not every student completed every reading. My fields notes were both descriptive and reflective, and they helped me think out loud over the course of this investigation. My field notes serve to provide an umbrella or holistic tie. Reading these notes before examining the notes of my students provides the backdrop against which their different responses can be compared.

5.1 Textbook Reading Prompts and an Outline of Context Explored

The following textbook reading prompts are samples that cover the expanse of the Reading to Learn Mathematics exploration. These are the prompts that the students were given, either verbally or on paper. The verbal prompts were also written on the board, while the paper reminders were mainly given when there was more than a day between classes, as on a Friday before the weekend, or when there were larger readings. The following are chronological and the independent reading prompts could be six or seven classes apart.

These prompts were for the individual student. That is, these are the prompts that students worked on independently; they read their own textbook and created personal written notes. They were allowed to talk to one another and ask questions. However, once they were given a reading task, they tended to be engrossed in their own work and ideas. There were often full class discussions following completion of independent reading tasks. Three of the following prompts are directed, and were not considered as data for this investigation. That is, there is a specific goal or focus to these reading prompts. However, I did read and consider them as they provided another window into the reading considerations of my students.

First Week

This prompt was given verbally and then written on the board:

> Read from the middle of page 273 to the end of 274 in your textbook. Create notes that explain what you feel are the important ideas on simplifying radicals.

At the end of the first week, the reading prompt introduced a friend to whom they were explaining their mathematics, and this friend became a fixture through the exploration.

This week the class also independently made notes on adding, subtracting and multiplying radicals. These notes could be considered for analysis as they were not directed.

Second Week

DIVIDING RADICALS

Name:_____

Date:_____

- Read from the bottom of page 286, 'Dividing Radicals', through to Example 3 on the next page (only consider a - c).
- Make notes FOR A FRIEND who missed this class and the notes I gave on dividing radicals
- As this is the first class they've missed in this unit, you can comment or connect to previous ideas and discussions we have had if this makes your explanations clearer.
- Note - you do not have to use the same format as the textbook nor its examples

Figure 5.1 Second Week Reading Prompt

Third Week

This week the Radicals unit concluded and the unit on Rational Expressions and Equations began. We started the new chapter with the explicit reading prompt given below. This 'intervention' prompt was provided to slow students down in their reading and note-taking and hopefully remind them to consider any connections to previous mathematics.

Read the following information on Simplifying Rational Expressions.
BEFORE doing any written work!

1) Read the entire passage first. Do not worry if parts do not completely make sense. Try your best to take in the point it is trying to make.

2) Summarize briefly, in your own words, what you think is the Big Idea of this passage. What is it trying to tell you/remind you/explain to you?

3) a) Note two facts or statements that seem obvious or familiar to you. These can be explanations or calculations.

b) Note two facts or statements that seem confusing or new to you. These can be statements or calculations.

c) Note any mathematical terms that you want to remember or understand, and try to explain them. You can use example to help explain.

Figure 5.2 Third Week Reading Prompt

Fourth Week

This week there were several independent reading prompts. In order to provide my students with a greater variety for reading about rational functions, I also photocopied pages from the Addison-Wesley Mathematics 11 (1989) textbook. Some prompts were written on the board such as "Create notes that explain to your friend (who missed class again!) how to add and subtract Rational Expressions". These reading prompts were 'pure' in that there were no explicit directions or strategies given to the students. There could have been reminders, as in the example below, to what we'd talked about during

class discussions after personal reading and note-making tasks. The page references do refer to the MHR textbook.

COMPLEX RATIONAL EXPRESSIONS

Name:_____

Date:_____

Make notes for a friend that missed the class on complex fractions. The example is on page 335. Consider strategies that we have talked about, such as reading the whole section first to find the big idea, and comparing simpler ideas - what we know - to what we are learning. What skills does your friend have that they will need to use?

There is a description of what is a complex rational expression on page 325.

Figure 5.3 Fourth Week Reading Prompt

Fifth Week

The following prompt was given at the end of the fifth week and before a four-day weekend. As usual, we did have a class discussion about their independent notes and what they'd learned once we returned from the long weekend.

Pre-Calculus 11 - Chapter 7

Absolute Value

Name:_____

Date:_____

- Read pages 360 - 362 in your textbook.
- Imagine that I gave you notes on this section of the textbook. What do you think I would tell you? How do you think I would organize this information? What do you think I would explain to you that is important for you to understand about absolute value?
- Make notes on these pages.
- It may help to imagine you are addressing the class and want them to understand what absolute value means mathematically. Please make notes that explain and address these concerns.
- As mentioned in class, I will not be directly teaching this section. If you are unable to make notes, then at least record questions and ideas that you have about absolute value. Try your best to make sense of this information.

Figure 5.4 Fifth Week Reading Prompt

This chapter also included Reciprocal Functions, and this week I gave the class the following directed reading prompt. As this was directed, it was not considered as independent reading data:

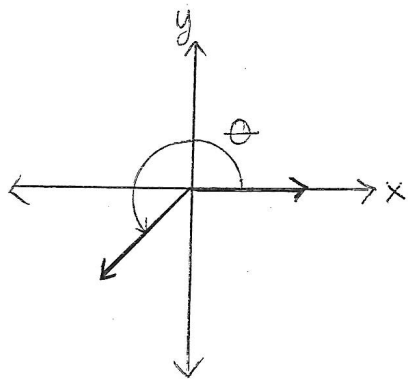
1) First - Read pages 394 - 395.
2) Use a table of values to graph $y = x$ and $y = \frac{1}{x}$
Use the textbook to help you get started, but try to make sense of what you're doing.
3) Answer the 5 'green' questions on these two pages.
Explain your answers.
4) How would you explain 'asymptote' to a friend?

Figure 5.5 Fifth Week Reading – Second Prompt

Sixth Week

Page references were not given with this prompt. To complete this reading task properly, students would need to read and interpret both text and illustrations.

Angles in Standard Position & Reference Angles



This is the only image you were able to send your friend before your phone battery died. However, it's a long weekend and because your friend is falling further behind, you decide to call them from the home phone. You know they have to complete this assignment.

What conversation do you have with them? How do you explain standard position and reference angles without the benefit of diagrams?

Use the back of this sheet to organize your thoughts. Record your conversation as a dialogue so that I can follow your thinking. Have fun! I will collect this sheet and your dialogue on Monday.

Figure 5.6 Sixth Week Reading Prompt

Seventh Week

Sine Law versus Cosine Law

Name:_____

MAKE NOTES FOR YOURSELF!!

As a way of thanks for all you've done, your thoughtful friend has given you their germs, and now you are missing math class! They are currently of no help to you because they are still trying to catch-up. Fortunately, you understand your textbook and know how to use it. You are determined NOT to fall behind.

Make notes on the sine law and the cosine law. You may think of this as a compare/ contrast essay. For example, when would you use the sine law versus the cosine law?

(Note: Please do not consider 'The Ambiguous Case' from pages 104-107)

Figure 5.7 Seventh Week Reading Prompt

The following reading prompt was another intervention prompt. That is, it did direct the students to use a certain format for their notes. Again, these student notes were not considered as data.

Solving Triangles

Name:_____

Read the attached copy on Solving Right Triangles.
Read page 118 in your textbook, Example 3 on 'Solve a Triangle.'

On the cue cards, write only one important fact or idea per card from these pages.

Arrange the cards in a way that makes sense to you. You do not have to use all the cards you wrote nor do you have to keep the arrangement or format of the textbook. The organization of your math ideas and facts is completely up to you. Your goal is to try to explain what 'Solving a Triangle' means. Attach these facts to a separate sheet of paper and bring to class tomorrow.

Figure 5.8 Seventh Week Reading – Second Prompt

Eighth Week

This task was given with several days' notice and with a long weekend approaching. Though it was not part of the content of the current chapter, it was a review on graphing that I would normally have given traditionally before we completed the last chapter on linear and quadratic inequalities, and the final exam. While we had completed the

quadratic graphing chapter traditionally, no-one used their lecture notes but rather, referred to the textbook.

> You are making notes to your Future Self about graphing linear and quadratic functions. Your mathematics teacher has been droning on and on since the beginning of the term about the importance of being able to graph, and you are worried that you will forget what you've learned. What ***is*** vertex form again? How ***is*** it different from standard form? Ugh - you are panicking already!
>
> Fortunately, you have a textbook and know how to use it. Vertex form, page 148. Standard form has key ideas on page 173.
>
> What notes do you make for yourself? What do you need to know to graph lines and parabolas efficiently? You do know that you need enough detail so that you will be able to follow your own thinking and interpretation in the future, but not too much that you overwhelm yourself.
>
> Your math teacher has also stressed the importance of the x- and y-intercepts to assist in graphing. For reference, she attached an example from a different textbook. You also know that you are starting Chapter 7 and look ahead. Yes! There are some linear intercept examples on page 370 and quadratic examples on page 372. They might help.

Figure 5.9 Eighth Week Reading Prompt

Ninth Week – Project Prompt

> You are now really questioning friendship. As if it wasn't enough that your friend got you sick and you had to miss several math classes, now your teacher is sick. To make things worse, it is the end of the school year and her Doctor has told her that she has to stay home for at least a week or this flu bug will go through the class again. But the worst part of the whole situation is that the TOC assigned to your class does not 'get' math.
>
> While it is no longer possible for your teacher to give you notes and test you on the very short last chapter, your teacher has a plan. She knows that you have mathematics textbooks and you know how to use them. This chapter will become a mini-project rather than the usual quizzes and test.
>
> Your goal is to create notes that are better than this chapter. That is, what do you really need to know about graphing linear and quadratic inequalities? What do you really need to remember so that your future math self is a graphing guru?

Figure 5.10 Ninth Week Reading Prompt

The reading and note-making for this chapter was independent, but I did sculpt the pages so that the task was not overwhelming. The first part of this project was to make independent reading notes. I did prompt them to 'Just read' so that they would just read to have an idea of what the chapter entailed.

Read the following pages. Just read.
- bottom of page 465 - 469
- page 478, 480, and 482
- bottom of page 489 - 494

Figure 5.11 Ninth Week Reading Prompt cont'd

5.2 Field Notes

The last weekend of our two-week break at the end of March, I broke in my new journal by writing about the factors that had influenced how I began teaching this Pre-Calculus 11 class and why I was now going to change my teaching approach. I wrote six pages that day, and the paragraph below is taken from the fourth page. This passage is taken directly from my notes and essentially summarizes some of my frustrations. When I read it, I can recount the feelings I had of wanting my students to engage with their mathematics and not rely on me for interpreting their work. I do notice that I was determined to proceed with this investigation despite my sleepless nights of worry as the first sentence occurs in the future; I was writing this passage a couple of days before March 29th.

> Why did I change on March 29? The week before Spring Break, I was noticing things about my Pre-Calc. 11 students. 1) When I talk and do examples - they listen, write, and think they get it - passive - like watching t.v. They are so quiet. Then when they look at homework questions, they stop when the question looks different, or is harder, than anything I've presented. They do not go further or push. 2) Not everyone pays consistent attention in class 3) They try to memorize rather than understand, but at this level they cannot memorize the variety of ways that questions are presented. They are not looking to the meaning of the question. 4) The final straw. Test question #1 - 11/16 students could not answer the question because they did not know the meaning of 'constant'. I thought this question was a given - we have talked about constants from Day 1 (it's in their notes!) and we have *used* this term throughout the last two chapters.

The test question I was referring to was the first question on a half-chapter test on solving quadratic equations by completing the square or by using the quadratic formula. It asked for the constant needed to complete the square and was taken directly from

their homework. From my entry above, I was clearly frustrated with what I saw as the lack of meaning-making my students were demonstrating. While ‘constant’ was a vocabulary term, it is one that they have been exposed to for at least two years, and one we had used regularly that term.

After reading their tests, I felt that I could not continue to teach them in the same manner. For some time now, I had been feeling uneasy, and honestly, a bit bored, by giving my students daily notes. I felt as if they were dependent on my interpretation of the textbook for them. That one question was the thorn that caused me to abandon my original plan of traditional teaching that semester.

5.3 Day 1 – Simplifying Radicals

This first day of reading the textbook was impactful for me. While I did not know what to expect, I did not expect the level of activity that resulted. At home the night I wrote ten pages in my field notes journal. Below are highlights that caused me to think and consider.

> After welcoming students back to school after their two-week March Break, I got right to the task at hand; “Read from the middle of page 273 to the end of 274 in your textbook. Create notes that explain what you feel are the important ideas on simplifying radicals. If you don’t have any lined paper, I have extra here.” I wrote this on the whiteboard, and turned to hand out paper.

In planning for this day, I had allowed approximately half an hour for them to read and make their notes. This was mainly in anticipation that many students would not want to read or becoming tired of reading and noting. Then I had allotted time to discuss what they had noted and consolidate their ideas into class notes. There would be time to complete some textbook practice questions, and then homework would be the next page as reading.

> It did not turn out like that at all.
>
> To say that I was stunned with their response to this task would be an understatement. Not only did they head right to work, the class was alive! There was talk and movement. There were questions and discussion and arguments. As mentioned above, I had found that the nature of this current Pre-Calculus 11 mathematics class was eerily similar to other Pre-Calculus

> 11 classes I had taught this past year; overall, they are quiet students. I had expected a quiet classroom. Reading, itself, is usually a quiet activity.
>
> They were talking, almost every single student. Questions were flying:
>
> “What is an index?”
>
> “What is a radical?”
>
> “What is the difference between a mixed and an entire radical?”

This was a surprise for me. They were asking each other questions and pointing at each other’s textbooks. Soon, they began to ask me the same questions as I circulated the classroom. My goal was for them to use their textbooks, so I didn’t answer. I wanted them to consider their textbooks and how to use them to find information. I was also curious to see how they connected this information to last year’s mathematics class. Would they recall any information on radicals as it was the topic of a chapter in Grade 10 mathematics?

> I continued to circulate as I spoke. When I reached the centre of the classroom, I casually flipped one student’s textbook to the glossary.

Their behavior was interesting and, perhaps it was due to the novelty of the task, but they jumped on the first definition they found, which was for radicand. They again asked me what radical meant. Another surprise, as for me it is a habit of a reader to scan around terms in a glossary. Eventually they located the term radical, and I wondered then why they did not look further to find the definition for the third term, *index*.

As new questions were bantered around the classroom, I moved to my desk to consider what had happened and to record a few thoughts. The class was back to the reading task and were asking questions loudly of each other in case someone had an answer. One question I heard asked from several students puzzled me, “Where did the little 2 come from?” I had no idea to what they were referring. Once I realized that they meant the exponent, I was aghast. Not only had we just completed the chapters on quadratic functions and equations, I felt that the terms ‘exponent’ or ‘squared’ were common enough that they should have been used. Other questions that I briefly had time to record were:

> “Is zero odd or even?”

> "What does it mean if the index is even but 4 – x must be greater than zero?"
>
> "What if there's no number in front (of the radical)?"

I continued to circulate while they talked and wrote. I had not considered that a positive byproduct of reading the mathematics textbook would be my ability to circulate and answer individual questions from students while they worked to understand the mathematics. Another byproduct was the time to support students who normally either avoided attention or were shy. A few examples that I had jotted in my notes:

> 1) I was able to focus one ELL student, notorious for avoiding work and often sleeping in class, to the task. I spoke with him and wrote on his paper "You need to be able to understand the math vocabulary" He used his translator to interpret, then nodded. He opened his textbook and got out a pen.
>
> 2) I was asked a mathematics question, for the first time that semester, from a different ELL student who never spoke in class.
>
> 3) A very bright, quiet student was able to ask me several questions during this time. I had taught this student for an extended period of time the previous semester as a TOC and had never heard her speak aloud in class.

I had not expected this type of activity from my students. After all, reading for me was a quiet, personal and individual activity. I was busy the whole time, circulating and conducting impromptu group and class discussions. Forty minutes into class, I walked over to my desk just to watch them and continue making notes of their questions. I sat and considered my next 'monitor and adjust' move.

> I was curious to know what they wrote this first day. I asked them to draw a line after their personal notes in order to keep our class discussion and mutual notes separate from what they had just experienced. Under the line, they could write down anything they wanted from the board and from our discussion. I welcomed any questions and observations, and our whole class discussion began from there. At bell, I collected their notes, and assigned the next page as reading homework, with the same instructions as today; 'Read page 275 and create notes on the important points that you think I would give you'.

This was day one and I was happy that I had not had a revolt on my hands. I was happy that I had heard voices of students I did not normally hear and I was able to connect with every student in the class, and more than once. I wondered if it was the novelty of the task that had changed their normal class demeanor? I now wondered what would

happen when I tried this again. Not surprising, a cursory glance at their personal notes showed that most of what was written was copied. Again, what would change when I next tried this?

5.4 The Next Few Days on Reading Radicals: The Evolution of Reading the Textbook

I did continue with daily textbook reading for the rest of the chapter on Radicals. My original plan had been to assign reading the textbook and note-making as daily homework, similar to a flipped classroom[3]. This meant that I thought I would have students read and make notes out of class and we would discuss and work in class. However, already by the second day, I realized that many factors worked against this. For one, reading a textbook was more challenging for students than watching and making notes from a video. Also, I was reading the textbook differently, trying to decide which explanatory sections were best for my students to read, and when or if I should intervene. I knew that I would not find every section of this textbook conducive to personal meaning-making. Plus, I was creating lessons based around this reading exploration as I went along. I was monitoring and adjusting continually which made a flipped textbook idea impractical.

Overall that first week, I noted that my students were copying what interested them from the textbook. I noted in my journal that if I had been teaching traditionally and giving them notes, they would be copying my notes from the board, so I felt there was no harm done. Collecting their reading notes had an added bonus; I could read what they were copying from the board, and I was surprised to see that many made copying mistakes.

Also, for the positive, not one of my students had complained about reading the textbook yet. By the third day I noted that I was amazed at the time I had to connect with, focus, and answer questions of different students. *I* had time to notice, and was able to notice unique and individual qualities and concerns. For example, one student was keenly interested in reading the green text questions that appear as thought prompts, located to

[3] Flipped classroom is a phrase that describes an instructional strategy where the traditional method of teaching content during class time and assigning exercises or homework for practice outside of class time are flipped. Thus in a flipped classroom practice occurs during class and content is reviewed or previewed outside of class.

the right of the black explanatory text. One of these questions asked “Why is the square root of 4 not equal to +/-2?” This question drove him crazy and I did not answer directly, but pointed to the page where the answer lay. I wanted him to continue thinking.

As an example of a reading intervention that week, I decided to model reading and thinking aloud with them. However, I started the class traditionally, standing at the front and saying I was going to give a lesson on multiplying radicals. Before writing anything on the board, I asked my students to read the textbook, page 284, and tell me what they noticed. They told me exactly what the textbook stated “multiply the coefficients and multiply the radicands”. I asked them what they meant and to give me examples. I wrote what they explained and we sculpted class notes that were in our collective ‘own words.’ In this way I was hoping to model how to pull information from the textbook and present or explain it in a more user-friendly way. Though I did not explicitly tell them, I was talking about personalizing notes and meaning.

On Day 8, I wrote something that seemed benign, but now I view it as powerful. In my personal teaching experiences, as well as through classroom observations of other teachers, classes are often started by answering homework questions. When there is a homework question to discuss, students ask the teacher to do a certain question from a certain page. The teacher then references the textbook and reads the question, and often reads out the question. This day I did not.

> I have taken to asking them to read the question to me, and I write what they read out loud on the whiteboard. As I write what they read and say, I repeat it to both clarify that I have it right and to further the sounds of math-speak.
>
> This day, one student was trying to get me to write 3 root 7, but he could only say “3, then that sign, or do I say the 7 first?”

This boy had a dividing radicals problem that he wanted further clarification on. However, when it came time for him to read the question to me, he wasn’t sure how to read the mathematics. I wrote further that I believed my students should do more of this. They should be communicating what they want to know and investing energy toward the solution by reading the question of interest. In a mathematics classroom, *they* should be communicating mathematics. I wrote next that there were small indications of change:

> There has been more ‘communication’ lately, or rather, joint (written inside a box) communication. I noticed that it is slowly (squiggled), very slowly, spreading, including the ‘fringe’ students, such as our ELL students.

5.5 Trigonometry – Day 28

The Trigonometry unit occurred approximately mid-way through the investigation. I include my field notes on this section for its timing in this regard, as well as the fact that its timing was poor. The month of May was disruptive and disrupted. To begin with, this particular May contained at best four-day weeks due to both Professional Development and Curricular Development Days, as well as Statutory Holidays. Added to this shortened month were a school carnival, the stress of several of my students writing AP exams, dance competitions, field trips, the flu continuing to make its way through our class, and unusual circumstances such as one of my students being in Ontario and medaling in the National Science Fair competition. As a teacher I found it difficult to create a smooth flow to the unit because of the relentless disruptions and the rotation of absent students that continued throughout the Trigonometry chapter.

We did read the textbook every one of the trigonometry classes. Some days were of the ‘read and make notes’ ilk, and other days were activities to look at or highlight different ways of reading mathematics. We did not necessarily read every section completely, nor read every section in solitude, but we read the textbook every class for the duration of this chapter. There were days when I thought I would just lecture and give notes because the disjointed feeling of the weeks bothered me. It felt like we were moving slowly while time seemed to be moving quickly. However, I found I could not return to lecture mode.

The Trigonometry unit was also unique as it held the greatest number of ‘interventions’. That is, we discussed different ways of reading mathematics text. Sometimes there were overt reading strategies, and sometimes there were simple discussions about reading mathematics. For example, my students completed activities that ranged from voting on “What did you notice that’s new on this page”, to using reading strategies such as ‘Clone an Author’. For me, these activities evolved naturally after I read their reading notes or listened to their thoughts on what they were reading. In what follows, I will note the flow and frustrations of this time, what I noticed and why we began looking closer at the mathematics text.

Exact Values and Special Triangles

The first trigonometry reading assignment was not one that I collected as it was an activity from page 76 in their textbooks. I have never done this before. I have re-written and sculpted activities from the textbook, for example, but this was a first for me as they read and worked on the activity directly from the textbook. Essentially, they were reading and following instructions.

While I thought that folding an equilateral triangle would be beneath Grade 11s, they focused on folding the paper to create their triangles. In hindsight, it was a lot of reading, but there were no complaints. As I circulated, I listened, observed, answered any questions, and noted which students focused on what type of text. That is, was anyone more comfortable following the words as compared to following the diagrams?

This activity provided some anecdotal evidence that supported my thoughts on the types of mathematical reading that students do; some students only read, or perhaps can only read or only know to read, alphabetic text, while others focus on the diagrams or visuals. Only two in this class were comfortable meshing the diagram directions with the explanatory text and completed their triangles easily. I was not surprised when one student said and wrote that he had a difficult time following the written instructions. This student always created notes that focused on algebra and numbers. I wondered if he had a difficult time reading instructions in English class, or if this was solely due to the mathematical register embedded in this text and its connection to the diagrams. For example, one of the instructions read: “Fold corner B so that BE lies on the edge of segment DE. The fold will be along line segment C’E”.

I was surprised that I was glad I chose this activity. There were many positive outcomes. I thought that my students would be frustrated with reading all the instructions, only to create a triangle. Yet my students persisted through the reading. When I reflected on the activity that evening, I noted that my students did not complain and easily accepted that they would complete this activity. Yes, they did talk and support one-another, but there was an overall environment of acceptance; that it was possible to read and complete the task. When someone struggled with their understanding of the text, others assisted. I did not have to instruct nor direct and was able to circulate and listen to their conversation

and their thinking. I wondered if this signaled that reading the textbook was becoming an accepted part of our classroom learning.

I was also highly surprised that they used their triangles as manipulatives and continued on with the activity, past the point I had expected. It was humourous, though I did not laugh, to see Pre-Calculus 11 students manipulate and reflect large triangles across the tiny coordinate axes that they had drawn in their notebooks. They were using them, and continued to use them over the next few days, to help visualize ideas such as reflections and reference angles.

I was also glad we completed this activity because it was a concrete activity that occurred before they read the more abstract definitions and ideas that followed. It became a great memory hook because it was a visual and tactile reminder. For example, we later read and made notes on special triangles and the exact values of their angles. Some students even wrote the exact values on their large triangles.

Trigonometric Ratios of Any Angle

The following is taken directly from my field notes:

> It continues to be a weird week. In a class of 16, it is noticeable when 3 students are away. One is at the National Science Fair and two others are away sick.
>
> Then today, 4 students are away: 2 with the flu and 2 others writing the mock English Provincial. Plus, one boy who is behind, wants to spend time catching up. That leaves 11.
>
> But it's going well. I like the fact that I can make notes on their notes. That is, I assigned the activity on page 89 for homework, and then I talked through it on the board. Or rather, they told me what to write, or add, to my diagrams. We collectively created class notes. I should have photo'd the post-note lecture notes!
>
> I find that there is more conversation. The two that normally answer still answer, but the previously quiet students are on task and able to answer when I ask questions (rather than shrug). Some are even volunteering. One of the best/most interesting things is that they are using correct terms when they tell me what they think.
>
> I also find that reading the textbook does stop with vocabulary. *But* some are trying to work through this. And sometimes it seems, to me, that it's the simplest stuff they've heard in previous years or classes, like 'trigonometric

> ratios', that stumps them. So, they are reading and are weeding through because the next day they are asking me what does this mean?"

Solving for an Angle Given Its Exact Value

My students began class this day reading and making notes from page 94. Solving for angles is traditionally a challenging section for students, so I had planned to go over the examples after they had finished making their notes. However, before I could begin the discussion by pointing out how I thought and made sense of the text, my students quickly entered and made it a conversation by adding their thoughts and questions. Little I wrote that day were my ideas. I did not use my pre-planned notes.

They were asking great questions and what I wrote in my journal was that, if I had been teaching traditionally, I would have given them ***a*** method for solving for an angle. What happened was triggered by a question one student asked, "Why did they drop the negative sign?" I didn't answer, Rather, I enjoyed asking back, "What if you don't?" Immediately they began to investigate. They compared processes and came to tentative conclusions. From this mini-investigation, we collectively created notes.

As mathematics teachers we all have our 'favourite' ways of explaining or demonstrating examples. This day, I did not have to subscribe to my way, but answered their questions based on their observations. They were prepared to hear and consider my explanations. As will be shown in §7.5, there were many different preferences as to how each student solved for an angle when the trigonometric ratio was negative.

My field notes that day are interesting. This is the first day I wrote in two different colours of pen. The comments I wrote in red stress the following threads of this entry; that I enjoyed that class and had fun teaching by responding rather than giving, and that my students were noticing details.

I also collected their notes, and quickly photocopied them so that I could return them. One boy said to me, "Oh, I didn't know you were giving them back so I took a picture of it already so that I could use it." His notes were his notes and useful to him. My third red notation in my journal that day was that, "they are using their notes."

Final Project – Day 45

The final project was born from several considerations I had throughout the trigonometry unit. First, I wondered if my students were using, looking at and reading their mathematics textbooks differently. If so, or even if not, would and could they independently read a larger body of the mathematics textbook? Finally, could they create sustained and coherent notes of this reading?

The last chapter in the MHR textbook was titled 'Linear and Quadratic Inequalities.' This chapter was based in large part on graphing linear and quadratic functions. My class had much experience graphing these functions through four previous textbook chapters. They had also created independent graphing summaries as well as a Desmos graphing project that focused on functions and restricting domain and range. In addition, they'd been assessed at great length on their graphing so I knew that they could graph and that reassessing this skill was not critical. In addition, these students had experience with inequalities through a chapter on linear inequalities in Grade 9.

My field notes reflect these thoughts throughout the trigonometry unit. On May 19th, before the May long weekend and in the middle of the trigonometry unit, I wrote:

> I am also going to have them read and make notes <u>for Chapter 9</u>. I have decided not to directly teach it and see how they do. I will also not test them directly on it. It <u>should</u> be a relatively straightforward chapter. Big idea – inequality graphing – but it is based on graphing lines and parabolas. I have to let them know that I'm not looking for quantity of information.

This final project concerned reading Chapter 9 of the MHR textbook and creating notes that they could use to understand the chapter. Chapter 9 is the last chapter and is one of the smallest chapters in the textbook. They could present their notes in any format that represented how they learned or understood or wanted to remember the mathematics of this chapter. I did not assign every page of this chapter and let them know which ones to ignore. In doing this, I was focusing on the mathematics they would need should they choose to take further mathematics. I was focusing on the big ideas, but not on every small detail of the chapter.

For three weeks this project had been evolving and developing in my mind and through my notes. On Day 45, I introduced the project to my students. I was again nervous as I was worried about their reaction. However, as I also noted on Day 34 in my field notes,

“Re-visiting my original question - Would they protest and revolt and complain about reading and making notes? Not yet, and this is Day 34.”

A pleasant surprise happened as I was handing out the outline for this project. The whole class was laughing by the time I got back to the front. When I asked why they were laughing, one boy read aloud, “You are now really questioning friendship. As if it wasn’t enough that your friend got you sick and you had to miss several math classes, now your teacher is sick.” They really enjoyed the connections between the independent reading prompts.

As they read over the outline, one student asked, “What does user-friendly mean?”, with reference to creating user-friendly notes. I was prepared, and had examples. To start, I compared their mathematics grade 10 textbook, a Pearson publication, to a mathematics 10 textbook from a different school district, a MHR publication. I asked why would there be different versions with the same content? We talked about a variety of reasons from order of topic presentation to personal preference. I said that part of this comparison was to note that organization and presentation involves personal choice. I also laughed and said that being different can get you in print, and the publishing business is big business.

Next, I gave a book talk on four different styles of mathematics textbooks that I had brought from home. From my field notes:

> It was fun! I enjoyed sharing the books and they laughed throughout the presentations on:
>
> *Algebra 13*
>
> *Girls Get Curves*
>
> *Trigonometry and Algebra*
>
> *The Humongous Book of Calculus*
>
> The books were vastly different in presentation and style. For example, the Trigonometry and Algebra book had no illustrations! Hard to imagine trigonometry without diagrams. I was able to talk a little about the history of mathematics and the Bourbaki movement. When I spoke about *Girls Get Curves,* the boys blushed. I asked what would be a counter-part title for them, and they continued laughing and said don’t ask. However, the point was made that for some female students, this style of mathematics text is appealing as it is less threatening and more inviting. A different example of user-friendly. Then I discussed how I really liked the Algebra 13 book as a high school student, even though it is dense with text and diagrams, and I

> could read and learn from it. We finally discussed the Humongous Book of Calculus and its similarities to a 'Book for Dummies.' This book had clear vocabulary with explanations in lay-man's terms.
>
> There was no one best talk that I gave; they all received questions and laughs and interest. The class did do a lot of laughing. But the points were made - 1) there are many ways of presenting factual information, even mathematics; 2) there are many ways of engaging your audience and; 3) there are many ways of organizing information.

After our book talk, it was easier to be clear that organization of information is personal, and that they did not have to stay with the format of the textbook.

5.6 Independently reading Chapter 9

I had created stages for this project. I recognized that, while they were now accustomed to being asked to read their textbooks, there was still too much text, diagrams and directions, even in such a small chapter that contained familiar skills and ideas. Though in line with their reading and note-making activities, this was novel and I did not want them feeling overwhelmed or frustrated, for example. The likely outcome of those feelings would be to copy the information for task-completion sake.

On this day, Stage 1 as I called it, their task was solely to read the assigned pages of the chapter. They were not restricted from talking, but I asked them to withhold questions until they had finished reading. As can be seen from my reading prompts in § 5.1, they were asked to 'Read, just read.'

Reading Strategy – Clone an Author

While I hoped that my students had internalized any one of the ways of reading mathematics that we had experienced since beginning this exploration, I knew that I could not leave this project completely open and without some structure. For this reason, the only structure that I imposed was that they read and begin making notes using the Clone an Author strategy.

This reading strategy was used once in the trigonometry unit. Basically, students record one idea on a card or, in our case, on a stickie note. It was a novel strategy then, yet an effective structure, as for some students it forced them to look at what constituted an idea, and helped to reduce verbiage and pull out key ideas, for example. I felt it would be

useful now as they were reading several pages. In addition, stickie notes could easily be reorganized or categorized, depending on the point of view of the student author. In this way the strategy provided the opportunity to be creative with the organization of the information, if the student chose.

Also, for this project, I gave them support reading and exercise materials, complete with answer keys. These pages were taken from the MHR workbook that can be used as a supplement to the textbook. I did want them to practice the mathematics of this chapter and acquire practical experience. These pages had summary sections that could also help in their understanding of the textbook, though I did tell them they could not copy these summaries for their notes. However, I did tell them that I would check that they had completed the work on these pages and, if they felt that one of the questions they completed was perfect to help explain a point in their notes, they could use it.

This day was a project work day in class. I was available as they began to work on the 'math' portion of the project. They could access anything I had or knew for this project. To be clear with them about personal preferences, I did talk again and relate that the dense textbook that I had given a book talk on that was full of words and diagrams, was my favourite. That is, to each his own.

What I liked about this day was that they were happy, as evidenced by their smiles and easy talk and engaged work. They talked with each other and asked questions of each other and myself. Again, as stated previously, I was able to circulate and push the students that avoided work. This time I was surprised when one very reluctant ELL student asked me a question, and then continued to work on his project after we spoke. That day these are a few of the statements I recorded:

> "Boy, this is the first time I ever taught myself math."
>
> "Wait - why can't I test (0, 0)?
>
> Reply from neighbour – Because it's on the line.
>
> An 'A' student, looking flustered and flipping through the textbook said to me, "It seems I've forgotten how to graph parabolas so I'm trying to remember how to graph parabolas..."
>
> Two girls laughing and pointing at each other's textbooks:
> And that should be greater than that ...

Yeah, Yeah,

But wait, isn't 2 > 1?

Yes, but that's how it's written! (written as 1<2 in textbook)

They continue on, talking and arguing, but also, being tactile about their mathematics. The best part of this day for me comes from a conversation with one student. She is having trouble as her test point seems to be a solution on both sides of the graph. Once we talk and I decipher that she is only substituting the '*x*' coordinate, she understands that it is the point that is a solution, not the *x* value alone. Solving takes on new dimensions for her.

As this student packed up to leave class she said to me, "You did it (this project) in stages because you wanted us to slow down and break it down and think about it."

Very true.

This was a short textbook chapter, but a large reading task. The milestones that I provided for this project gave my students freedom while working within a structure. Some of my student responses to this project are examined and highlighted in the next chapter, which contains the analysis of the written reading notes my students made in response to the prompts I created.

Chapter 6. My Students and Their Notes

"But those are *your* notes, not *my* notes." (Chloe, Day 50)

Five students were selected as exemplars as they represented the spectrum of change in this class. Reading a mathematics textbook and creating your own notes is hard work, and while all students invested a great deal of energy in their work, and all but two demonstrated tenacity with their reading and note-making, these five students reveal the breadth of reading engagement for this class. The work of the remaining students was also analyzed and small elements of each of their work was selected to indicate the range of outcomes in this class.

6.1 Chloe

Student description: Chloe was a popular and socially outgoing student who regularly attended Pre-Calculus 11 classes with a smile. She appeared to enjoy school life and its activities and she always arrived at class ready to share news and information with her classmates. I did not assign seating for my mathematics classes and Chloe chose to sit in the back row with four friends. From that day forward, her desk became her desk and was never used by anyone else that semester.

Initially, Chloe acted as the spokesperson for the back-row group of girls. If any of them were unsure of what I was explaining at the board, they would write on their papers and ask Chloe. If she did not know the answer, she would raise her hand and ask for them. It was common to hear Chloe's voice, and unusual to hear from any of the other four girls though I tried to engage them in the discussions.

As our class moved through the first unit of study, I became familiar with Chloe's classroom behaviour and mathematical abilities. She was typical of a student who felt that she was paying attention in class and to her mathematics. She was, after all, asking and answering questions. From the front of the class I could observe that when Chloe felt comfortable with the content of a lesson, she would focus on other things. She also felt she should be doing better because she completed homework. Through discussions, I learned she called her mistakes 'silly' whereas I recognized them as misapplying rules or not adapting her approach when a slight change had been made to the routine of the

question. Chloe tended to attack her mathematics from left to right and did not often look globally at the goal or meaning of the question. For example, multiplication and exponents were done in order from left to right regardless of what needed to completed first. One of her common mistakes in our Quadratic Functions unit was:

$$5(2)^2 = (10)^2 = 100$$

Study inclusion: I chose to include Chloe in my analysis for several reasons. She had identified herself as an active class member and an outgoing mathematics student comfortable with voicing her ideas, questions and concerns. I could hear some of her thinking through the types of questions she asked and the responses she gave to my clarification questions. That is, I was more familiar with her mathematical thinking compared to the quieter students. Also, she consistently engaged in classroom activities and, unless she was away sick or on a field trip during this reading exploration, Chloe completed all tasks.

Secondly, from my observations of her prior to this exploration, I felt she was not attending to her math. She simply completed her work. While she would ask questions when I was teaching, there seemed to be inconsistent transfer from what she asked to some of the work she completed. For example, after I reviewed and demonstrated how to expand perfect square binomials and then answered her questions on distribution, she would complete homework questions inconsistently, then ask why she was getting the wrong answer. For example:

$$2(x-3)^2 = 2(x^2+9)$$

$$2(x-3)^2 = (2x-6)(2x-6)$$

are types of distribution mistakes she made.

Finally, during this exploration, Chloe's reading work demonstrated a noticeable change. She took ownership of her mathematics. From the scattered, copied and distracted notes of the first days to the final personalized project, she began to notice more of her mathematics over the time of this investigation and went from restating the mathematical text to interpreting it.

Radical readings: first notes

Chloe's completed reading notes the first day cover the front of an 8 1/2 x 11 sheet of lined paper in her large script and contain copied sections from different spots across pages 273–274, as well as an image from the glossary. Some copied sections, such as:

Textbook:	"Radicals with the same radicand and index are called like radicals."
Chloe:	"Radicals that have the same radicand and index are like radicals."

are slightly altered, perhaps to try to use her own words, but are not changed enough to be called personalized. Another section that she copied, in omitting parts of the sentence, became an order or imperative with a mathematical phrase connected:

> Textbook: Isolate the variable by applying algebraic operations to both sides of the inequality symbol.

Chloe: Isolate the variable → inequality symbol.

Notice that she retained the capital from the beginning of the sentence and the period at the end, even though there is no meaning to what remains of this sentence and the intent of the original idea is lost. The initial fragment appears as an imperative, telling her what to do, followed by a term that may remind her what or when she has to isolate the variable. Much of her written work this day is like the above example. It is disjointed and the original intent is lost. As well, some of the numerical examples that she copied are out of context and do not relate to the text around them. For example, she copied an inequality being solved. In the midst of a page of radicals, however, it is out of place. There is no reference to restrictions on radicals, which was the purpose of this numerical example. Chloe was the only student who included the inequality out of context.

Another interesting aspect of her work this day is that she is only one of a three students who 'read' all that was assigned. Her notes include the first sentence from the beginning of the reading and end with the numerics from the bottom of the next page. While what she has written is copied, she created many syntactical problems by copying pieces of text. For example, at the bottom of her notes she wrote:

Radicals in simplest form. –contains NO FRACTIONS

This statement is out of context and in this case, it is also incorrect. Chloe copied these seven words from a group of twenty-five that have a different meaning and message.

Task completion and partial attention are characteristic of her written work the first few days. For example, the next reading assignment was given as homework and she was one of only four students who completed and submitted her notes. However, her reading homework was completed from the wrong pages. She partially copied a section on applying radicals to solve trigonometry questions. This was very unusual as we had not studied trigonometry yet. In addition, we had not started addition and subtraction of radicals yet she copied an example from another section of the textbook and placed it under the triangles with the trigonometry work. (see Appendix B, §1) While this subtraction question and the right side of the simplification is directly from the textbook, the left side under the question is Chloe's own work. It makes no sense and there is no indication why she has completed this nearly mirrored work where the left side has no radical sign. It's as if she's created an equality to solve, but doesn't know how to complete it. If she was trying to indicate that completing subtraction of radicals is comparable to the subtraction of polynomials, it is not clear. There are errors in her process and the equality sign is misused.

Indications of change

Two factors changed with Chloe's next notes. First, she began meta-commenting on her work, particularly the copied textbook example. Next, she tried to create her own numerical examples from the Your Turn questions (See below). She used the textbook examples as models, and, while she was partially successfully, there were several indicators that she tried to make sense of her work.

This page of her notes was eye-opening for me. This reading assignment was one of the few I collected, read, commented on and returned to the students. On her work I made three comments. One indicated a syntactic error, noting that the radical sign needed to extend over the whole radicand. I also made a comment to her, which included a large question mark, that said, "making 'entire' is not simplifying". As I read her work then, I

wondered why she had done this. I was in a hurry to read everyone's notes that day and I offered the quickest fix I could for her mistake.

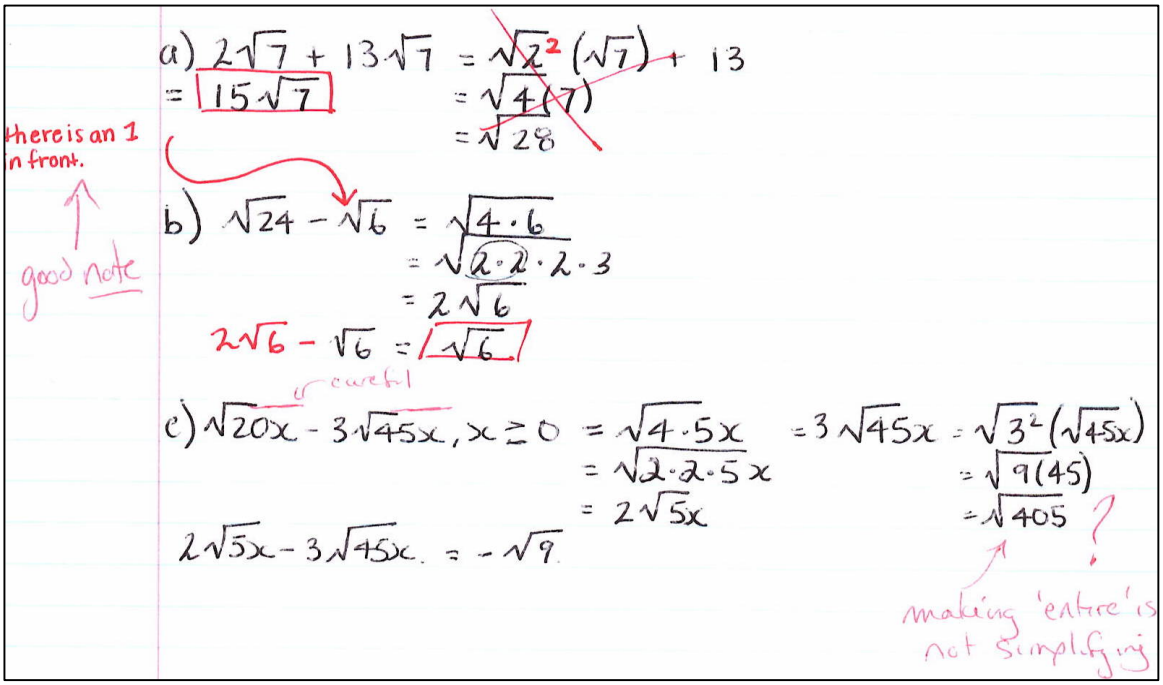


Figure 6.1 My marking on Chloe's work

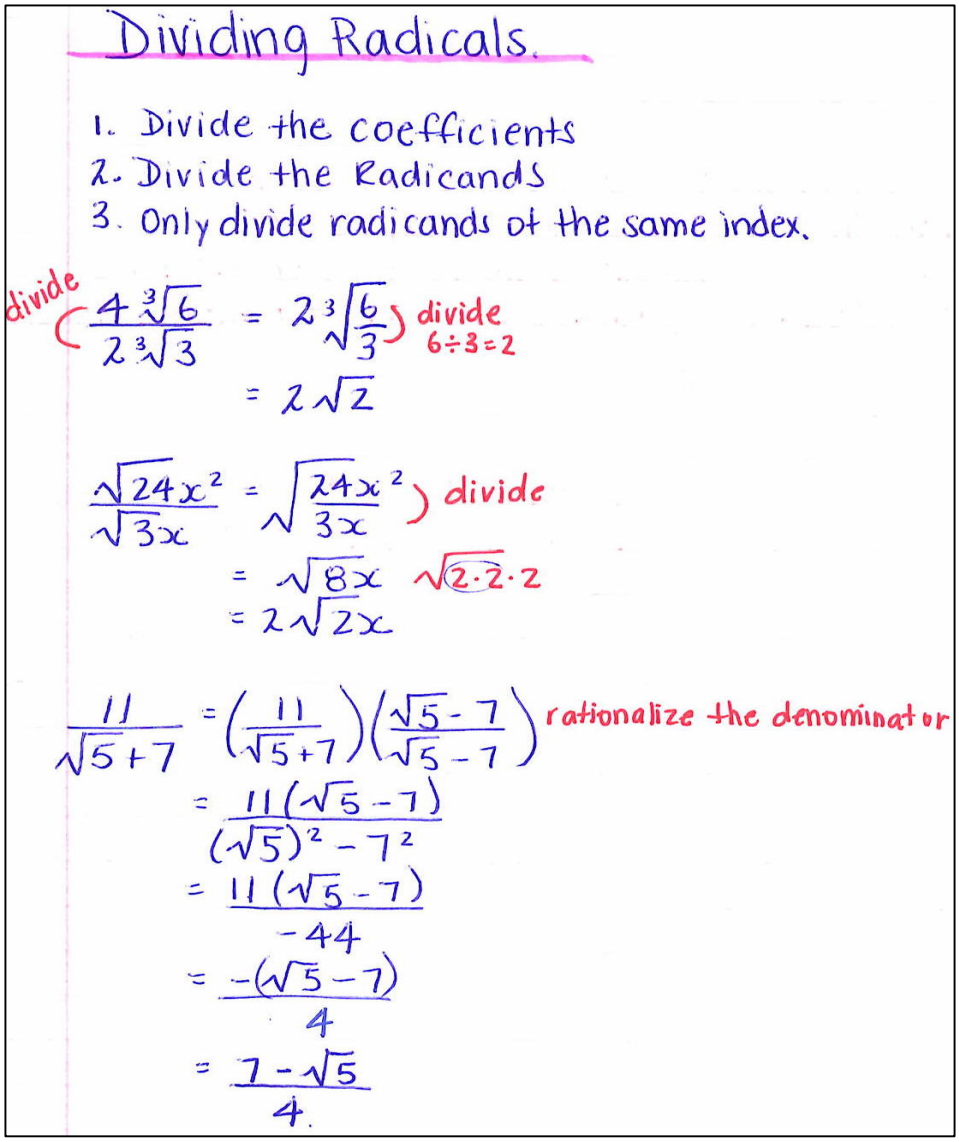
The third question is incorrect, as both her syntax and her inability to make sense of simplifying a radical with a coefficient cause difficulty. Chloe could see to simplify the first radical, but wasn't certain how to handle the coefficient of the second radical, so she tried something that she had been successful at before; she created an entire radical. She did forget to include the *x* in her entire radical of $\sqrt{405}$, but her numerical radicand is correct. She had reached a dead end, left it and did not use it.

It is noticeable here that Chloe had tried to make sense of her work based on her textbook readings and previous notes. She struggled and created an incorrect example and solution, but both her struggle and sense-making were personal. I could also see her created syntax, and what could cause potential problems for her. This was only the third day of reading the textbook, but she was trying to create. This day she copied approximately 25% of her notes and created 75% by completing her own examples. Rather than copying and addressing both text and examples, she chose to focus on numerics and add personal text.

Evolution of Chloe's written notes

Her next notes are also different and demonstrate what becomes indicative of her notes: they do not fill the page as they had in previous entries. This could indicate that she had become more comfortable with this new approach to reading the textbook, or that she was looking closer at the mathematics, taking longer to think through the work.

On the fifth day the notes she created are almost directly copied, with one slight change that becomes characteristic of her future notes. She took the textbook sentences and converted them to steps with imperative directions. These exclusive imperative directions, like divide the coefficients, for example, are taken directly from the textbook. That is, the textbook sentence read, "When dividing radicals, divide the coefficients and then divide the radicands." (p. 286) The numerical examples are directly copied, but she used colour to add her own personal meaning-making.



Dividing Radicals.

1. Divide the coefficients
2. Divide the Radicands
3. Only divide radicands of the same index.

divide

$$\frac{4\sqrt[3]{6}}{2\sqrt[3]{3}} = 2\sqrt[3]{\frac{6}{3}}$$ divide $6 \div 3 = 2$

$$= 2\sqrt{2}$$

$$\frac{\sqrt{24x^2}}{\sqrt{3x}} = \sqrt{\frac{24x^2}{3x}}$$ divide

$$= \sqrt{8x}$$ $\sqrt{2 \cdot 2} \cdot 2$

$$= 2\sqrt{2x}$$

$$\frac{11}{\sqrt{5}+7} = \left(\frac{11}{\sqrt{5}+7}\right)\left(\frac{\sqrt{5}-7}{\sqrt{5}-7}\right)$$ rationalize the denominator

$$= \frac{11(\sqrt{5}-7)}{(\sqrt{5})^2 - 7^2}$$

$$= \frac{11(\sqrt{5}-7)}{-44}$$

$$= \frac{-(\sqrt{5}-7)}{4}$$

$$= \frac{7-\sqrt{5}}{4}.$$

Figure 6.2 Chloe creating steps, using colour, and meta-commenting

As this unit progressed, students were given the option of creating notes for a friend that had missed the chapter and that could be used to complete the chapter test. Chloe did decide to complete this alternative reading assignment, and her notes are clear and address previous problems she struggled to understand. As well, she used colour. (see Appendix B, §2)

Mid-exploration notes

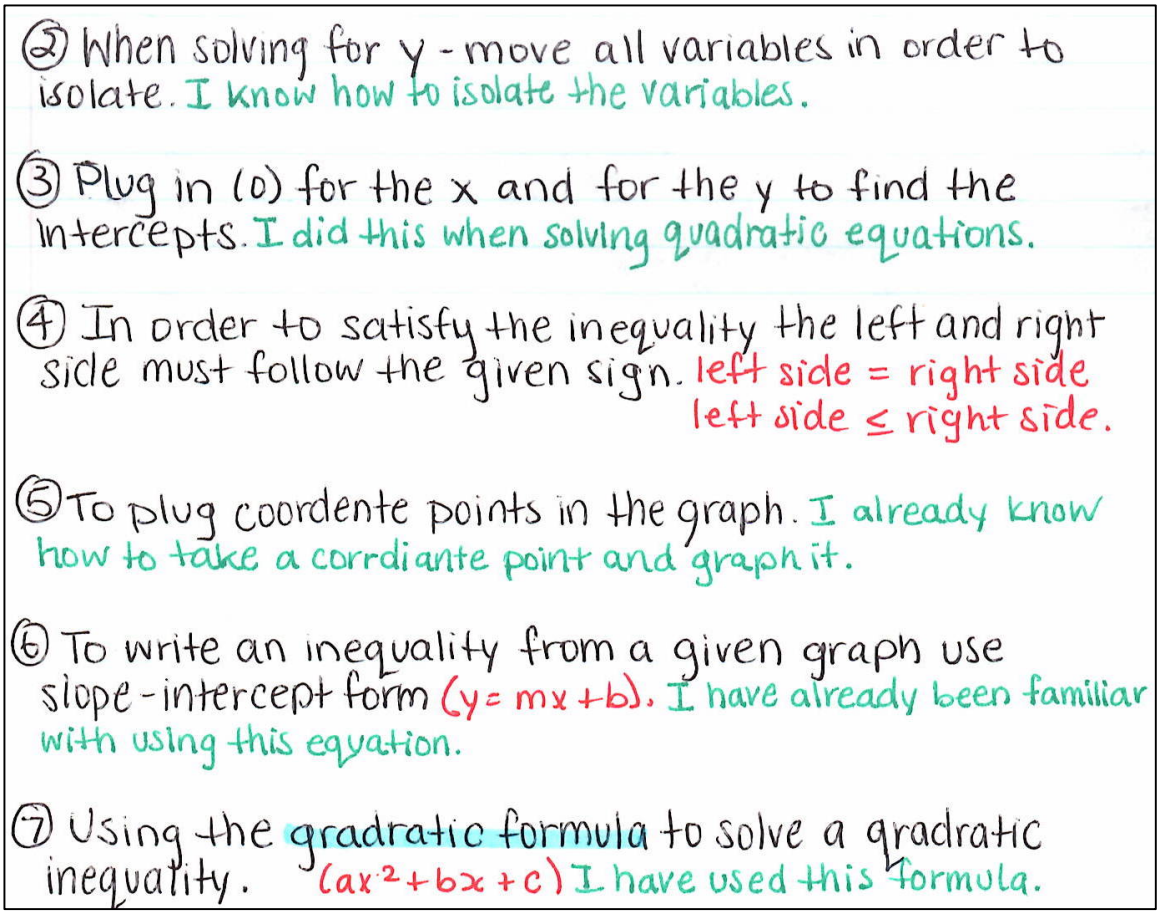
At this point in the exploration, Chloe was absent a great deal, but continued to read and make notes. What is evident now is that she had clearly adopted colour and steps to convey any meaning she wants to note. She also employed meta-comments. A meta-

comment indicates a shift in perspective. She stepped outside her work to add reflections and thoughts about what she was doing, continuing to copy most text and adding her personal touch through features such as arrows.

As an example, Appendix B, §3 is taken from her notes on absolute value. Note the seemingly simple reminder that “two neg. signs make a positive.” However, it is clear that Chloe was applying her past mathematical work and mistakes to her reading of the textbook, in order to make sense of current work. She copied ideas from the textbook, such as absolute value “should be treated like bracetes (*sic*)”, that were important to her and chose not to remind herself of order of operations. Another change in her written work this point in the semester is her tone of voice. During our trigonometry unit she wrote a dialogue where her voice has the tone of an expert (see §7.1). While she added colour, written text and numbers to the image sent to her friend, her dialogue explanation is that of an authority.

End of exploration notes and project

Chloe did invest a great deal of energy reading the last chapter, and she appeared to enjoy her work. She attended every class assigned to this reading and worked and talked happily with her classmates. The first written submissions that she wrote which listed the chapter material or ideas she felt comfortable with seem rushed and dismissive. Interestingly, the written text is her own and while most of it makes sense, it is not always clear what she meant, as with point 4 below. One thing that is noticeable and interesting is her spelling mistakes. Throughout her written work certain terms always seemed to be misspelled, such as ‘bracetes’ for “brackets” in her absolute value notes. It is particularly noticeable when a mathematical term is misspelled differently twice in one point, as in the misspelling of ‘co-ordinate’ in point 5 as ‘coordente’ and ‘corrdiante’, or ‘quadratic’ is misspelled as ‘gradratic’ and ‘qradratic’ in one sentence in point 7. Whether her spelling indicates haste, or part of her difficulty with mathematical terminology, is not known nor within the scope of this exploration.



② When solving for y - move all variables in order to isolate. I know how to isolate the variables.

③ Plug in (0) for the x and for the y to find the intercepts. I did this when solving quadratic equations.

④ In order to satisfy the inequality the left and right side must follow the given sign. left side = right side
left side ≤ right side.

⑤ To plug coordente points in the graph. I already know how to take a corrdiante point and graph it.

⑥ To write an inequality from a given graph use slope-intercept form ($y = mx + b$). I have already been familiar with using this equation.

⑦ Using the gradratic formula to solve a gradratic inequality. ($ax^2 + bx + c$) I have used this formula.

Figure 6.3 Chloe project thinking

This list is very personal as evidenced by the "I know how to…" and "I have already been familiar…" This contrasts the list of items that were new to her, where she asked questions and mainly employed the second person pronoun 'you' compared to only one single personal pronoun 'I'. For example: 'After graphing a boundary, you must choose a TEST POINT which is not included in the boundary'.

The presentation of the mathematical content of Chapter 9 was an independent choice and Chloe created a cookbook. Her cookbook was unique, eye catching and was titled, *The Recipe for Solving Liner (sic) & Quadratic Equations: The step by step guide to perfect results every time.* In the pages that followed, she expanded on her analogy of cooking and baking to graphing. On the first page her voice began as an authority, employing phrases such as, "You must know how". However, by page 2, she became an inviting author. All pronouns on the first page were 'you', but by page two they became 'we.' The complete page 2 from her cookbook is in Appendix B, §4, and below is an excerpt of her cooking and mathematics example.

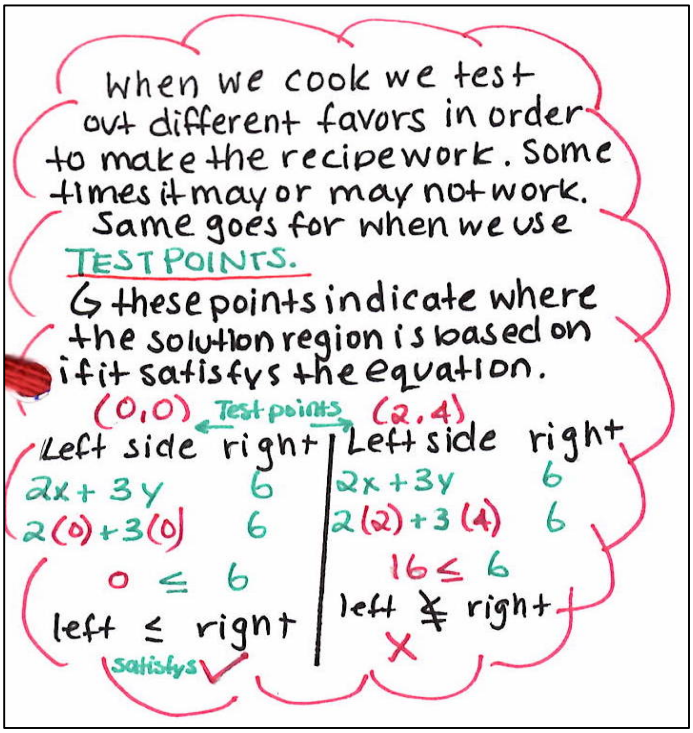


Figure 6.4 Chloe project thinking con't

Chloe began this study as a multi-tasking, vocal class member whose disjointed written notes contained many syntactical and mathematical errors. While she always completed her tasks, she did not attend to them. As the exploration continued, she began to read the textbook and went from restating the text to interpreting it. Her written work evolved to include steps that outlined a process, and these steps were followed by copied textbook numerical examples that demonstrated that process. On these copied examples, she meta-commented on ideas or vocabulary, for example. She created these comments and they were written in colours different from the copied text. Toward the end of the exploration, her voice also entered her written work. The authority that Chloe held in the classroom became noticeable in her notes through the use of imperatives and the pronoun 'you'. However, for the final presentation of her Inequalities notes, she created a cookbook and her voice changed. On the first page she talked to the audience through her use of the pronoun 'you.' For the remaining pages she was an encouraging and inviting author who worked with her audience through the pronoun 'we'.

6.2 Brianne

Brianne was another of the girls that sat in the back row of the classroom. While Chloe and Grace had their set seats, Brianne usually came in at lunch hour to place her books on the desk between these two girls. It was clear she wanted to belong and where she wanted to fit in. She was a difficult student to access personally. During mathematics lessons, she was quiet, not volunteering answers nor responding when called upon. If there were time to complete seatwork, she would leave to work in the support room. Her only spoken words would be to acknowledge that she was bad at math, no matter what I asked or said. Even simple comments that I made to her, such as 'Happy Friday', resulted in responses like "I can't do it".

Over the course of many meetings, it was recommended that Brianne change courses and take Foundations of Mathematics 11. Despite these conversations, Brianne decided to stay in my Pre-Calculus 11 class. Yet having made this decision, she continued to be very vocal about the fact that she 'sucked at math'. Her favourite greeting to me was, 'I tried the homework, but I don't get any of it". These statements were made loudly as she walked into the classroom during lunch hour, despite what I was doing or with whom I was working. I often felt that her loud statements were made as if as an excuse for not doing the work. Brianne was, in fact, weak at mathematics. She had only passed Mathematics 10 by taking it twice and even then just passing with 50%. To me, she presented as a student who was able to use this fact as a shield or defense against having to do and learn mathematics. Her homework was rarely completed and at this point she was not accessing help outside of class time.

Study inclusion: Brianne was an interesting student. To me, she was representative of the weak students who need mathematics for their future goals and have been able to get to this level of mathematics, but barely. Her loud, general statements and comments about her mathematical ability and her characteristic behaviour of never completing assigned work also seemed to indicate that she was afraid of mathematics, to the point that she would talk in circles and work around having to complete her work. Investing energy for potential failure when her previous results had been poor would not make her want to try harder. Her stated goal was to get through by completing and understanding enough to pass the course. However, memorizing steps was not working for her as she could not understand the purpose of her work. For example, on a solving quadratics test,

she undertook several steps to complete the square, but then answered many questions as this one: $b - 2 = 0.77$. Judging by such actions the problem's purpose, namely here to solve for b, seemed not to be understood.

Brianne's reason for taking this course was "to keep her options open", a phrase used by students who want future university choices. However, the goals of just passing while trying to keep her options open were in conflict. She was included in this study for many reasons: Brianne was a weak mathematics student who struggled to keep her mark at 50%; she daily indicated her frustrations with mathematics; she did not engage in the mathematics classroom before the exploration; she did not or could not work to understand her mathematics; her classroom behaviour and reading notes provided a different window into this reading to learn mathematics exploration.

Radical readings: first notes

Unlike her vocal neighbour, on our first day of reading the textbook, Brianne remained quiet. If she had had any questions, they were asked timidly to her friends or she would raise her hand to call me over and then ask me directly only once I was next to her. To date, I had only ever heard Brianne raise her voice when she let me know that she could not do her homework. Now, doing seat-work, Brianne was demonstrating the typical mathematics classroom demeanour that I had witnessed so far: quiet, and appearing hesitant and uncertain.

Brianne did work hard in that class, asking questions and making notes. She completed a full 8.5 x 11 page of notes, to which she added another half-page later that class when we held a class discussion, sharing insights and ideas. Her work was neat and easy to read, and the volume of content she wrote also surpassed many of her classmates. Brianne's reading notes of page 273 were very nearly a replica of page 273. There were only three slight aberrations. First, she took the title 'Like Radicals' and equated it to the copied words of the sentence below using a shaded arrow. What I did not know at this time was that Brianne used arrows a lot, often to represent 'equal to', but she was not consistent with this application as the course progressed.

The next difference is that Brianne created a unique example to provide a definition for the terms *index* and *radicand*. These terms were a concern for the whole class, but unlike her classmates, she did not copy the glossary definition. Rather, she used

numbers to define these terms. However, as can be seen in the image, her ‘definition’ for index surrounds the radical sign. It is not apparent from this example whether the ‘2’ that appears as a coefficient is actually meant to indicate that the index of this example is 2. As the meaning is not clear here, I counted this interpretation as a syntactic error.

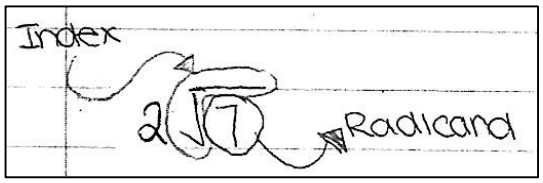


Figure 6.5 Incorrect definition of index

As with many of her classmates, Brianne also copied the worked example that explains restrictions on radicals. The words she copied to explain this example, however, are syntactically incorrect. The textbook sentence read: “Isolate the variable by applying algebraic operations to both sides of the inequality symbol,” of which Brianne abbreviated to, ‘Isolate the variable to both sides:

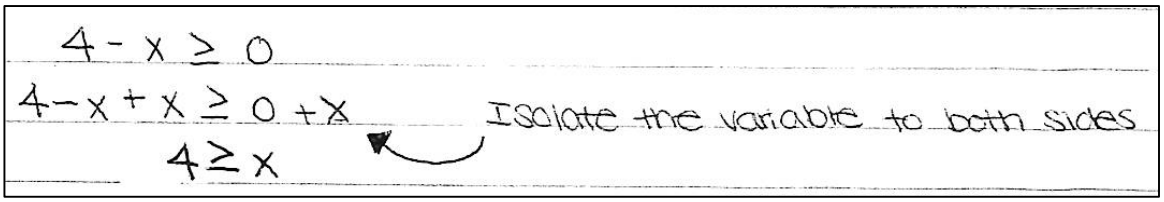
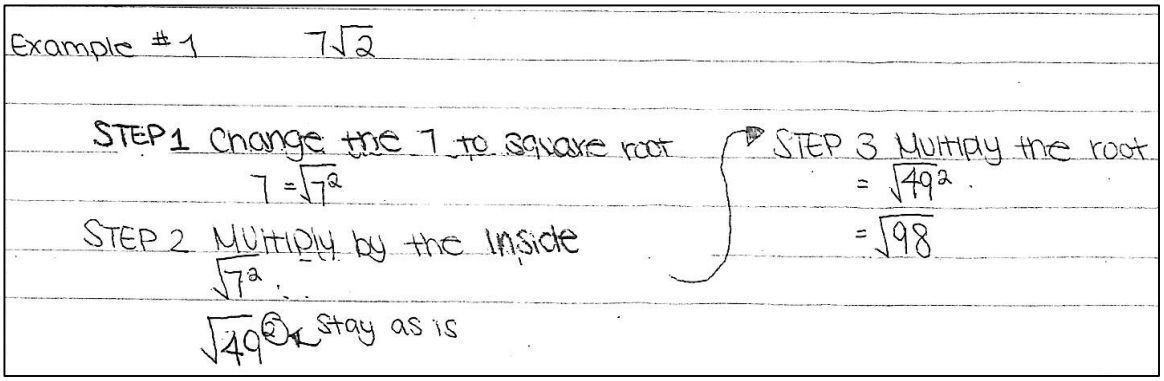


Figure 6.6 Incorrect syntax

In the context of understanding, ‘isolate’ means have the variable restricted to one side of the equals sign, as in the final line. To have it ‘isolated to both sides’ does not make sense. Yet Brianne has directly copied the three lines of algebra and the green text that began next to it. Perhaps working algebraically with variables was difficult for her and thus she noted it as such.

As Brianne moved to page 274 to create notes, greater syntax errors appeared as she tried to mesh written text with a numerical example. While it is interesting and noteworthy that perhaps she had already moved to synthesize this text and is placing it in what appeared to be a meaningful way to her, it contains mathematical errors. For example, ‘STEP 2 Multiply by the Inside’ is not clear. The original number 2 of the question seemed to be lost, but then reappeared in the line below where Brianne wrote ‘Stay as is,’ yet wrote it as an exponent.


Example #1 $7\sqrt{2}$

STEP 1 Change the 7 to square root
$7 = \sqrt{7^2}$

STEP 2 Multiply by the inside
$\sqrt{7^2}$

$\sqrt{49}$ or stay as is

STEP 3 Multiply the root
$= \sqrt{49^2}$
$= \sqrt{98}$

Figure 6.7 Further syntax errors

In Step 3, to see the '2' written as an exponent on the '49', and then to see the next line equate this to 98, is problematic. The mathematics is not correct as she's written it. If Brianne has been speaking to herself like her meta-comments and steps, her math-speak and computational skills are at odds.

This **was** her first attempt to create notes by reading a mathematics textbook. Despite the amount of text Brianne 'read' that first day, she only noted 4 of the 8 vocabulary terms. One of these terms was incorrectly addressed, despite the whole class interest and involvement in looking at the glossary and talking about the meaning of the term index. Brianne did also copy 6 of the 9 numerical examples, though in simplifying the textbook text, she created several syntax errors and mathematical meaning was lost.

The next reading assignment was for homework and Brianne was one of only four students to complete it. She did not attend to vocabulary but directly copied 4 of the 5 worked examples. Placed side by side, Brianne's work and the format of her work is a near copy of the textbook. (see Appendix B, §5) This is, however, the first homework assignment that Brianne has completed for this class. The only syntactical mistake again happened when she only copied parts of a sentence.

Indications of change

As the Radicals chapter progressed, Brianne stopped leaving class to work in the support room. Her notes evolved, first with mainly worked examples accompanied by her meta-comments. Note the misuse of the term *divide*. While 'divide' may be used to make sense in the first line, it does not in the second. This simplification of mathematical terms becomes a characteristic of Brianne's written work.

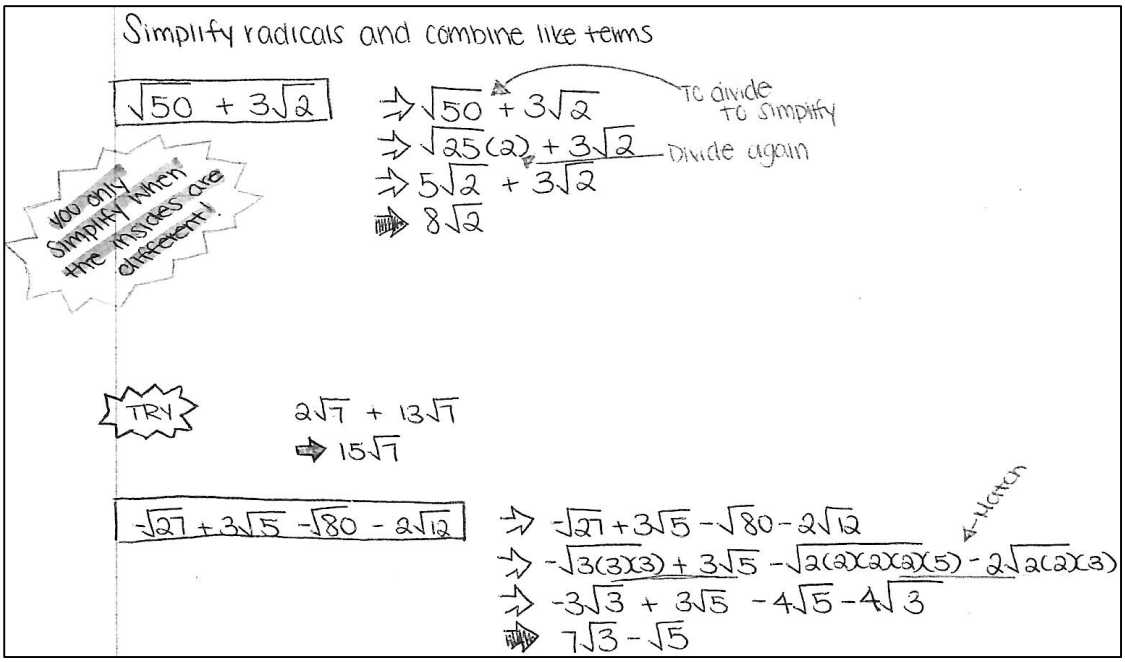


Figure 6.8 Development of personal style and use of meta-comments

What also becomes characteristic during this time is her use of imperatives and steps. Brianne copied textbook examples, created steps from sentences, and added warnings in thought bubbles.

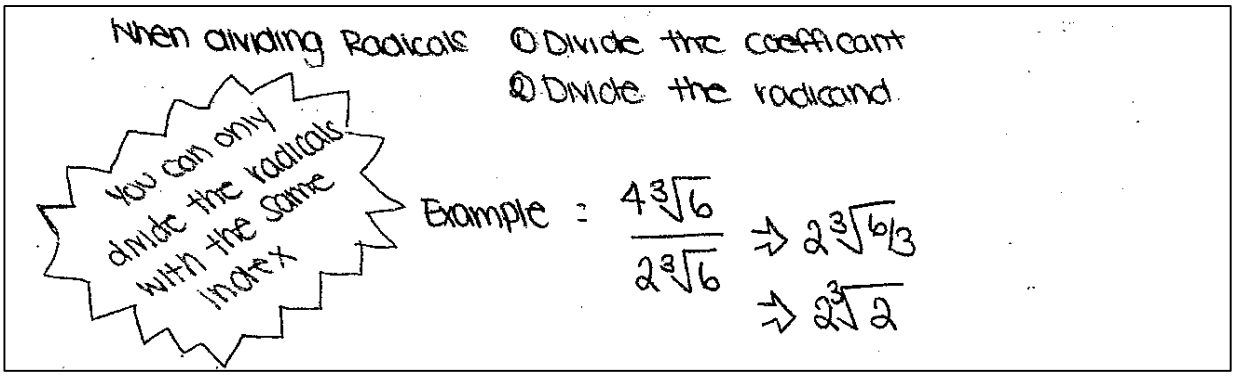


Figure 6.9 Creating imperative directions

Steps 1 and 2 above are from the first sentence on page 286, while the warning is the second sentence. The numbers are directly copied, though with arrows rather than equal signs. Unlike others, Brianne did not copy the algebraic definition of dividing radicals. By the end of this chapter Brianne's notes were set in this format. Her focus was on steps and directives and it seemed that she worked through mathematics in procedural manner that relied on memory. Vocabulary and terminology were either not a priority or they were a difficulty for her.

Mid-study notes

I had asked the class to make notes on the important ideas they felt I would have given them on Absolute Value. For Brianne, that appeared to be everything. The only new vocabulary on these two pages was the expression *absolute value*, and Brianne wrote this seventeen times. Of the ten numerical examples, Brianne copied all ten. By the second page of Brianne's notes, her signature procedural steps with imperative directions were evident. What is noteworthy on this page is the meta-comment Brianne added concerning the absolute value of a negative number. This detail is explained using an idea from the previous page. While it was a small detail, it demonstrated some noticing after copying a page and a half of notes.

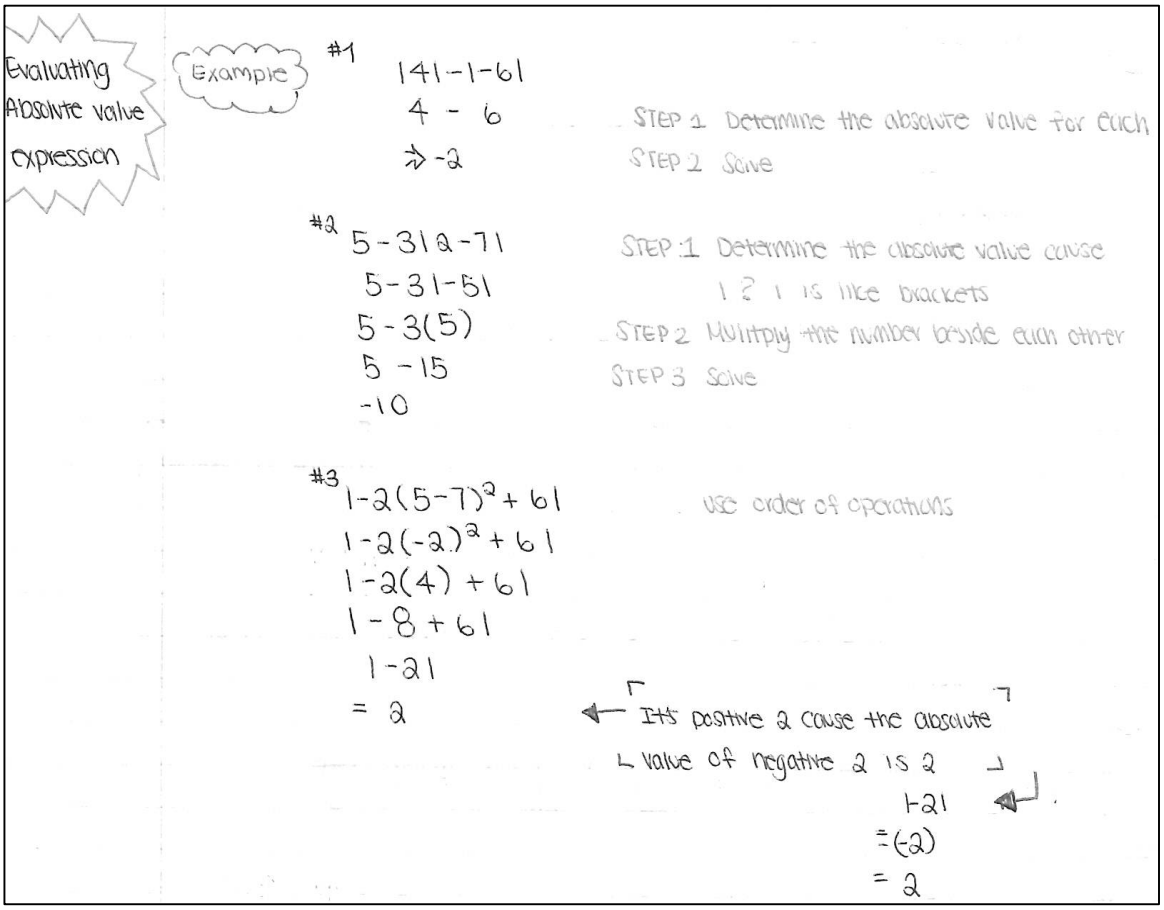


Figure 6.10 Noticing and use of meta-comments

The next development in Brianne's written work occurred in the trigonometry dialogue. As before, she did reduce the mathematical register in her writing, for example, from terminal arm to end arm. However, it is clear she is talking to a classmate as the 'Okay, so,' and 'Pretty much' indicate. There are sections of her dialogue that do not make

sense mathematically, and that again comes from copying parts of the textbook text and fusing them. It is also not clear that Brianne recognized or understood what a reference angle was. For example, she wrote, “The reference angle is how much the angle is at the nearest quadrant. The reference angle is acute btw (*sic*).”

While it is not clear that Brianne understood what a reference angle was and how it related to an angle in standard position, she has made some relationship sense in her dialogue. She has worked with the image and the textbook text to connect quadrants to terminal arms and established a relationship between names and between calculating angles. Brianne’s personal voice is here, when her friend exclaims, “A terminal what?” and she replies, “It’s the ending arm. You know, the other side of the arrow.” This statement, too, is not necessarily clear, but it is Brianne’s way of connecting the first arm to the ending arm by the arrow that indicates the angle of interest. Whether this angle is called the reference angle or something else may not be of importance to Brianne. To this point, vocabulary has not been a priority in her notes. However, the fact that Brianne noticed the ‘acute angle’ part of the reference angle, and stressed this, may indicate that Brianne is reading, but only notes what language is familiar and friendly to her.

End of exploration notes and project

While Brianne is still a tentative mathematics student at this point in the semester, she has become an active and involved class member. Over the past few chapters, Brianne has actively engaged in class activities, such as cutting and folding equilateral triangles, to reading and discussing the textbook. She has contributed her thoughts and ideas to class discussions and even raised her hand to do so. Not only does she complete her reading and note-making assignments, she has begun to complete and submit textbook practice questions. In addition, she is asking for help in class. There is a distinct change in Brianne’s behaviour.

As the class began the reading of the last chapter, Brianne made lists, and this time she could list things she knew, unlike her complaint before this exploration that she knew nothing. Brianne’s sentence structure and tone are different for the compiled list of things that were familiar versus unfamiliar to her. For example, on the list of things she knows, her voice is definitive and to the second person ‘you,’ as well as plainly written. For example, “You can always use the quadratic formula” and “You always need to solve for

y". In contrast, her new list contains items like "The set of points that satisfy a linear inequality can be called a solution region" or "An inequality in two variables describes a region in the Cartesian plane". It is clear her new list is copied: for example, the first statement is taken from page 465. It could be that things mathematically new to Brianne sound mathematical, whereas what she feels comfortable with seems more conversational. For her final project, Brianne completed her work well and her presentation was as her notes in that they were complete, full of steps, exclusive imperatives and spiky think bubbles.

Before the reading to learn mathematics exploration, Brianne was a quiet student who did not remain in the classroom once it was time to complete computational work. Even though she left daily to seek assistance in the support classroom, she never completed her homework assignments. Her work and assessments indicated she was a weak student and her behaviour indicated low self-esteem in the mathematics classroom. Once this exploration began, Brianne began to complete homework. As the locus of control had been shifted to her, she could keep pace with her classmates as they were all independently reading and creating personal written notes. In addition, over the course of this exploration, her classroom presence changed. Brianne no longer left the classroom and she became an engaged and active class member who was able to both ask and answer questions during class discussions. Her written notes show that she read the textbook as she translated terms, such as denominator, into terms more comfortable to her, like 'bottom.' What is unique and interesting about the development of her written work over the course of the reading exploration is that she created exclusive imperative steps, in the manner of the textbook voice, to explain her numerical examples.

6.3 Michael

Michael was a quiet but friendly student. His customary greeting as he entered the classroom at the end of lunch was to nod hello to me, and greet the few students he knew with a large smile. Michael chose to sit near the front of the class with two other boys that he didn't know well at the beginning of the term. They rarely spoke but would nod politely to each other at the beginning of every class.

During class, Michael remained quiet. He never volunteered his thoughts or answers during lessons, but would respond succinctly if I addressed him. The answers he provided were thoughtful and considerate of the work being done or presented, and he would elaborate if I questioned him further. From the questions I asked, he appeared to be completing his work and listening to discussions.

Michael, however, could only listen so long. He was my marker. He was the visual that made me feel guilty when I taught traditionally. Approximately twenty-five minutes into any lecture, he would begin to yawn. He would continue to yawn if I talked further, and approximately ten minutes after that, his eyes would begin to cross and he would struggle to stay awake. This happened daily.

On assessments, Michael was struggling. Where his spoken words were clear and complete, he never seemed to finish a test. There was always something missing from his assessments. His mark before this exploration was hovering around 65%. I spoke with his mother about my concerns and allowed him to rewrite a few assessments. I was beginning to wonder, for example, if he had difficulty sustaining attention, or if written work was difficult for him.

Study inclusion: Michael was included in this study for many reasons. Before this exploration, he wrote on his term reflection that he made good notes that he could use and understand. That his understanding was not being reflected by his test scores made me wonder about the notes he wrote. On this reflection he also stated that he had a difficult time using correct mathematical terminology. He was aware that his understanding of the mathematical register was weak and wanted to improve it.

The notes that Michael made during the course of this investigation were definitely unique and not like any other student in the class. Also, Michael most often wrote half a page of notes, no more, and it didn't depend on the amount of time he was given. Whether he was noting what was important to him or what was enough for him to understand was intriguing. Also, over the course of this investigation, his mark improved, though not drastically. I wondered what else academically, if anything, did change? The other reason for including Michael in this exploration is that his classroom presence altered and his relationship with his neighbours developed once we begin reading the textbook. What struck me was that Michael changed differently from what I would have expected.

Radical readings: first notes

Michael was in attendance first day of this exploration. What struck me then was that his behaviour, when given the opportunity to read and make notes, was different. First, he talked freely with his neighbours, discussing the mathematics on the textbook pages. While he did not ask me any questions, he looked around, talking to those seated next to him and those behind him. He was animated compared with his usual still presence.

Despite the classroom conversations and sharing of information, Michael did not attend to any mathematical vocabulary in his written work that day. In addition, he did not make any notes from the first assigned page, only from the second. Like his classmates, Michael's notes are literally copied. However, his written notes are of the three worked examples from p. 274. He also copied the first sentence of two written at the top of this textbook page. This sentence is an imperative direction for the worked examples. (see Appendix B, §6)

Michael's notes are short and cover less than half a page. However, aside from his notes being unique in that he was the only student to copy solely numerics, he wrote a few short directions for himself. These simple directions are modified from the textbook explanations that accompany each of the three examples in the textbook. All three textbook directions are formatted similarly and, for example, state, "Write the coefficient 7 as a square root: 7= $\sqrt{7^2}$" He translated this to, "Write outside number as a square root". His focus is on the numbers and not the vocabulary, as the change of use of coefficient to 'outside number' evidences.

It is curious that Michael did not note anything from the first page. Visually, the page has more text. The only worked example on that page deals with finding restrictions on radicals and he did not copy this worked example. Whether Michael did not understand this idea or could not work to understand it, is not known. However, that he did not include the second sentence from the top of page 274 concerning restrictions on variables in the radicand after copying the first sentence, is interesting. The second sentence connects the explained ideas of the first page to the examples of the second page. In addition, he did not make steps or any notes on the sentences following each example that involved restrictions. Whether Michael avoided what he did not understand is not clear. Is it that he does math and has created notes of what math means to him?

Two days later, the class was reading and making notes from the textbook on adding and subtracting radicals. What struck me as I circulated was that Michael had again noted three numerical examples. He worked independently on this assignment and did not ask questions nor for help. Later, when I looked over his reading notes, I noticed the similarity to his first notes: There is a copied title and then a copied directive, followed by three numerical examples. The only change Michael made to the copied title is that he changed the imperative ‘Add and Subtract’ to ‘Adding and Subtracting’ and in this way made the title active, though it is not known if this was purposeful.

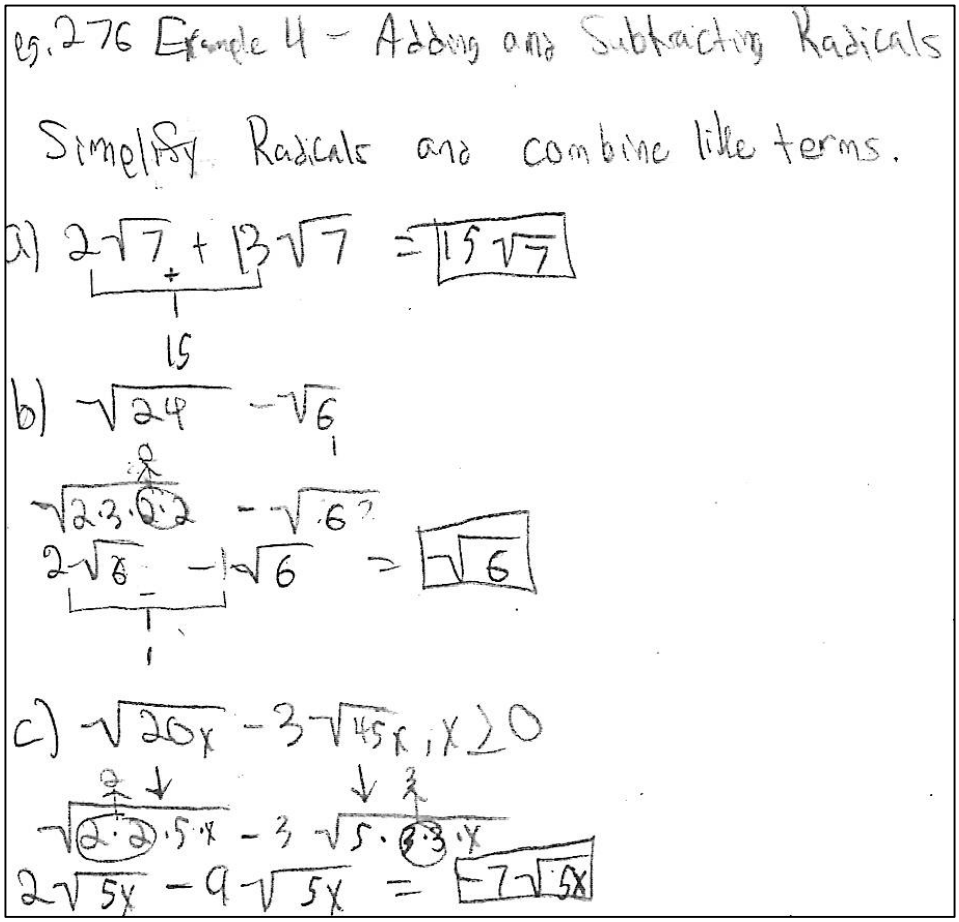


Figure 6.11 Creating notes on day 3 and explaining through numbers

Where other students copied the textbook examples, Michael did not. Rather, Michael used the ‘Your Turn’ questions and created examples, modeling them after the worked textbook examples. He completed them correctly. Again, he gave himself reminders on how he arrived at the answers. None of these reminders are from the textbook or from the textbook examples. For example, Michael connected the coefficient 2 to the coefficient 13 by a line with an addition sign that has the total 15 written underneath.

While his answer has the 15, it is the idea that he noted only the coefficients are added while the radicand is unchanged in the process.

In these examples, Michael explained his work without the use of words or mathematical vocabulary. His work is clear and easy to follow. He even noted the coefficient 1 in the second example as a reminder that there is a term to be subtracted here.

Indications of change

It is on Day 4 of this study that Michael confirms his affiliation with numbers. The class was to read and make notes for a friend that had missed the class on dividing radicals. Michael's notes again have a copied title, and this time he copied both sentences that follow the title. He may have copied both as they deal with the action of dividing radicals, and not on a more abstract concept, such as identifying values of a variable for which the radical does not exist. He then has his now signature examples and brief meta-comments, such as 'same so good' with reference to the indices being the same. However, his examples are not completely copied: he does not finish two of the three.

What Michael has not addressed yet in his written notes, terminology, appears to be what halts his work. Half a page of this assigned reading on dividing radicals focuses on rationalizing the denominator. Michael did not address this idea nor what it means. However, two worked examples require rationalizing, and this is what Michael does not complete. One of the green thinking prompts accompanying the third example uses the term 'conjugate' but Michael did not note this nor ask questions that day.

It is of interest that Michael did not copy the textbook solutions and only went as far as he did. Did he not complete these examples because he did not understand what it meant to rationalize the denominator and could not read to make sense of it on his own? In the context of dividing, rationalizing the denominator was a multiplication step that was perhaps out of context for him. Michael's classmates that completed this reading assignment copied the rationalizing text and its examples to varying degrees.

The other unique feature on these notes of Michael's was his inclusion of an example that defined the terms coefficient, radicand and index. This example is squished to the left of his work and runs into the task instructions, but it is the first time that he is addressed terminology, and it is from day 1.

Figure 6.12 Definitions through numerics

What is difficult to read is that next to index, in the brackets, he writes $\sqrt{\ }, \sqrt[3]{\ }, \sqrt[4]{\ } \ldots$ These are Michael's first definitions, and probably noted because the two sentences that he copied in his notes used these terms to explain dividing radicals.

Mid-exploration notes

Michael's notes to this point still predominately focus on numerical examples with meta-comments that are predominately arithmetic as opposed to written text, but they are expanding to include copied text and definitions. For example:

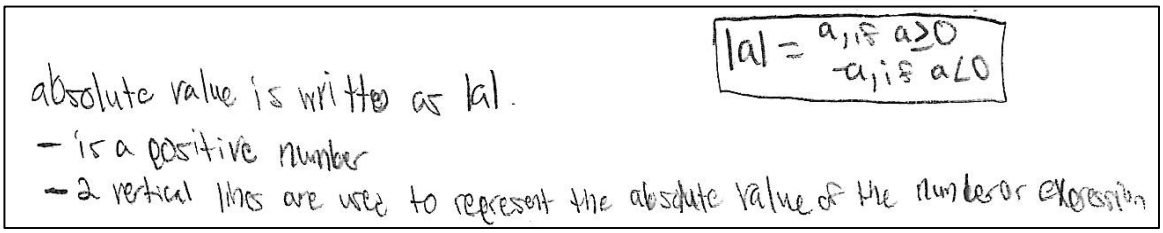


Figure 6.13 Adding text

The above is taken from two lengthy sentences at the top of page 360. The only new term in this section is absolute value, and Michael notes that plus states what it is.

For his trigonometry dialogue, Michael wrote a paragraph to explain angles in standard position and reference angles. It was not an explicit conversation as the reading prompt storied. However, this was the most text that he had ever written in a mathematics class and there were no numerical examples attached to his work. Also, he did not directly copy the textbook definitions, but modified them to produce his paragraph to describe what these angles are. His longest sentence detailed where to find the four quadrants. As usual, Michael wrote half a page of notes. From this task forward, written text has a place in Michael's work. (See 7.1 for further analysis on his dialogue.)

End of exploration notes and project

The work Michael completed for the final project did demonstrate that he read the textbook. The lists that he wrote after reading the chapter explain what was familiar to him compared to what was new material and his work is predominately written text. The statements that he wrote connect the ideas within the chapter as well as connect this chapter to previous chapters. For example, on his list of things he knows, where before he would have provided a numerical example to show checking, he wrote:

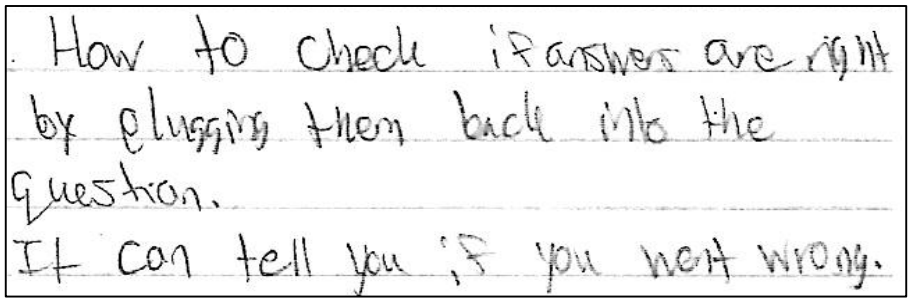

How to check if answers are right
by plugging them back into the
question.
It can tell you if you went wrong.

Figure 6.14 Explaining big ideas

In previous chapters, such as solving radical equations and solving absolute value equations, we had to check to verify our answers were acceptable. In this new chapter, checking points occurs in every section. Michael does include a numerical example, but it also included text and gave a direction:

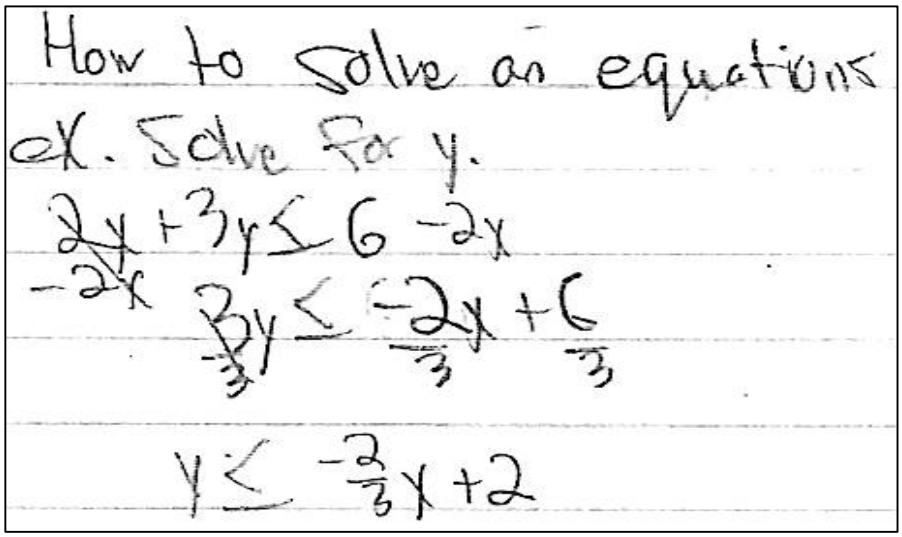

How to solve an equations
ex. Solve for y.
$2x + 3y \leq 6 - 2x$
$-2x$
$\frac{3y}{3} \leq \frac{-2x}{3} + \frac{6}{3}$
$y \leq -\frac{2}{3}x + 2$

Figure 6.15 Explaining ideas through numerics

Also, on Michael's know list is a statement and rationale that connected to our work before the exploration. He had stated in early March that he knew he needed to understand mathematical terminology better. On this current list he wrote, "How to read and understand what a question is asking you. Understanding vocabulary and steps to get the answer of a question is the key to understanding and solving a problem." Michael is the only class member who noted this.

On his list of new considerations, Michael asked questions, thought aloud, summarized big ideas and used visuals. For example:

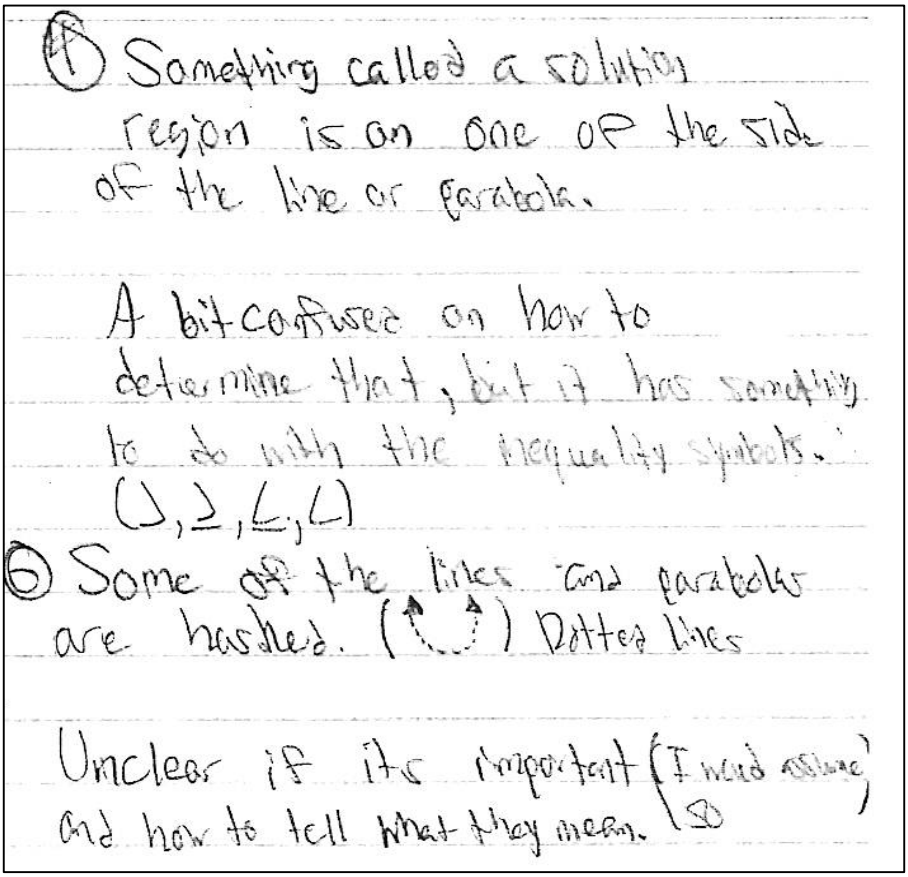


Figure 6.16 Questioning new ideas

He also wrote: "The line is called the boundary and it separates 2 different sides of the equation. This was the first thing I noticed and see that the chapter revolves around it." Michael indicated that he has noticed this more than once.

His written notes and final presentation of the chapter are uniquely Michael. They are brief and predominately 'mathematics.' That is, they mainly contain algebraic solutions

and graphs. There is some written text, but it is often included as meta-comments. Part of his section 9.3 notes are in Appendix B, §7. While every student in the class included all versions of the algebraic representation of quadratic inequalities, Michael was the only one to succinctly summarize them, as noted at the top of that page.

Michael began this term as a respectful class member who would answer questions if called upon. However, he never volunteered his thoughts or ideas to our discussions and his engagement with the class and his mathematics seemed minimal. Michael's classroom personality changed on the first day of this reading exploration. While he was still respectful, he became engaged and vocal, and over time he became comfortable walking into the classroom and talking to others. From day 1 his written notes indicated that he saw mathematics as numbers as his reading notes only involved numerical examples. His written work also indicated that he worked to understand the textbook examples and, if he could not complete a textbook example on his own, he would leave the example unfinished. As this exploration progressed, Michael did begin to include some mathematical terminology in his written work but defined most terms through numerical examples. He also began to contribute his ideas to our classroom discussions and tried to use correct terminology whenever possible, or he would ask for the 'right word.' What became interesting about his final written work is that his text became as conversational as he had become, with more declarative than imperative statements.

6.4 Grace

Grace was one of the five girls who sat in the back row of the classroom. She usually sat in the middle of this row and appeared at ease chatting and laughing with the pairs on either side of her. During math time, however, Grace was quiet. Even when called upon, she preferred to shrug rather than create an auditory sound. She was one who most often asked Chloe questions during lesson time rather than ask them herself. While I was glad she was working to understand the mathematics, her method of clarifying the current work and what I was saying had the effect of keeping her out of sync with the lessons; she always seemed to be working an example or two behind.

By the fourth week, Grace began coming in for extra help. Her mechanical skills were weak, which also slowed her down as it affected her surety in her work. In contrast to her quiet class demeanor, one-on-one Grace was clear spoken and forthright. She also had

a quick wit and clear sense of purpose. When she came in to ask a question, she had a question to ask, and was not hesitant about writing her work on the whiteboard. As soon as she saw where her error or misunderstanding was, she would nod, thank me, and unless she had another question, leave. During these sessions, she was open to suggestions and demonstrated a desire to understand. It was fun working with Grace because she could laugh at both herself and my bad sense of humour.

The difference between Grace's behaviour in and out of class led me to believe that lack of confidence in the mathematical arena was one factor inhibiting her progression and her work. Out of class she was capable of self-effacing behaviour and spoke with a quiet strength that came from a self-confident person. That she could not speak or ask questions in class, using an intermediary to pass on her questions indicated a concern with making mistakes, looking foolish and drawing attention to herself in this setting.

Student Inclusion: Grace demonstrated a lack of mathematical confidence but a strong desire to achieve. Before the exploration, her assessment scores ranged from 25% to 77%. She was methodical and often wrote every step when completing a question. This example is taken from an assessment several weeks before the exploration

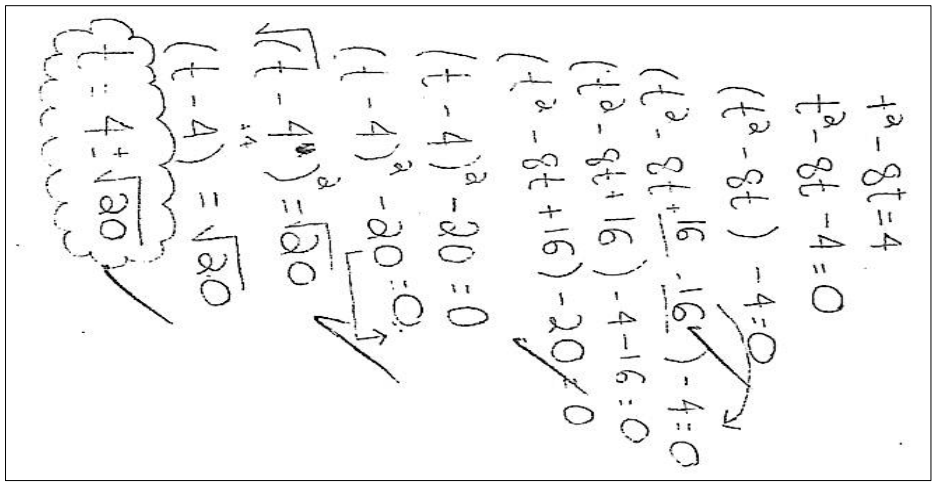


Figure 6.17 Grace prior to exploration

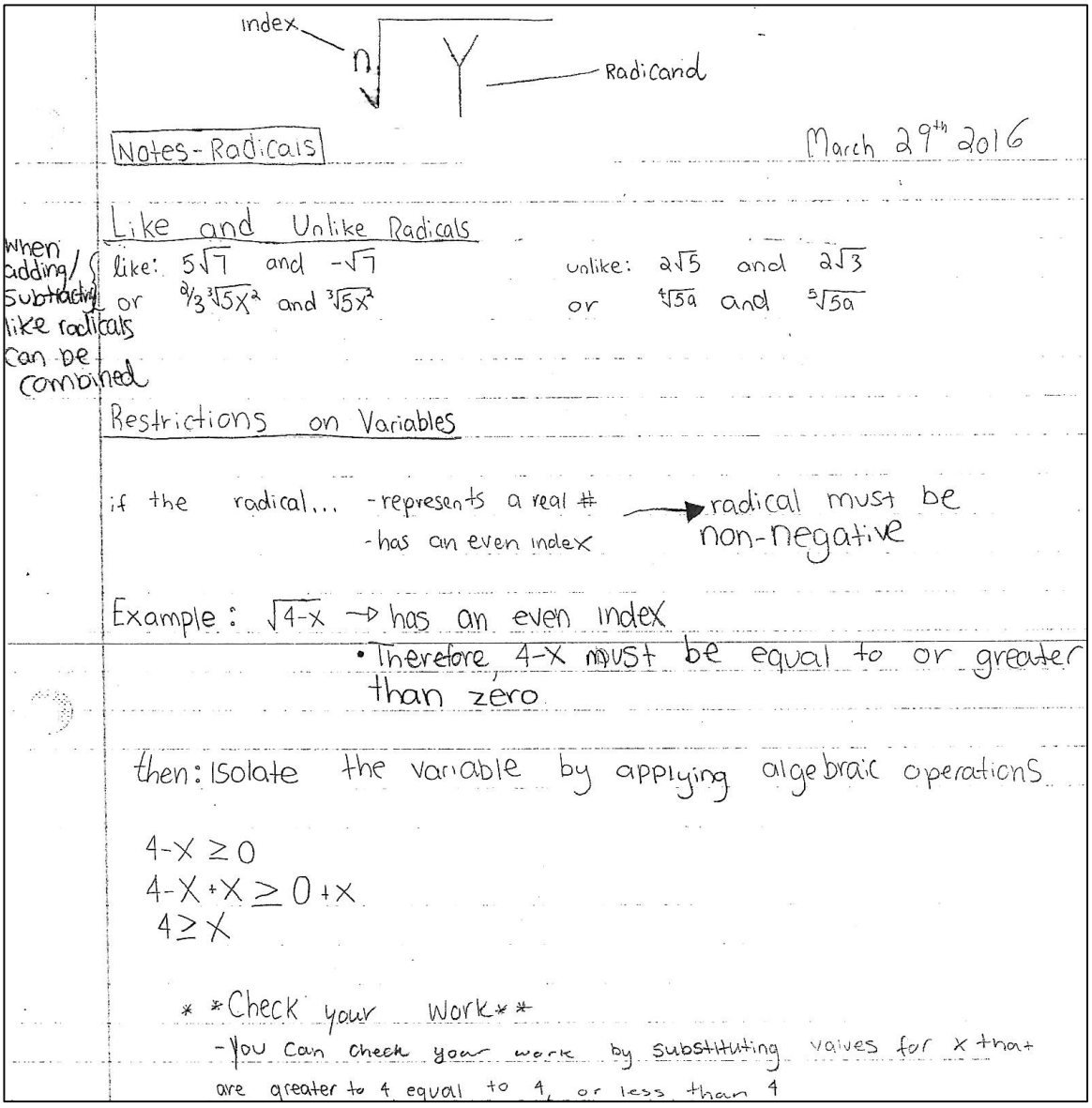
Grace successfully completed several questions on this quiz by writing sideways so she could include every step. No other student wrote in this manner nor included as many steps. While she may have done this as a tactic to avoid mental mistakes, it also served to frustrate her. Grace took more than twice as long as her classmates to complete this quiz, and this was characteristic of her assessment writing. This slowness did bother her

and she spoke to me about it on a few occasions. She also wrote on her term reflection that she wanted to improve in the area of “writing quicker tests”.

During the exploration, Grace changed. Once we began reading our textbooks, she stopped coming in for extra help. Instead, she became a more active classroom member. Also, Grace did show steady academic improvement. Third, her confidence in her abilities changed as will be shown below. Finally, Grace’s development as a reader of the mathematics textbook developed to reflect her personal voice.

Radical readings: first notes

What is first noticeable about Grace’s reading notes is her printing style. Words were printed in different sizes, even the letters within words. I learned over this exploration that Grace only used pencil, but my initial concern was that Grace could be losing accuracy and exactness through her penmanship. For example, below is a section taken from her Day 1 notes:



index

$\sqrt[n]{Y}$

Radicand

Notes - Radicals

March 29th 2016

Like and Unlike Radicals

when adding/subtracting like radicals can be combined

like: $5\sqrt{7}$ and $-\sqrt{7}$ or $2/3\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$

unlike: $2\sqrt{5}$ and $2\sqrt{3}$ or $\sqrt[4]{5a}$ and $\sqrt[5]{5a}$

Restrictions on Variables

if the radical... - represents a real # - has an even index → radical must be non-negative

Example: $\sqrt{4-x}$ → has an even index

• Therefore, 4-x must be equal to or greater than zero

then: Isolate the variable by applying algebraic operations

$4-x \geq 0$

$4-x+x \geq 0+x$

$4 \geq x$

Check your work

- You can check your work by substituting values for x that are greater to 4 equal to 4, or less than 4

Figure 6.18 Grace day 1, front of first notes

On closer look at her first written notes, it was not clear if Grace was exaggerating certain text on purpose. For example, she wrote an example to define radicand and index in the border above her title and date. The example itself is large in comparison to her notes below, but the capital letter ‘Y’ representing the radicand is three times larger than the letter ‘n’ representing index. Mid-page, she also wrote ‘non-negative’ much larger than the other words in the phrase it was attached to. Also noticeable are the last few lines. The words get smaller as she wrote, even though she was several lines from the end of the page. Was this purposeful?

Grace's words and numerical examples that day are copied from the textbook. Like other class members, she does not copy complete sentences. Her work, however, does retain meaning and make mathematical sense. She used arrows and hyphens to connect the parts of sentences she copied rather than parse bits of sentence fragments together. While she did copy seven of the nine vocabulary words and eight of the nine examples, it is not clear if she understood what her work meant. It is clear, though, that she could read: the pieces she copied and placed on her page retained the message of the page.

The following example does not come from Grace's reading notes on day 1, but from what she copied that day from the board. While these are not reading notes, they help in understanding her work. Grace did not copy much from the whiteboard, but written in the margin she did include a numerical example of index, with a comparison of what is not an index. She does not call it a coefficient, but a 'not index.' Here her penmanship is clear in that the 'not index' is large while the index is reduced. Following this, however, is a noteworthy example because it draws attention to either Grace's penmanship and/or her misunderstanding of rational exponents.

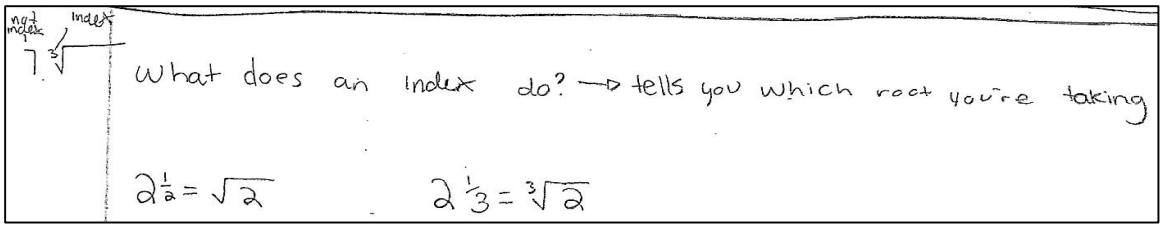


Figure 6.19 Incorrect copying from the board

From this example it could be stated that Grace did not always understand or interpret correctly what she was copying, or copied correctly. The left side of both equalities should be read as mixed numbers. The right side is an irrational number, and they are not equivalent. The next day Grace copied this from the board:

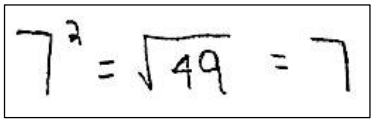


Figure 6.20 Further Incorrect copying from the board

I did wonder then about her copying from the board. Was there too much for her to attend to and process all at once?

Indications of change

On the third day of reading the textbook, Grace changed her approach. While she still copied, she did not literally copy it but created steps. For example, the textbook said, "**Add and Subtract Radicals.** Simplify radicals and combine like terms." (p. 276)

Grace changed this to:

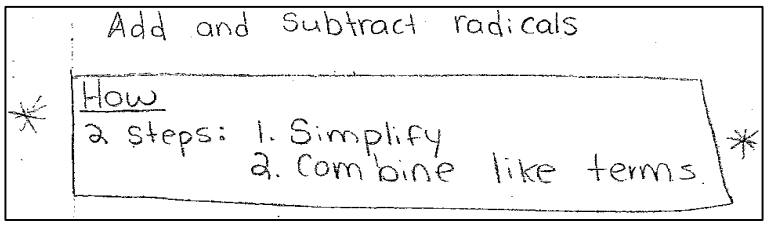


Figure 6.21 Changing textbook text to steps

She copied an example from the textbook and wrote next to it, 'Problem – the Steps – Solution' in a conversational way. It is as if she were trying to explain the work to someone through a few personal words. Grace did omit the equal sign until the solution, but it appeared important to her to have it there, to clarify the problem from the solution.

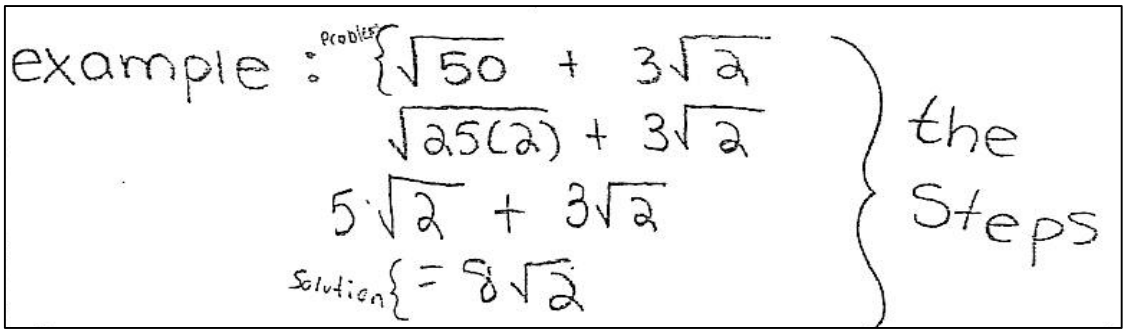


Figure 6.22 Developing personalization of notes

From this point in the exploration, Grace continued to use steps, and began to clarify terms next to any terminology. Her notes now became 50% copied and 50% created:

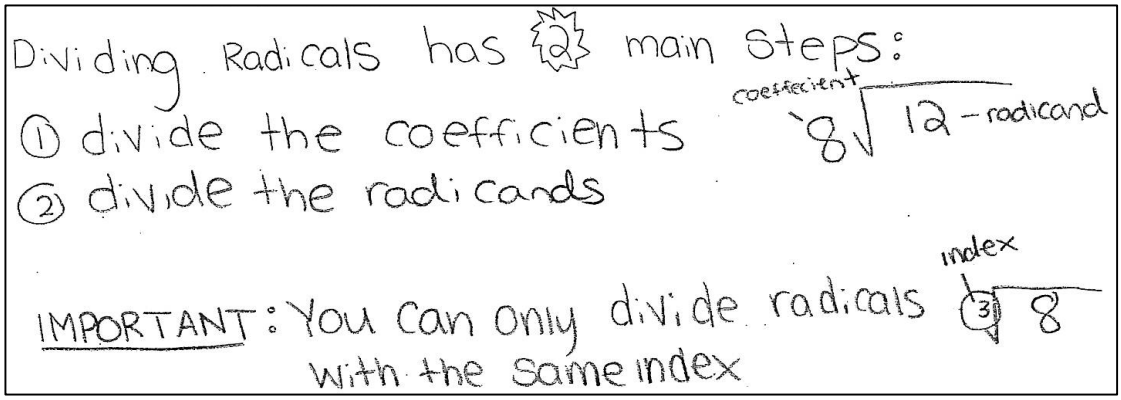


Figure 6.23 Further development of steps

She also began to include conversational questions and examples:

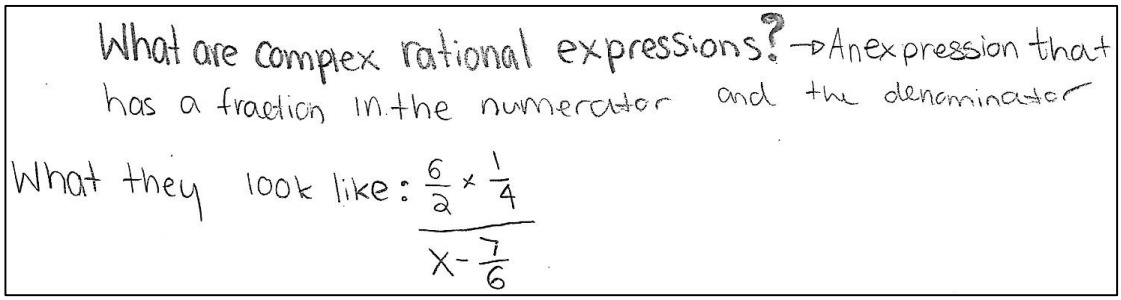


Figure 6.24 Inviting the reader

Grace's intended audience is not yet clear. She did rarely use the pronoun 'you' (as above), but more often she explained her steps using the exclusive imperatives of the textbook. Even on longer copied numerical examples where she wrote her steps next to the work, her steps began with imperatives such as, 'Find', 'Rewrite' and Flip'.

Mid-exploration notes

Every student that created written notes on the first Absolute Value reading copied the algebraic definition, Grace included, though she called her rules.

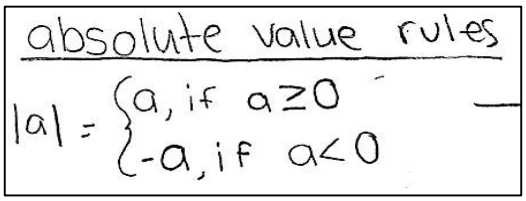


Figure 6.25 Copying algebraic definitions

When I spoke to the class about this, no-one could explain what it meant or why they thought it was important. Chloe said she included this because it made sense, though

she couldn't explain what sense it made. I asked her, "If Grace had missed class and you wrote this to explain absolute value to her, what do you think she'd do or think?" To this, Grace's eyes widened. I continued by saying it was a definition, but how do we read it? What does it tell us? For the first time, Chloe did not answer, but Grace. She said, "Actually what confused me on first read is if absolute value is always positive, then why does $|a| = -a$ if a<0?"

Grace was not only talking, she was asking questions! Her question indicated consideration of what she had been reading, though this particular question was not written on her sheet of notes. Her question revealed that she had been trying to make sense of the mathematical notation. Grace had read and contemplated written, numerical and mathematical notation.

Rather than her particular notes being important that day, this class was pivotal for her as a mathematics student. She continued talking and explained to the class that she drew a number line because, "Absolute value is the, well, count the spaces. This idea makes more sense to me". Grace's presence from that day forward was different. Her thoughtfulness also came through in her trigonometry dialogue. As discussed in § 7.1, she considered having her friend ask the teacher a question. For me, this bit of script indicated both sides of Grace. One is her character, considering and interpreting what would happen if the initial and terminal arm were both on the positive *x*-axis. The friend was the original Grace in math class, asking someone else to pose a question.

End of exploration notes and project

As the class read the last chapter of the textbook and made notes, Grace wrote several pages. She began with a cursory summary of 'what is old' and how it connected to 'what is new', and then expanded on these ideas. What is very clear here is that Grace is reading and connecting ideas. For example, below is part of her brief overview:

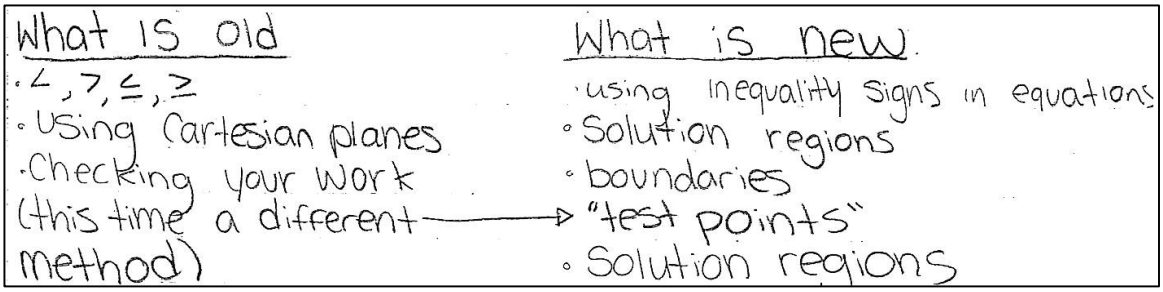


Figure 6.26 Connecting old to new

Grace continued and created lengthy lists, but her ideas connected and compared new terms and ideas to her work across the semester. One theme on her 'know' list were strategies to avoid mistakes. For example, point 3 referred to understanding the shape of graphs because "knowing what you graph should look like prior to graphing is a great way to double check your work and catcha (*sic*) lot of mistakes" and point 5 was titled, "Getting in the habit of checking you work." What was noticeable about her New Ideas list was the interrelatedness of the mathematical ideas. For an example, see Appendix B, §8, and notice points 3, 4 and 6.

Grace continued past these lists to create another titled "10 Mathematical Ideas" where she summarized the big ideas of this chapter, such as, "Since this is a graphing unit, you must be very comfortable with the layout of a graph/how to read them." While her notes weren't perfect, they were thoughtful and she did build on them for her final project which she titled, "Math Party." Like her lists, Grace's Math Party was inviting and conversational. She invited the reader into her work with sentences such as, "Let's say the inequality we have is 3y-12x<12! So what I like to do first is find the x and Y intercepts because you only need two points to graph a linear inequality (or any linear graph)." (see Appendix B, §9) She employed all kinds of pronouns, like 'I', 'we', 'you' and 'our'. She also asked many questions of the "How do I do It? and "How do we know…?" variety (see below). It was clear that Grace was asking the questions that made her wonder about the unit, and by doing this, made it easy for the reader to follow and understand. For this project, her meta-comments were inside different stickers. Overall, Grace's project was comprehensive, clear and an enjoyable read.

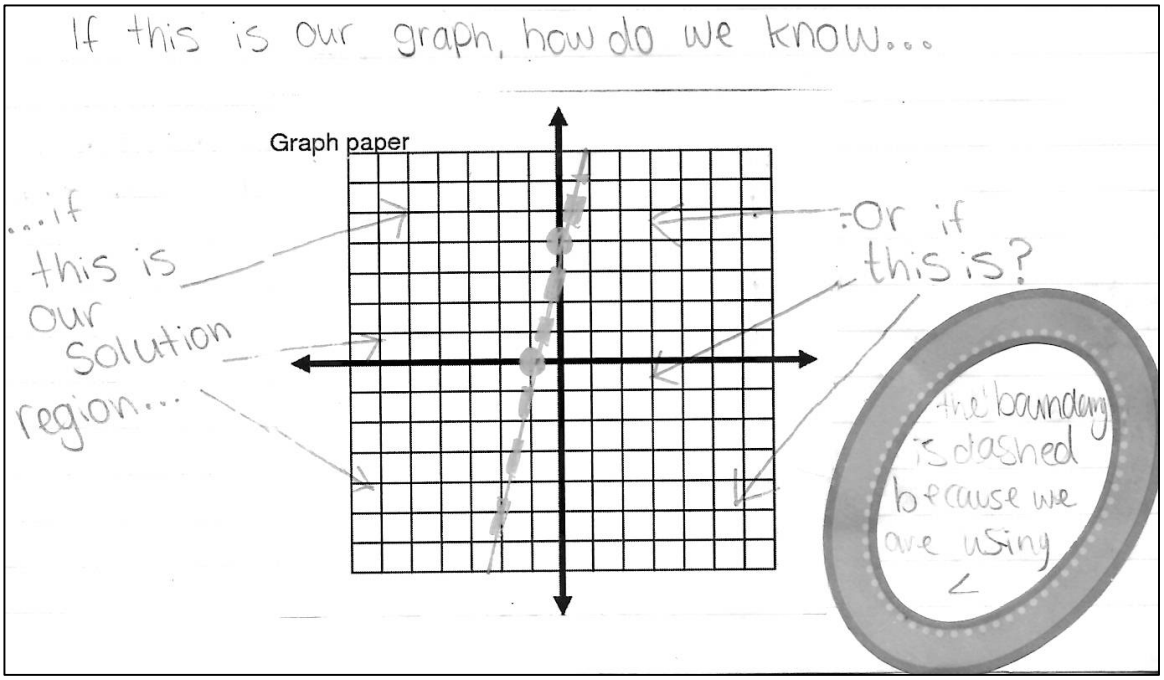


Figure 6.27 Final project – using questions and pronouns

Before the reading exploration, Grace was the quiet class member who would not answer if addressed. She spent a comparatively lengthy amount of time completing her work and assessments because she was afraid of making 'silly' mistakes. As the reading exploration began, it was clear that she could read. Unlike her classmates, Grace did not make syntactical copying errors. While she did copy bits of sentences, for example, the pieces she connected did retain correct meaning. As the exploration progressed, so did her ability to communicate her work and understanding of her work. Initially, she developed an implicit dialogue with the reader, and by her trigonometry dialogue she had explicit conversations. Toward the end of this exploration, Grace's written notes contained all types of pronouns, most commonly you and I, but she also used others such as we and she. In this manner she tended to avoid exclusive imperatives such as 'Graph the parabola' and invited the audience through "There are many ways to graph a parabola." Equally important as her written and mathematical development was that she became an active and engaged class participant, speaking mathematically comfortably.

6.5 Landon

Landon was thoughtful in his approach to everything: friends in the hallway, classmates, and his course material. He was always approachable, polite and never negative. He

considered carefully before speaking and was a supportive class member. He was also a strong student. Those were only a few of his attributes.

Landon was an interesting case because for all his personal and academic strengths, he was perpetually behind. Over the course of the semester, he would work in school, but never had time to complete assignments outside of school, either due to work commitments, friends, or, in his words, “I just didn’t have time”. During this exploration one constant thing for Landon; his ability to complete and submit out-of-class work on time.

Before the reading to learn exploration began, Landon wrote on his Work Habits assessment, “In general, I always grasp an understanding of the basics in a topic, needing time to fully understand the work”. When the time would arrive, was not clear. To his credit, Landon would admit this when directly addressed about his homework issues. During a conversation with me he summed up his stance, “Honestly, I haven’t been working, but I know I get it”. In my mind there was no doubt that he was academically capable, but I felt that he had reached the point where getting by without working was not going to attain the marks he wanted nor was capable of achieving.

Study inclusion: Landon was included in this exploration for many reasons. Predominately, he was and remained a thoughtful student. This exploration provided time for his in-class thinking and personal investigations. He modeled thoughtful reading behaviour. Once he began reading the assigned pages, he did not talk unless it was to ask a clarifying question, and he did not ask many. He was one of the few who read quietly, and was not distracted from his textbook. He always set to work immediately, alternating between reading and making notes. He appeared pensive at times, looking up at the ceiling as if to consider what he was reading and what to note.

Landon was interesting because he was a dichotomous student. While thoughtful in his approach to his work and appearing independent in class, he was dependent on others to organize his work and agenda. While he worked hard in class, he did not work on mathematics outside of class. While his written notes were neat, clear and easy to read, they were not always complete. What did change positively for Landon over this exploration was his development of the mathematical register and his understanding of why things worked. Landon developed a classroom presence as the one to ask the

‘green’ thinking questions. Over the course of this investigation, Landon’s written notes did evolve and change. His written work was singular.

Radical readings: first notes

On the first day of reading the textbook and after more than 50 minutes of making notes, Landon was the student who wrote the least next to his classmate Michael. While Michael focused on mathematical examples, Landon focused on terminology and examples that supported these terms. Landon literally copied the textbook this day. (see Appendix B, §10)

Landon’s notes focused solely on the beginning of the reading, which comprised less than half of the task. Interestingly, of the 115 text words from the page he focused on, not including the text prompts, Landon only copied 28. Most of his effort went into considering the meaning of what he read. His questions in class concerned clarification of the terms *radicand*, *index* and *like radicals*. His notes reflect this concern. In addition, rather than create one image and label it with *index* and *radicand*, he created one for each. Whether he repeated them to reinforce their individual meaning and relationship is not known.

There are two unique clarifications that Landon wrote on his notes that day that are not from the textbook but from discussion. He copied that the radical $\sqrt{4-x}$ has an even index, but that did not make sense to him. He questioned me about this while he was reading the textbook. He was confused as traditionally ones are implied, but one is not an even number. He was comparing the meaning of index, the new focus, to the older idea of coefficient. From our discussion, he noted that the index of a square root is 2.

The other clarification concerned why the radicand must be greater than or equal to zero. Landon personalized what he heard during our class discussion to fit what he had copied from the textbook. He gave a concrete numerical example where the radicand would be –1. Next to this, he wrote an “ – E – “ to indicate error on his calculator. However, he had not yet written about restrictions, though he had copied the work.

It was during the class discussion on this reading that Landon defined restriction as well as several other mathematical terms. While the following are not his independent reading notes, but copied from the board, they provide further insight into what

interested him. He did not copy everything that was on the board. He focused on terminology, and supported this terminology with numerical examples. Notice he underlined terms new to this reading, and used arrows to remind him of terms that we had used previously, though perhaps he was not clear on their definition.

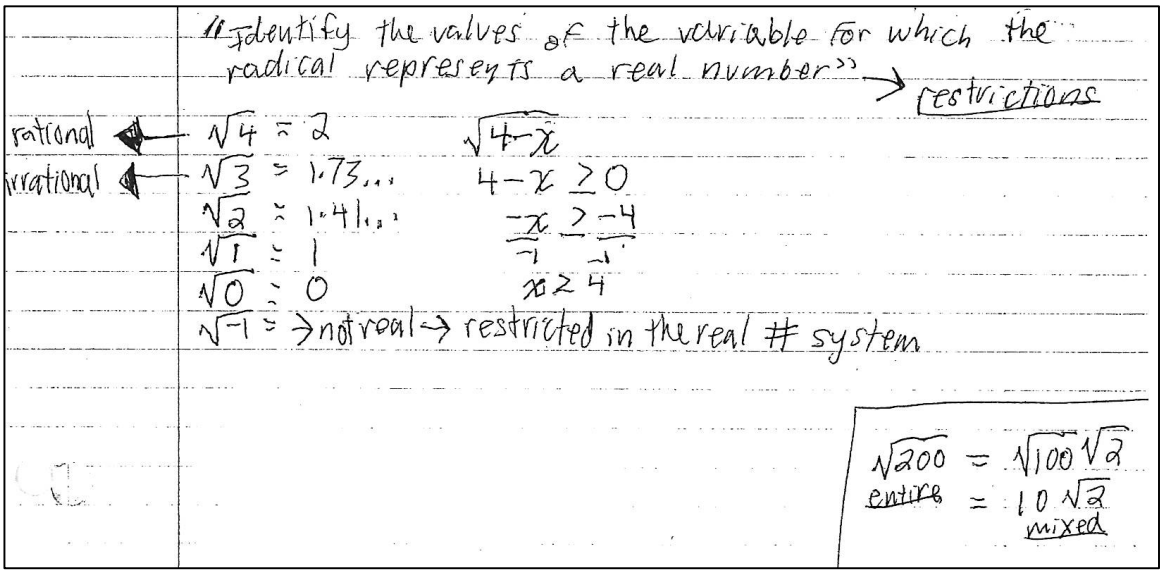


Figure 6.28 Defining vocabulary

Landon copied my explanation on how to find the restriction on the variable in the radicand. He also copied this for his notes from the textbook. However, I provided a different approach as I wanted to remind my students about dividing by negatives when solving inequalities. While Landon copied the division by negative one, he did not circle the inequality sign as I had, nor did he flip the inequality. His two restrictions look the same, but mean different things. He did not notice his mistake or it may also be that, when he compared the two solutions, he thought they were the same. The inequality sign reads as 'greater than or equal to' for both, but in the textbook the variable on the right.

Indications of change

The next reading assignment was for homework and Landon did not complete it. Rather, as I was circulating and collecting notes, he hastily copied the first few examples from the textbook. His work that day is literally copied, both from the textbook and from the board. This was not an indicator of change as, previous to this exploration, Landon would come to class without his homework completed and, while I was lecturing, would

finish it. Landon did let me know at that time that previous teachers allowed late homework submissions, no matter how late. However, what I had noticed before this exploration was that when Landon arrived to class without his computational homework completed, he would finish it while I was lecturing. Because of this he was working on past ideas while partially attending to new ideas. It did snowball on him as there were times he was more than a day behind on his homework. He was often working on tests to learn the material.

The next day I assigned a textbook reading on adding and subtracting radicals. Landon, again, copied all the textbook information, which comprised one directive and three numerical worked examples. He also went on to create notes by successfully and independently completing the Your Turn examples. What was different this time was that Landon meta-commented on all his worked examples. His main focus, as indicated by his comments, was to understand like terms. (see Appendix B, §11) He was singular among his classmates for this, and also because his meta-comments were prompted by the green thinking text to the right of the textbook examples.

As the chapter progressed, Landon began to create his own notes and examples. Often he would copy an algebraic definition, clarify what he needed to understand, and then demonstrate the ideas with numerical examples. For example, below are the brief notes that Landon created on dividing radicals. The only piece that is directly copied is the first line, left side of the equality sign. The mid-section of that line Landon created, perhaps to emphasize the relationship of the division of rational numbers to irrational numbers, as well as to create a step in the process of dividing mixed radicals. He then created a text box that stressed his way to understand and/or remember the different look of division of radicals. The text he wrote explaining ‘k’ he paraphrased from the text (for *k* is not labelled as the index there). Landon has applied his understanding and terminology that he has learned. The numerical examples that follow are not from the textbook or questions from his homework, nor Your Turn questions. Landon created his own examples.

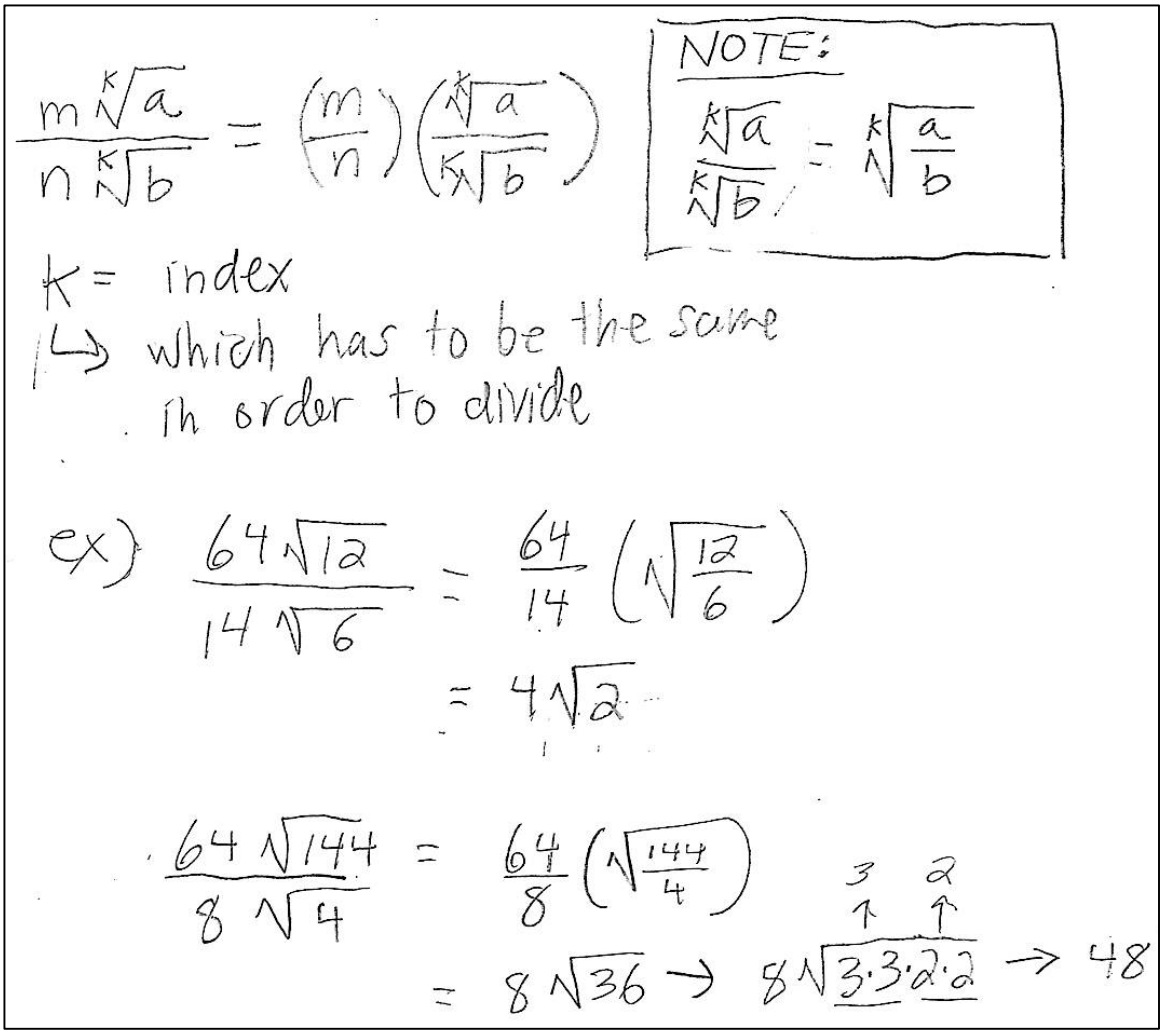


Figure 6.29 Creating examples and expanding on algebraic definitions

Mid-exploration notes

Landon continued to be involved in reading his textbook and creating his own notes, but inconsistencies began to show between his reading notes. He used class time well to make sense of his work, but was still not completing work outside of class. When we read the textbook and made notes in class, Landon was thoughtful and original. From the questions he asked me and his focus on the green questions, it was clear he was working to understand why his mathematics worked the way it did. However, of the few reading assignments that were to be completed out of class, Landon either did not complete them or he literally copied the textbook. Landon continued with meta-comments throughout the exploration. That he did not create steps also spoke to his drive to understand rather than simply to memorize, adding his personal opinion and voice. For example, at the end of the Rationals unit, he argued that working with

variables was simpler than working with numbers. This was quite important to him: he came over to where I was helping another student to show me this work, and he added his opinion to his notes:

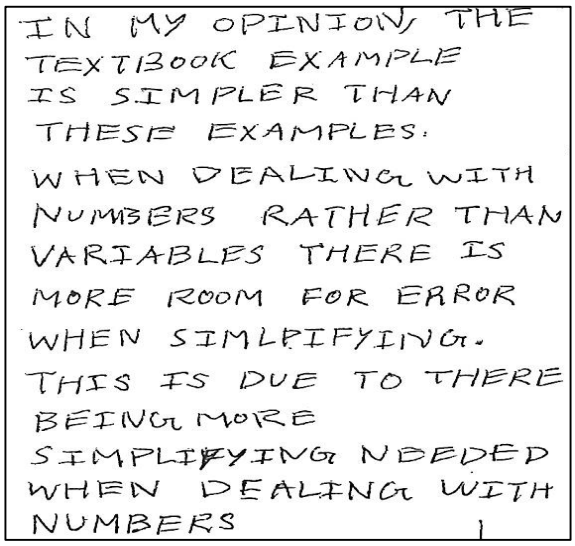

IN MY OPINION THE
TEXTBOOK EXAMPLE
IS SIMPLER THAN
THESE EXAMPLES.
WHEN DEALING WITH
NUMBERS RATHER THAN
VARIABLES THERE IS
MORE ROOM FOR ERROR
WHEN SIMLPIFYING.
THIS IS DUE TO THERE
BEING MORE
SIMPLIFYING NEEDED
WHEN DEALING WITH
NUMBERS

Figure 6.30 Adding personal opinion

It was clear that Landon was a capable student from the work he was creating. However, he was falling further behind. A week later I assigned a textbook reading on absolute value that was the only homework to be completed over a long weekend. Landon missed class the day it was due. At the end of that school day, Landon arrived to show me his notes. They were brief notes, and he was back almost literally to copying the textbook.

After this, Landon began to miss classes. When I spoke to him about coming in for extra help, he stated that understanding was not the problem; he wanted time to complete the work he was behind on. Of the classes that Landon attended, he was actively involved in all reading tasks. However, he was completing less and less and missing more and more, and as a result struggled with the content of the trigonometry unit. This was a chapter test that he did rewrite as he could not complete its final assessment.

End of exploration notes and project

When Landon found out that our final chapter was a structured but self-directed reading assignment, he attended class on the days that were scheduled for this project. He

always came with a smile and worked well, asking occasional questions. He read his textbook quietly, but he worked while chatting with his neighbours.

Landon's reading and writing work on the inequalities chapter was singular. He created a flow chart to show the pathways to understanding how to graph and solve both linear and quadratic inequalities. While he did struggle in his own right to complete the course properly, he chose to complete the independent reading project rather than invest his energy making up missed or poor assessments. (see Appendix B, §12)

Over the course of this exploration, Landon went from an unengaged, distracted class member to an active class participant who regularly contributed to class discussions. He added thoughtful responses to our class discussions and, as the exploration progressed, he began to use correct mathematical terminology. Like the focus of his notes, the questions he asked during these discussions were 'green'; they were thoughtful and deep and worked to the meaning of what we were discussing. Landon's reading notes moved from literal copying to creation of syntax and examples. However, Landon's nature of being perpetually behind did catch-up to him and by the end of the semester, though his final notes were unique, he was embarrassed that he could not successfully complete the last final assessments.

6.6 Notable Ideas from the Rest of the Class

This exploration involved all students in the class. From day 1 of reading the textbook to the last day of classes, every student in this class read their textbooks. As they did not know I was collecting data, they evolved, grew and learned to their personal capacities and as if it was 'math as usual.' What each student can tell us about their learning through their work and how different strategies affected or influenced them, is important as it can inform both teaching and further research. While the notes of the following students were not chosen as exemplars, it is not that some of them could not have been; it is that the exemplars that were chosen involved and could represent more aspects of the learning profile of a mathematics student. The following analyses of the rest of the class provides further information on the impact of this exploration, be it from the social, critical thinking, language, communication, personal, and mathematical viewpoints, to name a few.

Amy

Amy was a late addition to our Pre-Calculus 11 class. She had recently arrived from China and it was nearly half way through the semester. I met her and her aunt as the counsellor introduced us outside the doorway to my classroom. According to her aunt, Amy's command and understanding of English was extremely low. To summarize the conversation we had, she was placed in my Pre-Calculus 11 class because doing mathematics was easy for her and would provide her with the high mark she would need to compensate for her struggles in other subjects that depended on English language.

She began in our classroom as a quiet, shy and focused student. If she had any questions, she would only ask if I walked over to check on her during work periods. Her math skills and background were strong, but the language of mathematics in this class was challenging for her. A few things changed as we approached term break. She was now sitting with the other ELL students. Soon enough, she began trying to sleep in class. On the first day of our textbook exploration, she directly copied the first three sentences from page 273, though did not finish copying the third sentence. Then she flipped to the next page and copied the three numerical examples without copying the accompanying written text. Lastly, she completed the 'Your Turn' numerical questions accurately and completely. Aside from the 3 copied sentences, Amy completely ignored written text and defining vocabulary or terminology associated with restrictions and like radicals. It is not clear from her written work whether she understood these ideas, which were a large focus of this section. However, her computational skills were strong.

On the third day of this exploration, Amy submitted numerical notes. She did not write any text. This time she created her notes from the Your Turn questions without copying any of the textbook examples. As I circulated, I spoke to her about the purpose of the task, to make notes for a friend who had missed class to help them understand and complete their homework. After that conversation, she did add to her notes, drawing a line, under which she added a section that showed how she simplified her radicals.

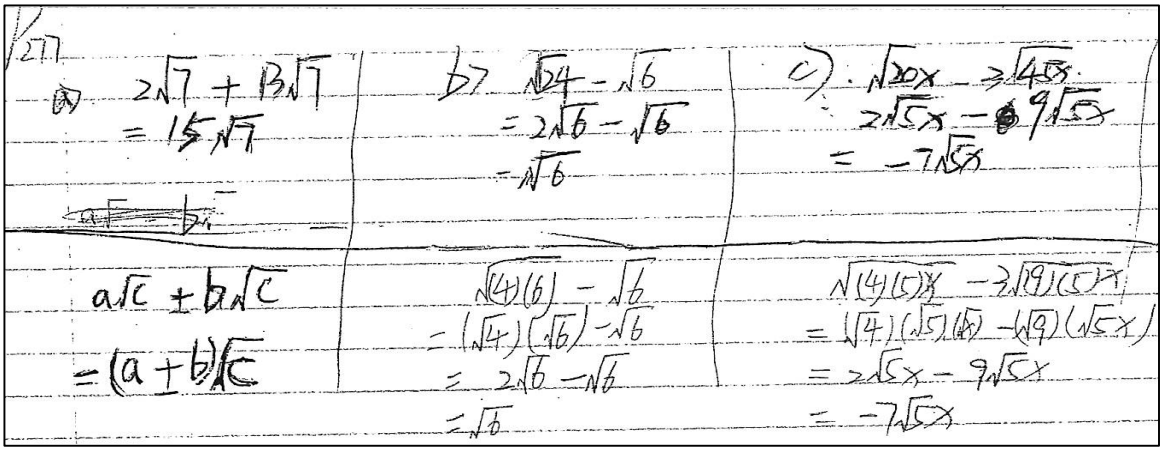


Figure 6.31 Explaining numerics with numerics

Amy's communication was through numbers. While she expressed a desire to develop her mathematical register, her focus was numbers and algebra, and this continued throughout the exploration. Perhaps this was her experience with mathematical communication. It is clear from her first expression on the second line above that she knew and understood this work. This expression is not from the textbook.

Mid-way through the reading exploration, Amy was still attending class, but was struggling with external social pressure. Her inclination to follow her friends versus her obligation to succeed at school were in clear conflict. Her ability to complete Pre-Calculus 11 computations was problematic as that was her survival strategy; she could complete calculation questions on assessments to pass tests, but could not interpret the questions independently. In addition, she was not completing any homework. In class, she would complete some reading assignments, but I often felt she was frustrated. Any written notes she completed after reading a section in the textbook focused on numerical calculations, and were often accompanied by algebraic explanations:

Near the end of May, Amy faltered. She had been skipping classes for all her subjects. There were school meetings and conversations. She did come back in June and tried to work to include mathematical vocabulary in her notes, as well as complete some readings where she copied written text rather than numbers. This was a start toward her goal of understanding and learning terminology. All of these written notes were directly copied and her work tended to copy declarative rather than imperative statements. Sadly, there was only a week left in the semester.

For Amy, the struggle was real. She entered this classroom able to do computational mathematics and could easily represent her understanding algebraically. For the first half of this study, her written reading notes predominately contained copied or completed numerical representations of mathematics, communicating that mathematics was computational and algebraic for her. Throughout her time in the course, she did state a strong desire to familiarize herself with vocabulary and written text, but social acceptance prevailed. While she did try to complete the final reading assignment and focus on the English language, she did not succeed in personalizing this work.

Elisa

In a class of quiet students, Elisa was one of the quietest. What set her apart was that her demeanour was really a cloak of invisibility. She always sat in the desk near the back door and slid in and out of her chair at bell, never speaking unless to the friend that sat next to her. Whenever I walked over to her to offer help, she would only shake her head no. Despite my efforts, I felt as if this would be a student I would never know. Elisa always used her class time to complete what tasks she was given, though the mathematical skills and understanding she demonstrated on assessments were weak. Once the exploration began and I was able to read her notes, I found that she copied as much as she could, both from the textbook and from the board. It was as if by the sheer force of copying that she would know. While she did copy numerical examples, most of her notes were written text. In particular, Elisa focused on language.

As her written notes evolved, she began to add meta-comments to herself and mainly as directives. For example, she told herself twice to ‘think’ within five examples. (see Appendix B, §13) She did continue with meta-commenting on her work throughout the exploration, and on occasion tried to create steps. The steps she created used imperatives like the textbook, even when a step did not need an imperative. (see Appendix B, §14) It was also vocabulary that Elisa added to her notes that made her notes unique. They stressed her placement of mathematical terminology and words as important. Even her trigonometry dialogue asked, “What do they mean?”.

While Elisa’s notes remained lengthy, some of Elisa’s voice found its way to her writing and into the classroom. Over the course of this exploration she began asking questions as I circulated the class. These questions were based on what she was reading and interpreting, not on what information I had given them. Elisa was happy to organize her

work and address issues or ideas that interested or concerned her. Her written work did not include pronouns but did incorporate the exclusive imperatives of the textbook. As the reading the textbook exploration continued, Elisa's writing went from literal copying to loosely transcribing the text. As with her writing, she also began to relax in mathematics class. At that point, about mid-way through our study of trigonometry, Elisa's laugh entered the mathematics classroom. Though Elisa stated at the end of the semester that she would not be taking any future mathematics courses, she did thank me for making a stressful, difficult subject accessible and approachable.

Serena

The female equivalent of Landon, Serena was bright, quiet, and at ease in class, and laisse affairs about her coursework. For the first six weeks of this course, her homework was never completed but, unlike Landon, she did not care to make up the work. She was also frequently absent. Her marks remained average and this frustrated me as I could see glimpses of strong mathematical skills on her assessments.

As we began the reading exploration, Serena literally copied whatever was in the textbook or on the board: no apparent attempt to personalize her work. What was different about what she copied compared to her classmates was that she was not consistent in what she attended to; sometimes her notes were completely written text while at other times they were completely numeric. In my field notes I wrote that, "She is not trying to make sense, what she writes is copied, she even copied the green prompts". However, she made no attempt to answer the green thinking prompts. During the second week of the exploration I spoke to her about trying to explain her mathematical thinking, and she wrote the following:

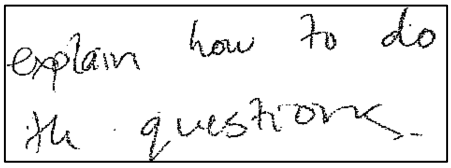


Figure 6.32 Serena copying my words

While it made me laugh, I was also frustrated that she had noted what I had said to her at the bottom of her paper, copied the question from the textbook, created three meta-comments, one of which was from a different example, incorrectly tried to rationalize the denominator, and then copied the correct answer. (see Appendix B, §15)

On the occasion where Serena did complete the Your Turn questions, she did them correctly where most of her classmates struggled. However, she only did this twice. Serena had an ease with mathematics that she was not developing. Perhaps copying the textbook was enough to inform her, but this could indicate that developing mathematical understanding was not important to her and that mathematics was really only about successfully completing assessment calculations.

Overall, Serena did not change or develop her written output or the form of her written work during this exploration. Even her final project was a literal copy of the student workbook information. Serena was the perfect characterization of studenting, (Liljedahl, 2013b) where she went through the motions of being a student. It was not until the last day of classes that Serena talked to me about my course. She stated that this was the first time she had had to think in a mathematics class, and was not happy with her final mark because it did reflect the effort she had applied to her work. In the future, she "wanted to work on the vocabulary so that she would not have to worry about memorizing everything". This made me wonder if copying was her memory tool. She also acknowledged that for her mathematical history, she had been able to get away with little work, but now recognized that post-secondary is a concern because she had not learned how to work to succeed.

Gabrielle

Gabrielle was one of the ELL students in this Pre-Calculus 11 class that was interested in developing her English skills, as well as working to succeed. For this reason, she often chose to sit alone and preferred to sit near the front of the class. From early on, Gabrielle let me know that she was going to be a surgeon. Thick anatomy and biology textbooks often accompanied her to class. She was hesitant to speak aloud during class discussions but, like Kira, I could often see her mouthing responses. Gabrielle and Kira also shared other traits: they wrote down as much information as they could, whether from the textbook or from the board; they only wrote in pencil; they always filled every page with information; and their written work looked rushed and often messy.

How they differed was that Gabrielle had had previous experience in Europe with some of the mathematical topics that composed our curriculum. There were topics she had no exposure to, such as factoring, but overall her background and mechanics were strong

and she was keen to work and learn. She was also keen to develop her mathematical vocabulary, and this was evident in her notes. Most often she wrote the words or phrases she wanted to learn to the left on her page. Even though she copied sentences and phrases from the textbook, she both re-ordered and re-organized them. Also, she would define vocabulary with both text and numerical examples. (see Appendix B, §16) demonstrates that while some of the textbook work was copied, it was not copied in the same order as the textbook, nor is every number or word copied.

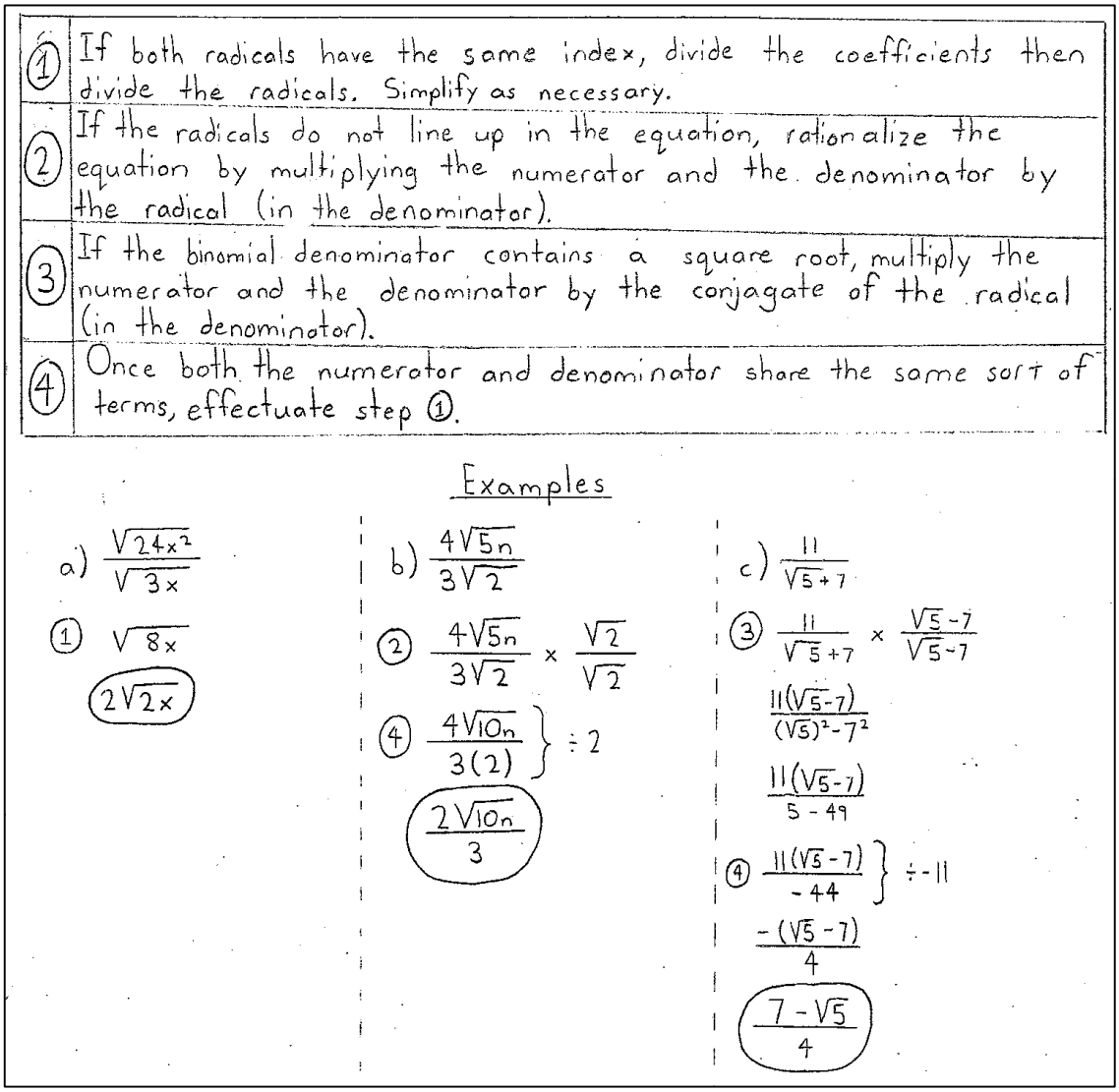
What was also evident from Gabrielle's notes was the difference between creating reading notes for a lesson versus summarizing notes for larger tasks. For example, when she read and wrote notes during class, there were exclusive imperatives and, depending on the section, she would copy the pronoun 'you' if the textbook employed it. However, for tasks such as the trigonometry conversation, Gabrielle was conversational, clear, and her written work did not have the rushed feel of her daily notes. (see Appendix B, §17)

This portion of the dialogue demonstrates the use of the pronoun 'you' in an inclusive manner. While there are aspects of the conversation that evidence background abroad, such as 'graphics section,' Gabrielle took time to describe the Cartesian plane clearly. She also connected the graphics section to new information. (see Appendix B, §18) Gabrielle's conversation was comprehensive and covered three written pages. While the handwriting may have been difficult to read, the conversation was not and it was accurate, informative and very inclusive.

Gabrielle was sick and absent for most of the last month of class, but kept up with reading and note taking as it was an accepted part of our classroom routine. Even when absent, she used different strategies, like Clone an Author, to make her work interesting and engaging. As an example, for the sine versus cosine prompt, she wrote a question or term on a stickie note, and the response beneath. Despite only having used pencil for all her assignments, Gabrielle used colour and a heavy card stock for her final project. When there were directions on her final project, she used exclusive imperatives as the textbook. When there was explanation, she was inclusive and concise.

Bryant

A confident student who often arrived late to class due to his many extracurricular activities, Bryant was polite and friendly but when he was in class, his attention was given to his academics. In particular, his attention was given to his AP Science courses. While Bryant knew that he required mathematics for his future in Science, he made it clear that he had little time to develop his mathematical skills and understanding. During lecture time before this exploration, he would have at least two notebooks open; one for recording any new mathematics, and another for his current Science assignment. When I commented on this to him, he let me know that it was not my fault, it was just that he hated mathematics. He did plan to obtain a high mark and he let me know that he was fairly certain he could do that. What made this comment interesting for me was that Bryant told me on several occasions that he did not review nor study for mathematics tests. Initially his assessment scores were high and only contained simple mistakes such as 4 x 5 = 15. However, as the course progressed, Bryant began to stumble, particularly in understanding what questions were asking. For example, on a quiz before the exploration began, Bryant could not answer a question because he did not understand what perfect square meant.



1	If both radicals have the same index, divide the coefficients then divide the radicals. Simplify as necessary.
2	If the radicals do not line up in the equation, rationalize the equation by multiplying the numerator and the denominator by the radical (in the denominator).
3	If the binomial denominator contains a square root, multiply the numerator and the denominator by the conjagate of the radical (in the denominator).
4	Once both the numerator and denominator share the same sort of terms, effectuate step 1.

Examples

a) $\frac{\sqrt{24x^2}}{\sqrt{3x}}$

1 $\sqrt{8x}$

$2\sqrt{2x}$

b) $\frac{4\sqrt{5n}}{3\sqrt{2}}$

2 $\frac{4\sqrt{5n}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

4 $\left.\frac{4\sqrt{10n}}{3(2)}\right\} \div 2$

$\frac{2\sqrt{10n}}{3}$

c) $\frac{11}{\sqrt{5}+7}$

3 $\frac{11}{\sqrt{5}+7} \times \frac{\sqrt{5}-7}{\sqrt{5}-7}$

$\frac{11(\sqrt{5}-7)}{(\sqrt{5})^2-7^2}$

$\frac{11(\sqrt{5}-7)}{5-49}$

4 $\left.\frac{11(\sqrt{5}-7)}{-44}\right\} \div -11$

$\frac{-(\sqrt{5}-7)}{4}$

$\frac{7-\sqrt{5}}{4}$

Figure 6.33 Example of Bryant's notes

Bryant was absent the first week of the reading exploration, but reading a textbook and creating written notes seemed to fit well with Bryant's style of learning. The previous image is the first reading task that Bryant completed and further written notes from this exploration followed the same format.

Bryant always created a flowchart style of procedures where at least one step could loop back to another. Numerical examples, that were copied from the textbook, were only included to clarify particularly new ideas. They were never used to define terms or clarify vocabulary and appeared to be included only when necessary. Bryant had been away sick for the chapter thus far, so he copied the above three textbook examples.

As the reading exploration progressed, Bryant did not have two different subject notebooks open on his desk concurrently. Rather, he would concentrate on reading his mathematics textbook and making his notes. If there was a class discussion, he would contribute his ideas, particularly when he knew the answer. While Bryant seemed less distracted by his Science courses, he was still continuing to 'do' his mathematics.

Even with his reading notes, he seemed to be completing rather than working for understanding. Interpreting questions or instructions still caused him difficulty or meant that his written notes were half complete. In examining the scope of his work, it is not the fact that he did not use proper terminology in his writing, it is that he did not take time to define or clarify what terms or key words meant. These were always impediments to his understanding and higher assessment scores. All of Bryant's written notes were factual, declarative statements. He did not use pronouns, even when writing notes for a friend.

Antonio

He arrived in Canada and started at our school the semester before this exploration. I became acquainted with Antonio as a TOC teaching Grade 10 mathematics. Until his mother arrived, he was alone to acclimatize to Canadian living. The habits he developed were not conducive to a successful academic career. Once his mother arrived, we had several meetings and discussions with him to try to alter his career path. While these meetings were fruitful in that his mother understood the situation and her son's behaviour, they were never enough to change his attitude toward engagement with his mathematics learning. I was surprised when I saw him walk into my Pre-Calculus 11 classroom as he had barely passed Grade 10 mathematics. He was one of the students who sat at the back of the classroom and tried to avoid work. Worse, he was never prepared for class and always tried to use class time to catch up on sleep.

The reading exploration was positive in that it afforded me class time to nudge Antonio in forward motion, which allowed him to remain current. When it came time to read the textbook and create his own notes, he was a minimalist, and his notes reflect this. His written work was never longer than a quarter of a page. There were no pronouns nor personalization of the mathematics on any of his first notes, and the bits of text and examples that he did copy were not connected nor complete and did not make sense. His initial refusal to try to learn some vocabulary was problematic.

What I found in hindsight after considering Antonio's notes collectively was that the more direct the reading tasks were, the better his written work. For example, his responses to the explicit reading prompt on simplifying rational expressions was thoughtful and complete. On the prompt sheet, he squared and underlined some words, such as 'Big Idea,' and 'mathematical terms.' Perhaps it was a comfortable structure, or perhaps it was focus that he needed that other students did not, this cannot be known. However, as in the example in (see Appendix B, §19), he had read a textbook passage and responded simply to the prompts I gave. He also responded in complete sentences.

Antonio did remain a 'problem' student, skipping classes as the weather became warmer. However, I learned from his written work and classroom engagement. When I taught traditionally, he did little, never completing homework nor taking notes from the board, but mainly trying to sleep in class. When we began the reading exploration, he would use his textbook grudgingly and made simple notes that were often messy with lots of scratched out work. However, he did become more engaged and part of the classroom culture by engaging in this task. When the reading tasks became structured and less independent, he experienced some success. I found that even his printing changed and there was less or no crossed out work. He also talked to his audience through the voice of the textbook, 'You can check..." to becoming personal with 'I didn't know..." In addition, from the beginning of this exploration to the end, he did complete the reading tasks, and some more completely than others.

May

Like other ELL students in this mathematics class, May was resistant to speaking English and speaking it aloud. She demonstrated strong mathematical skills. May started this term and began most classes with what appeared to be a scowl on her face as if she was not pleased to be there or told what to do. However, she did try, even when she was unsure or frustrated. When we first began reading the textbook and making notes, she would not start. I spoke to her and mentioned that it was okay to make a list of terms that you want to know with their translations. She did and her first written sample listed some vocabulary. (see Appendix B, §20)

She also read my prompts for information, and would circle key words. Following this, she would endeavor to write sentences, though her English was developing.

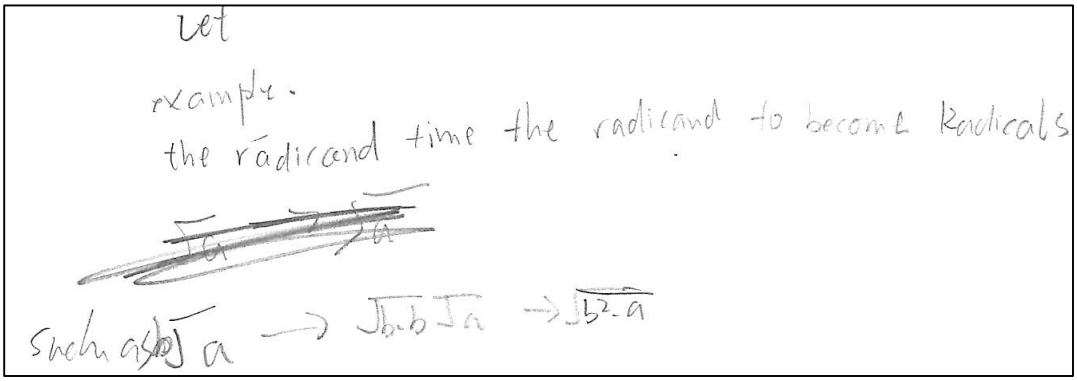


Figure 6.34 May incorporating written text

May missed a great deal of class this semester, but when she was in class, she worked hard to read and interpret the textbook. She did not hesitate to ask questions when clarification was needed, and when she began speaking in class, it was with a smile.

Kira

A very quiet, high-achieving student, Kira never spoke aloud in mathematics class even when you could see her mouthing the right answer. She would answer if you called upon her, but very softly and hesitantly. Kira consistently worked hard and maintained an over 90% average. Her goal was to become a ballerina and through her parents I learned that she was extremely good, but part of this meant that she danced at least 4 hours a day. She used her class time wisely to attack and complete anything that was assigned. She usually sat alone, and worked continuously.

Kira's work for this exploration was complete as she was never one to fall behind nor miss an assignment. She only used pencil to complete her mathematics work; never anything else. Visually, she worked to complete her assignments, not to have them published. Her written work was tiny and her page was always full, sometimes messy. The one thing that differentiated her written work from many of her classmates was that she wrote a lot. It seemed that everything was important to her.

As the exploration progressed, Kira's notes remained lengthy. A few characteristics did evolve: for example, she began to add questions to her notes, like 'Why use prime factorization to break it [a rational expression] down instead of crossing out values through division?" She was examining a process different from hers and questioning it. Her notes also evolved to be formatted like a double-entry journal. She would copy the textbook example on the left side of the page, and then explained the process or what

needed to be done or even defined important terms on the right. This was not a format that I spoke about, nor modeled. Over time, her process evolved and her explanations became shorter.

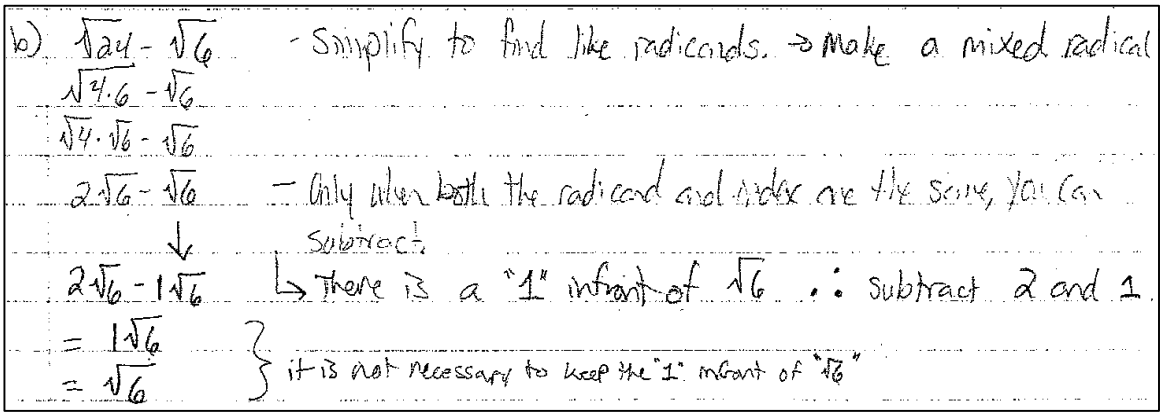


Figure 6.35 Kira's notes separating written text and numeric examples

What is also interesting about reading Kira's notes is that the tone is very removed. With the exception of a couple of copied 'you' pronouns, she did not use any pronouns. Even the question above that asked why prime factorization would be used, is not addressed to anyone. That is, it is not clear if she was thinking, "Why would I use…?" compared to "Why would the textbook writers use…?" Her written work tends to follow the pattern, "To find/solve this, do/use this."

Kevin

Both during class discussions and independent work, Kevin was focused and did not talk nor share his ideas – another quiet class member. He did demonstrate strong mathematical skills and was a meticulous worker who ventured a few words to me when he wanted to understand where his work did not agree with the textbook answers. As the semester progressed, I found that he was a warm and sensitive student who was embarrassed by weaknesses. In this mathematics class, he felt his weakness was both spoken and written English.

From Day 1 of this exploration, Kevin was like his classmates where he read the textbook and created notes without question and completely on task. Unlike his classmates, Kevin created his own numerical examples. His first written notes stated or paraphrased textbook text but gave unique examples. As the study progressed, he began to ask questions of the textbook text on his notes while he continued to create his

own examples. As the exploration ended, he began to state his personal thoughts and opinions. The following example from his work is not taken from an exploration reading prompt, but a separate, photocopied reading where my students were questioning mathematical vocabulary and how to explain it.

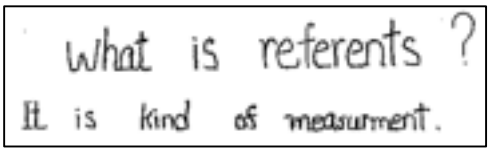


Figure 6.36 Kevin incorporating written text

What is interesting is that he compared what was stated to what was not stated. He ended this idea with:

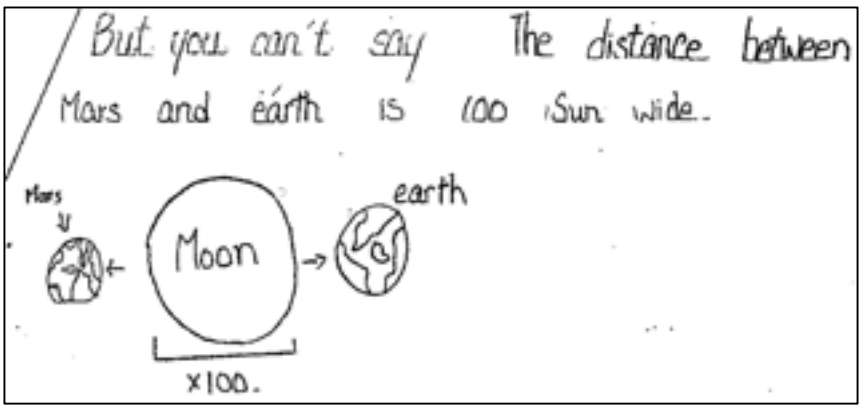


Figure 6.37 Kevin creating an example

For Kevin, it was not his mathematical abilities that changed, it was his communication and consideration of them that did. His work became very conversational, using declarative statements to describe and explain ideas, and he often used the pronoun 'you' as if he were explaining his work to his friend who kept missing classes.

John

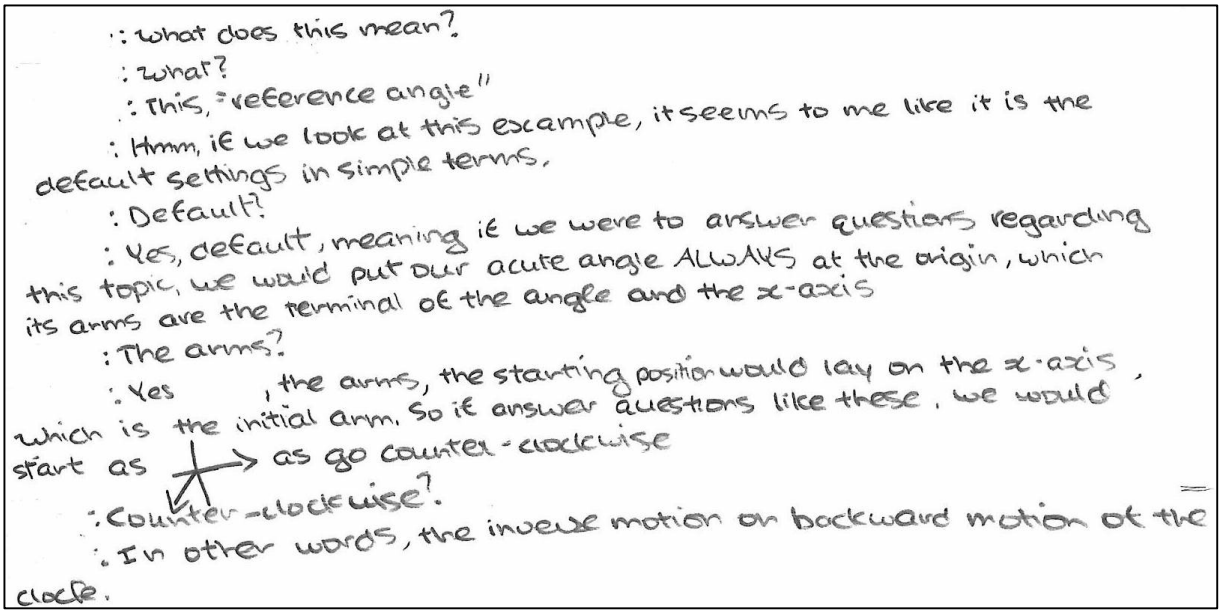
In a classroom of quiet students, John was not. He was enthusiastic and engaged every class. He always volunteered ideas, even if he was just guessing. When he did know the answer, I would often have to curb his enthusiasm to give his classmates thinking time. While he was not shy, he did not have any friends in the class but Ryann, who he sat beside every class. Like Ryann, John was academically minded, but his course load this semester was heavy. In addition to this Pre-Calculus 11 class, he was enrolled in 3 Advanced Placement (AP) courses; Chemistry, Physics and Psychology. While he

enjoyed this mathematics class and became quite the cheerleader for the reading and work that we were doing, his AP exams soon took their toll on his ability to complete his mathematics to the level he wanted. However, he was never deterred and while he knew by the end of this exploration that he was not doing his best mathematics, he did stay current and completed all of his reading tasks.

As with most of the class, John began this reading exploration the first day by literally copying the textbook text and examples. He continued to copy the textbook examples and words the next day, but he changed the exclusive imperative directions and the verb tense. For example, from “Use Prime Factorization” and “Express the radicand as a product of prime factors” to “We can solve by Prime Factorization by expressing the radicand as a product of prime factors.” This became characteristic of his work; he copied most text but added ‘we’ to the instructions to make exclusive instructions feel inclusive and changed the verb tense to indicate current work.

What was also characteristic of his work was his focus on formulae. He was the only class member who stressed formulae and attention to it. Whether this was an extension of the work and study he was doing for his Science classes is not known, but John did like to memorize formulae. Even in his trigonometry dialogue where he clearly talked to his friend, he told them the formulae for finding reference angles. (see Appendix B, §21)

John was also like other class members in his concern with understanding vocabulary. In his written notes, he would often copy definitions but did try to personalize the meaning. For example, in his trigonometry dialogue, which was one of the clearest examples of his voice and precision, he began by having his friend ask:



: What does this mean?
: What?
: This, "reference angle"
: Hmm, if we look at this example, it seems to me like it is the default settings in simple terms.
: Default?
: Yes, default, meaning if we were to answer questions regarding this topic, we would put our acute angle ALWAYS at the origin, which its arms are the terminal of the angle and the x-axis
: The arms?
: Yes, the arms, the starting position would lay on the x-axis, which is the initial arm. So if answer questions like these, we would start as as go counter-clockwise
: Counter-clockwise?
: In other words, the inverse motion or backward motion of the clock.

Figure 6.38 John's voice

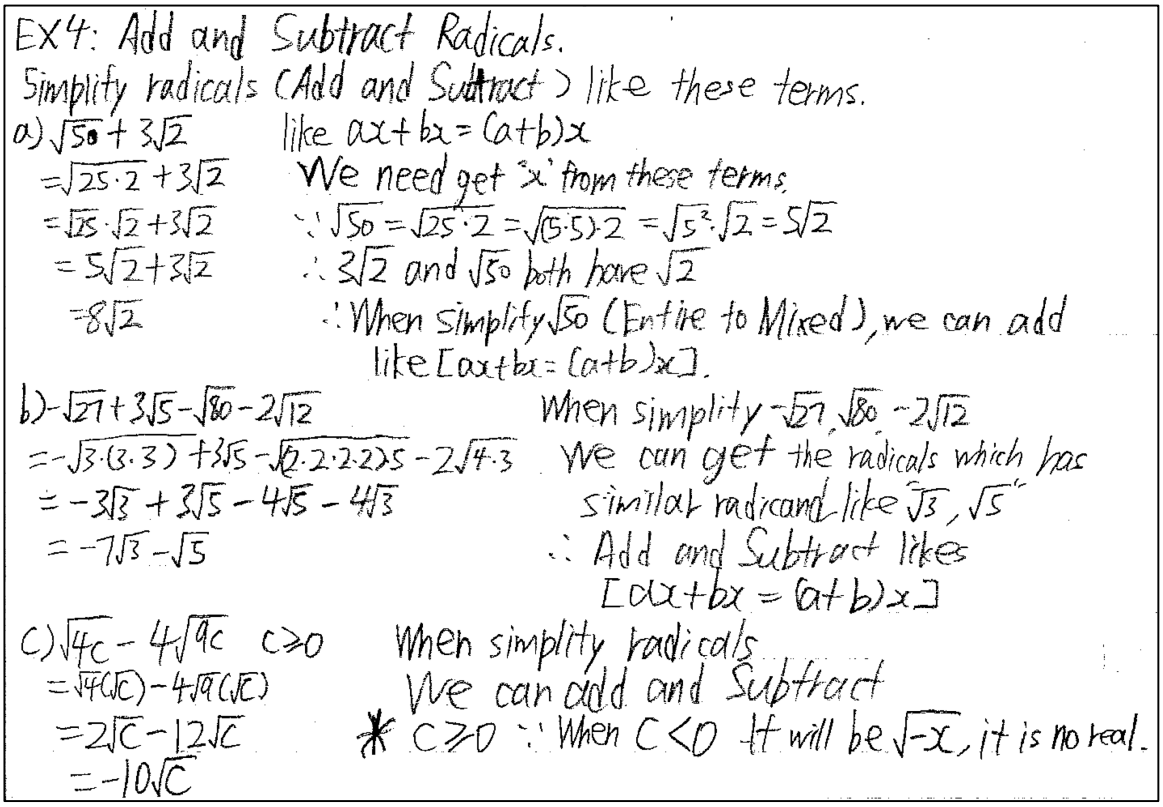
While John did write many of his notes to be inclusive and with a focus on terms and algebraic representations, it was clear he was personalizing the meaning of the mathematics.

Ryann

A punctual, responsible and attentive class member, Ryann always completed every assigned task. If he was away or unsure of an assignment, he would come in after school to clarify that he knew what was due next day. He had to sit at the same desk every day and always sat next to his friend John. While he was mathematically strong, he was socially weak, not understanding social cues and often John clarified for him. Ryann was also an ELL student, wanting to understand both English and the mathematics register. On his term reflection he stated that, "I will improve my English than (*sic*) I can know the terms." Once we began the exploration, he wrote, "I need to prasice (*sic*) how to explain the statements clearly, too". He was committed to learning and improving.

Ryann did literally copy the textbook for the first reading assignment. Unlike his classmates, he copied all of the first two pages plus he accurately completed the Your Turn questions that day. His ability to write that much while successfully completing independent work indicated his comfort and ability with mathematics. What changed

quickly for he was his inclusion of personal voice. For example, while he copied the text and numerical examples the next day, he included his own explanation (see below).



EX4: Add and Subtract Radicals.

Simplify radicals (Add and Subtract) like these terms.

a) $\sqrt{50}+3\sqrt{2}$ like $ax+bx=(a+b)x$

$=\sqrt{25\cdot 2}+3\sqrt{2}$ We need get "x" from these terms.

$=\sqrt{25}\cdot\sqrt{2}+3\sqrt{2}$ $\because \sqrt{50}=\sqrt{25\cdot 2}=\sqrt{(5\cdot 5)\cdot 2}=\sqrt{5^2}\cdot\sqrt{2}=5\sqrt{2}$

$=5\sqrt{2}+3\sqrt{2}$ $\therefore 3\sqrt{2}$ and $\sqrt{50}$ both have $\sqrt{2}$

$=8\sqrt{2}$ $\therefore$ When simplify $\sqrt{50}$ (Entire to Mixed), we can add like [$ax+bx=(a+b)x$].

b) $-\sqrt{27}+3\sqrt{5}-\sqrt{80}-2\sqrt{12}$ When simplity $-\sqrt{27}, \sqrt{80}, -2\sqrt{12}$

$=-\sqrt{3\cdot(3\cdot 3)}+3\sqrt{5}-\sqrt{(2\cdot 2\cdot 2\cdot 2)5}-2\sqrt{4\cdot 3}$ We can get the radicals which has similar radicand like "$\sqrt{3}$, $\sqrt{5}$"

$=-3\sqrt{3}+3\sqrt{5}-4\sqrt{5}-4\sqrt{3}$ $\therefore$ Add and Subtract likes [$ax+bx=(a+b)x$]

$=-7\sqrt{3}-\sqrt{5}$

c) $\sqrt{4c}-4\sqrt{9c}$ $c\geq 0$ When simplity radicals

$=\sqrt{4}(\sqrt{c})-4\sqrt{9}(\sqrt{c})$ We can add and Subtract

$=2\sqrt{c}-12\sqrt{c}$ * $c\geq 0$ $\because$ When $c<0$ it will be $\sqrt{-x}$, it is no real.

$=-10\sqrt{c}$

Figure 6.39 Ryann creating written text to explain his work

When Ryann copied, his written text was sometimes inaccurate as he pieced together bits of sentences as, for example, his trigonometry dialogue. However, reading the textbook provided time for him to ask questions of his friend John and of me. As an ELL student, it also gave him time to read, consider, translate and assimilate the mathematics. For a high functioning mathematics student with language and social-cue barriers, reading before class discussions facilitated his ability to understand and follow the post-reading discussions. In this way, he began to contribute his answers to our discussions, and being able to speak in class let him practice both his English and social skills.

Chapter 7. Emerging Themes

"You can't avoid thinking when you write." (Alber, 2014, p. 2)

It is evident from the student analysis that reading the mathematics textbook in this research not only afforded students an opportunity to use their mathematics textbook for more than a source practice and homework questions, but it also allowed them to use the textbook as a learning tool. All students participated in this exploration by reading their textbook and creating written notes independently. The spectrum of written responses to these readings ranged from the significant personalization of textbook information to the literal copying of textbook text.

What was unexpected were the resulting changes in classroom dynamics, discussions and mathematical talk, as well as how individual students handled the responsibility of creating their own notes from their textbook readings. In an effort to understand these varying experiences, the emerging themes will be discussed and connected to educational theories. These themes - of personal mathematics reading, of literacy as inherent in the mathematics classroom, of increased student involvement and student autonomy, and of increased use of the mathematical register, development of community and student voice - are examined here.

7.1 Addressivity

What is the voice of the mathematics textbook? Whom does it address and how? How do students address one another in a mathematics classroom? In their written work, whom do they address? What audience do they write for? How do these audiences compare?

As my students wrote notes over the ten weeks of this exploration, their written work evolved and, for many, their written voices also changed. The trigonometry dialogue task was notable, as it proved a pivotal moment of change for many students. In the writing of these dialogues, students were 'freed' from the structure of the textbook and created text that was not anchored to its shape. While I had told my students, and even written reminders on some of their prompts that they did not have to follow the format of the

textbook, they most often did. When they wrote their dialogues, the main similarity of their work was the topic and the tense, not the format other than that of a dialogue.

How did the addressivity of the textbook compare to the addressivity of my students? Were their audiences common? Addressivity is a term coined by Bakhtin (in Pimm et al., 2008, p. 1) to describe his claim that every human utterance is necessarily addressed *to* someone. In what follows, I will examine the trigonometry dialogue task to understand who my students were addressing. For example, were they addressing me (the teacher that assigned the task) or were they talking to a friend, to themselves or to the whole class. To evaluate this written task, I will cross-examine the dialogues for discourse features such as pronouns, deictic markers, modal elements, hedges and verb tenses, as well as some of the dialogue features like the 'friend' responses. I will also consider the extent to which my students adopted the communication style of the textbook.

Addressivity of the textbook with respect to the dialogue

Briefly, the textbook pages that my students read for this task to create their dialogues had the following characteristics. The explanatory information for this task was on pages 77 to 78, though I did not dictate these pages. The preceding pages also held information and diagrams on angles in standard position that some students did read and include. For example, Elisa directly copied two of the 'Focus on' statements that appear at the beginning of this section, on page 74.

With respect to the literary features of pages 77 and 78 in the textbook, the first thing that is noticeable is the lack of audience. For example, in the 20 lines of written text, there is only one 'you' and it occurs in the very first sentence. There are no 'I' or 'we' pronouns. This first sentence also has the high modality term 'can' as in 'you can generate'. This does swing the polarity of the idea to the positive. The lack of first- and second-person pronouns in these pages is interesting when you compare it with the fact that the adverb 'always' is used twice. The lack of pronouns removes the human element.

Following this first sentence, the text becomes very factual and clear, repetitive, and non-contextual. For example, from page 77:

> The starting position of the ray, along the positive x-axis, is the initial arm of the angle.
> The final position, after a rotation about the origin, is the terminal arm of the angle.

These sentences have no pronouns and few deictic markers. As with many sentences in this textbook, there is a parallel structure; both sentences begin with reference to the position of the ray, and they both end with "of the angle." It is like the following pages, 79 and 80, where the given solutions for Examples (a) – (c) are, and formatted the same as, "Since _____, the terminal arm of θ lies in quadrant __. The format is repeated and nothing is left to question. This reminds me of the phonics approach to reading; repetition to learn a skill. Or, as Ostler (1997) found, the textbook was providing the blueprint for the students to use to find success.

I find the use of high modality words in this section interesting. For example, if my students read the short, declarative paragraph at the bottom of page 74 that does not include any pronouns, they would have read the last sentence that states, "In this chapter, all angles will be positive." This is very authoritative and official and, again, it is a certainty that all angles will be positive. For me, this statement reads as question as the authors clarify that it is 'in this chapter' that angles are positive. When are they negative? And why this chapter? Was this sentence necessary? Who was it addressing? Was it to draw the line for the teacher so that they know only to teach naming positive angles? Or was it for the students? One of my students, Johns, may have read this sentence as he said to Ryann, "Yes, reason being is that if we go clockwise, our angle is negative, which we do not want". However, Gabrielle did not seem to respond to this, as she was happy describing and letting her friend state, "If the arrow is clockwise, then the angle is negative". To which Gabrielle replied, "You've got the idea, yes". What the textbook really seems to be saying here is that, "We know more than you, and for now this is all that you are allowed to experience because you are not ready for negative angles quite yet".

Student submissions not employing a dialogue structure

Twelve students were in attendance when the trigonometry dialogue task (see § 5.1, week 6 prompt) was assigned before a long weekend. Of the twelve, eight completed the task using a traditional dialogue structure where it was clear that there were two

different parties involved. The conversations from these students were explicit. The work of the other four students was different.

Three ELL students did complete the task by defining terms and attempting to make sense of the diagram in English. The two examples below are by the same student. The top diagram, taken from the prompt sheet, shows his clarification of the Cartesian plane being divided into the quadrants and how this relates to angle measure. The diagram below was on the back of the sheet and contains new vocabulary. That sketch was copied from page 77 and the phrase was copied from the Key term definition of ‘angle in standard position’ given on that same page.

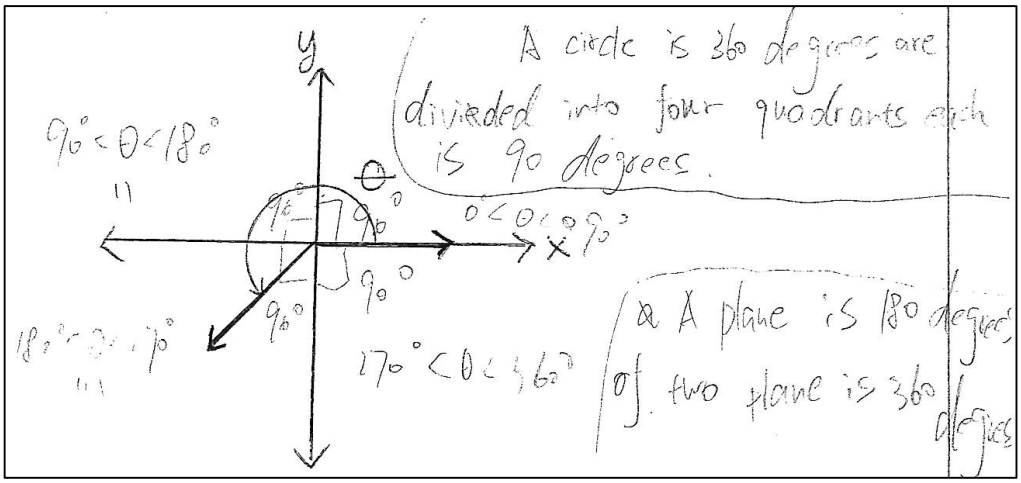


Figure 7.1 Understanding the Cartesian plane

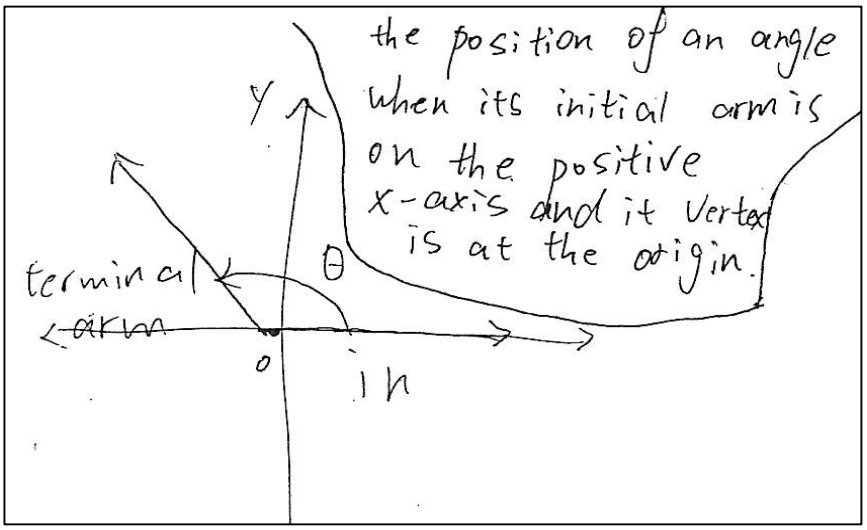


Figure 7.2 Copying text and diagrams

It is not known from the written work of the ELL students whether they misunderstood the task. If they only interpreted the first sentence of the second paragraph that asks, “What conversation do you have with them?”, then they may not have realized that this was to be in the form of a dialogue. This meant that they could address someone in person. For example, could they have been trying to explain the angles on the diagram of the Cartesian plane to a friend who was sitting next to them? This group of students had changed, as noted in the engagement section, in that they were now talking and pointing at one another’s work. Could this also be the case here?

In his written work, a third ELL student, Ryann, used the pronoun ‘we’ three times in seven lines of written text. However, as he had used ‘we’ in his written work previously, it is not clear whether this was intentional and in line with an implicit dialogue or not. What is interesting about Ryann’s work this day was that after his written text, he did abbreviate the textbook ideas into simple, hyphenated points. He labeled two diagrams with his ideas and used the diagrams and points to define two terms. It is a clear and singular departure from his usual copying of text and use of complete sentences. In addition, where his written text did not always make sense because it included partial or fused sentences, his diagrams here were clear and accurate.

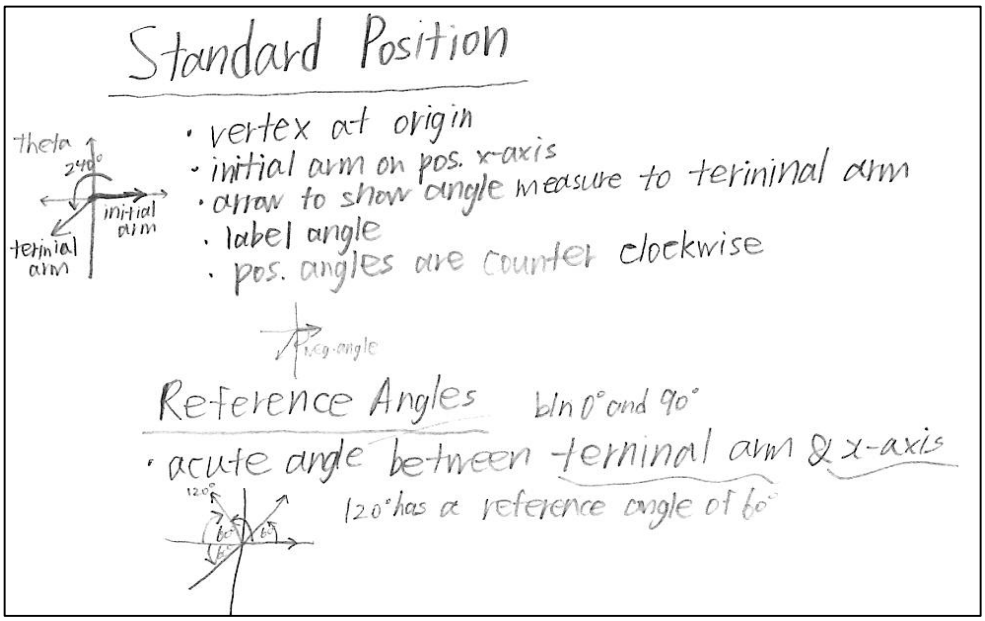


Figure 7.3 Ryann summarizing new terms

The fourth student who did not employ an explicit dialogue structure was Michael. His written work this day, however, was very different from his notes thus far, since it *was*

written text. Up to this point in the exploration, Michael's focus had been on numbers. From this task forward, he did include more written text in his notes.

What was also interesting about Michael's notes this day was that they could be read as conversational. He could be talking to a friend on the phone, but we could only hear his explanation of the diagram. His text is declarative, like the explanations he had read in the textbook. His conversation took place in the present and he used 'is' six times, and 'be' and 'are' twice each. He did incorporate modal words, like 'must' and 'can.' This indicates his belief that it was certain, for example, that, "One part of the angle must be on the positive side of the x-axis and have y = 0". This sentence and the sentence that follows, "The other part can be in any section of the graph", have deictic features, which means that his audience has understood his conversation and what he referred to. That is, 'one part' and 'the other part', for example, refer to the initial and terminal arms of an angle, respectively, and for the recipient to understand the meaning of these phrases is dependent on the context of the subject they are talking about and their shared mathematical experience. Michael's written work was his own and while his work was approximately the same length as any of his notes thus far, he had addressed the task as well as an audience.

Real dialogues

There were many interesting similarities and differences in the explicit dialogues written by the other eight students. On first comparison, what is noticeable is that each student's dialogue and organization of content was particular. For example, the textbook section is titled "Angles in Standard Position" and that is the focus for four pages until reference angles are defined. John's, however, began by having his friend open the conversation by asking about reference angles. He went from the small, acute angles, then to describing the motion needed to rotate the terminal arm to how to calculate find angles on the Cartesian plane. His organization was a complete departure from copying the textbook and his connection of the static image to the motion of a clock was singular.

Unlike John's, the remaining dialogues began with the authors, my students, opening the conversations. While Chloe began her conversation politely, she took an authoritative, teacherly stance, "Today I'm going to explain to you ..." and began with standard position. Kira took the approach of a teacher by referring to the picture and explaining that there are two parts to it. In this way, she allowed her friend to choose the direction of

the conversation, or learning experience: “Friend - Let’s start with standard position, what is it?” Serena, in contrast, began by detailing quadrants. Her conversation was definitely with a friend, as her language sounded like students talking, involving many ‘so’s’ and ‘Okays’. ‘Yeah, so you see the other arrow?”, to which her friend replied, “Yeah, what about it?”.

As I continued to read their conversations, I noted that five of these eight students wrote a whole page of dialogue. While John’s page read as a complete conversation, Chloe’s did not. In taking an authoritative stance, Chloe copied much of her work directly from the textbook. However, as with her copying of radicals on the first day, she meshed parts of textbook sentences together and, in the process, several lost their meaning. For example, “The final position that the terminal arm rests is the acute angle”. She did end the conversation by directly copying two sentences from the textbook. Her only inclusion and intrusion into these sentences were also authoritative:

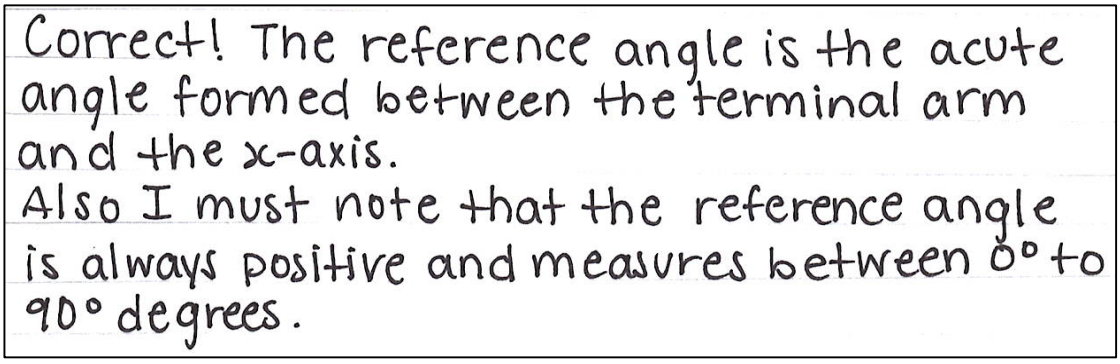

Correct! The reference angle is the acute angle formed between the terminal arm and the x-axis.
Also I must note that the reference angle is always positive and measures between 0° to 90° degrees.

Figure 7.4 Chloe copying text and adding her voice of authority

Following this, her friend was reportedly very grateful and let Chloe know that she could make sense of the diagram now. Despite their mistakes or misunderstandings, what I did notice about these dialogues was that the format allowed for a great range and variety of approaches, as illustrated above, for my students to explain what they had read and noticed in this section.

The friend

One striking similarity across the eight dialogues was the way that the ‘friend’ served as a foil, or a prompt, for my students’ conversations. I was curious about this. To compare the contribution of the friend with these dialogues, I counted the total number of lines of dialogue written by each student and then the number of lines granted to the friend. For

reference, the typical lined page that my students wrote on was thirty-four lines long. While each of my students would have a different number of words per line due to varied writing styles, it was more informative to see the relative proportion that the friend contributed to the conversation.

In agreement with their other written notes, Kira and Gabrielle wrote the longest dialogues. If I considered Michael's implicit conversation as a dialogue, he wrote the least, which also matches with his work in general. Overall, my students' written dialogue length was in keeping with their written work thus far. What was initially most interesting about this task was the consistent way that my students made use of their friend. That is, they predominantly utilized their friends to prompt the conversation, posing questions that the authors could then and would answer. In the table that follows, the amount of space given to the friend's voice in these dialogues is dramatically similar. The table is organized from most written lines to least:

Table 7.1 Comparing student dialogue lengths and contribution of friend

Student Name	Number of lines of dialogue	Number of friend lines	Number of questions asked by friend	Percent - Lines of friend to total dialogue lines
Gabrielle	105	18	8	17%
Kira	48	8	8	17%
Grace	42	12	7	28%
Elisa	34	9	7	26%
John	29	9	4	31%
Chloe	25	8	3	32%
Serena	20	6	6	30%
Brianne	18	5	2	28%

Even with the varied dialogue lengths, my students were similar in the amount of space dedicated to the voice of their friend. Six of the eight students dedicated just under a third of the dialogue to the voice of their friend. The two students who consistently wrote the lengthiest notes allotted less space to their friend, closer to a sixth of the dialogue.

What was also noticeable is that their friends predominantly asked questions. While the numbers are not completely alike, the dialogues themselves show that the friend was not there to talk, but to provide breathing space for and encouragement to my students, and to prompt the conversation. Overall, of the line space allotted to the friend, roughly 60%,

were to ask questions. The remaining 40% were mainly phrases or short sentences of 1–4 words such as, “Oh, I see” or “Ok, Ok, go on”.

From the questions the friends asked, it would be possible to know which student wrote the dialogue. For example, up to this point in the investigation, Elisa’s written notes highlighted and focused on mathematical vocabulary. Her friend only asked questions to clarify vocabulary, such as, “What do they mean?” and “Okay, that makes sense, but what are initial and terminal [arms]?” Grace’s dialogue included a question where she asked her friend to ask the teacher a question. This is in keeping with her classroom presence and the fact that she was shy about asking questions. This question was also thoughtful and matched the way that Grace tried to make sense. For example, in the dialogue, she wondered if the terminal arm could be where the initial arm had to be. Then she stated, “Because I don’t know! Wouldn’t that not be an angle anymore?”

To contrast the thoughtfulness of these dialogues, Serena’s was minimalist, in keeping with her written work thus far. While Brianne’s dialogue had fewer lines, she tried to address both standard position and reference angles, the purpose of the task. Serena did not include reference angles in her dialogue, but wrote six lines describing where quadrants I – IV were located. Her conversation reads as relaxed and easy going as she is, “The quadrants are on the graph where its split in four sections [...], then it goes left down into the bottom left”. However, eleven of the twenty lines she wrote were single ones that do not explain much, such as, “Okay, so what’s next?” and “Is there anything else?”, which was repeated twice.

John’s dialogue is also clearly by John. His involvement with his Science courses, and his drive to be admitted to Waterloo in the Sciences, was always reflected in his written notes. In his conversation with his classroom seat-mate, John used terms such as ‘default settings’, motion, and focused on formulae. Even though John’s numbers, for example, only indicate that his friend asked four questions, the remaining five lines were of the nature, “Oh I see”, “Ohhhhh” and “Yes, I see.” These utterances were used to illustrate that his friend was part of the conversation and that they both understood the explanation. John and Ryann did relate like this in class. In general, any lines the friends spoke that were not questions demonstrated support for their friend’s explanation and familiarity with the speaker and with the content.

Addressivity of the student dialogues: some highlights

In contrast to the textbook, these are noticeable features that my students included in their dialogues. One thing is clear - that they are addressing someone. Chloe, for example, was addressing the class as an authority; Grace was addressing both herself and the teacher; Elisa was addressing herself. Overall, my students were talking to a clear audience, whereas the textbook audience is not clear.

Chloe's dialogue allows her to demonstrate her knowledge. She speaks with authority, and much of that comes from her direct copying of textbook text. The few words that are hers are also of high modality like the textbook. For example, before copying the textbook, she adds, "Also, I must note […]". Chloe also gave her friend the voice of confusion and inferiority: "Oh thank you, I'm so confused" and "Yes! I can see that from the image that you provided. Thank you so much. I can make sense of it now."

In contrast, Elisa's audience was also Elisa. She too used high modality words, for example, "First, you have to […]" However, while this reflected the authoritative tone of the textbook, it was in the context of Elisa's lack of confidence and her focus on mathematics having to be learned through memory. For example, she wrote, "Make sure you memorize them". Elisa was also the only student to copy two 'Focus on' statements from the beginning of page 74. This indicated that she had read more than most of her classmates and also it is a reminder to herself of what the big ideas are that she needed to know to pass this section.

Gabrielle tended to use lower modality words such as 'should' and she used them often: "You should also have it on your angle […]" and "You should also label the angle". In this section, and in most sections, of the textbook it did not use 'should'. The use of these terms tended to soften her dialogue. Gabrielle had a confidence with mathematics that Elisa did not, and Gabrielle's voice was not authoritative, but it was knowledgeable. What was also very interesting was that Gabrielle asked as many questions as her friend in this dialogue. She encouraged conversation. While Gabrielle's audience could have been a friend from class, as they shared common knowledge and connected their conversation to the classroom, Gabrielle's audience could also have been the whole class. At times, she offered both the encouragement and the voice of a teacher.

All student dialogues incorporated first-person pronouns. They contained the human element. Kira, who wrote a lengthy conversation, employed the smallest number of personal pronouns. Her audience was herself. She defined everything in context and left nothing to be guessed. For example, “Well, on a Cartesian plane, a graph, you can make an angle by rotating arms from the origin, x = 0, y = 0”. Kira wrote a lot, typically, and she had learned to put definitions into her own words. However, when she directly copied, it was because she liked to memorize. To indicate that she was talking to herself, and to memorize the sentence, she wrote, “Remember, if the vertex of an angle is at the origin of the coordinate grid and its initial arm coincides with the positive x-axis, the angle is in standard position”. This sentence is copied from page 77, though the last phrase is first in the textbook.

Grace’s dialogue was interesting because she was talking both to herself and to me. Her dialogue contained the most hedges, words that indicate a degree of uncertainty or tentativeness, which very clearly reflected Grace’s personality in the mathematics classroom. Sentences such as “Ummm, Well, it could be along that negative x-axis …” indicated that it was possible, but that she was unsure. Grace’s conversation also contained the most deictic markers, making it clear you had to be part of the conversation to understand the context. For example, “What about the other arm?” requires the context of an angle rather than a body. “Or it could have no intercepts and go up or down on a slope that isn’t 0, something like that” can only be understood if you knew they were trying to understand angles that are not in standard position.

7.2 Engagement

According to The Glossary of Education Reform (2016):

> In education, **student engagement** refers to the degree of attention, curiosity, interest, optimism, and passion that **students** show when they are learning or being taught, which extends to the level of motivation they have to learn and progress in their education.

Summarily, and from my perspective as an educator, engagement means being there and being present in attention, attitude and thought to either carry out the thought or the task. One of the themes that comes strikingly from this data is that of engagement. Both from my field notes and from the fact that all students completed all reading tasks when they were present in the classroom, (and even when they were away some would read

and give me notes without being asked) speaks to engagement. In addition, the increase in contributions to class discussions from class members without prompting was a dramatic change in this class of quiet students. Even the students I called upon, like our ELL students who were not comfortable responding verbally, but nevertheless did, was a clear departure from their dozing during class when I taught traditionally.

It is important to restate here that my students did not know that I was interested in their reading of the textbook or their written notes: that is, their engagement with their textbooks was not influenced by my research, because they did not know that I was exploring reading in the mathematics classroom. This was a blind study and my influence as a teacher during this exploration was simply that of a mathematics teacher. As mentioned in Chapter 5, I was concerned that they would resist or refuse to read their textbooks: after all, reading a textbook and making notes is hard work. Yet every student participated and not one considered note-making to be *my* job.

As described through the individual student analyses, the level of engagement of my students changed. Overall, they were more engaged through reading their mathematics textbooks and creating written notes, as compared with when I taught traditionally for nearly the first half of that semester. Engagement manifested itself in many forms: students stayed in class rather than working in the support room; quiet students began to speak in class, both during discussions and during work time; more students asked questions during and outside of class time; class discussions involved everyone; as the exploration progressed, the class as a whole became so engrossed in reading their textbooks that on two separate occasions when other teachers walked in, they thought my students were writing tests because they were so focused.

How does the textbook speak to engagement? As with any book, it is as engaging as the reader's interest in it or in its subject. Mathematics does not tend to be a popular subject and, on its own, its textbooks are not often a reader's choice. However, the authors of the MHR textbook try to engage students in a few ways. Pronouns were not used frequently throughout the textbook and, though the pronoun 'you' was rare, it could be used to engage, as for example, "What do you notice about this calculation?" (p. 99). When found in the Investigate portions of each section, it was being used to attempt to engage students in those tasks. However, most of these tasks in the textbook required teacher direction, explanation and/or organization.

Colour photographs were also used to attract attention and there were at least one for every two-page spread, with more photographs to be found near the beginning and end of each section. In addition, every section of every chapter began with photographs of content related to but outside of the course material, followed by a paragraph or two of explanatory text. These pages were meant to convey interesting facts about the textbook mathematics that followed as it is applicable outside of the textbook. Also, to create movement and student engagement to mathematics outside the classroom, web links provide further information on some topics.

Class engagement before the exploration

Before the reading exploration began, there were only two students who voluntarily engaged in what might loosely be called 'class discussion'. Chloe and John would always respond to a question or say something, especially if the extent of quiet time lengthened. When I called upon other students, like Grace, she would never take a chance and would typically shrug her shoulders rather than say anything. While she was a strong student outside of mathematics, Grace was afraid of making mistakes. Michael, while he would say something if called upon, would yawn and start to fall asleep if I talked from the front of the class for too long. Students like Antonio, hidden at the back of the class, would just fall asleep. He did not like mathematics and only wanted to pass. He could get by with doing minimal work. Bryant would work on two different subjects, while Landon tried to catch up on mathematics work from the day before and take notes concurrently. Even more serious students, like Kira, would just write down everything that I said or wrote on the board, but there was no reason for her to engage in the community of learning. While she was gaining information, she was only passively receiving what I was telling.

It is a juggling act even for the consummate teacher to focus every student's attention consistently and constantly for the whole lesson while teaching. The behaviour of my students was normal. While there are different combinations of students every year that make some classes more verbal than others, it is still most often a minority of students who are consistently engaged in a lesson. Many of my students were studenting. (Liljedahl, 2014b). Through their educational experiences, students have learned to look like students. They know to look at the board, but that does not mean they are paying attention or listening. They know to use their pencils every so often, but that does not

mean they are writing notes or considering what has been said or written. When the teacher says, ‘Try this’, they know to look down and at least write the question slowly because, soon enough, the teacher will finish the example with the correct steps. Even when a teacher circulates to prompt students to try, there is no possible way to spend time ensuring every student will try.

Math, after all, is tough and complicated, and mathematics teachers feel they have to provide a variety of examples so that students have models for their homework questions. I have supported in several classrooms where teachers, using either tablets or whiteboard to offer notes and examples, wait the ‘required’ few minutes before completing the examples they have asked the students to engage with. While I was there to support certain students, I had the opportunity to circulate and talk to many others. I was surprised that even competent students did not often try to complete questions before the teacher didi. One young woman said that, “I want my notes to look good, so it’s better to copy”. Another said, “Oh, I know how to do it, but it’s more fun to talk while we wait. Besides, there is time to write it when the teacher does.” One teacher, to gauge their students’ comfort level with the work, would ask for a show of thumbs. Confident students confidently showed their thumbs up or thumbs down. Others would look around to assess the broader class view before voting.

Class engagement during the exploration

As mentioned in my field notes, I was surprised by the level of activity on the first day I asked my class to read their textbooks. They were independently engaged with looking at their textbook and reading it to consider what information they themselves would deem important to record as notes. They were also engaged with their neighbours as they asked one another questions, as well as working with me as they asked me questions. I was circulating and students such as Antonio stayed awake because I was not situated in one spot. Michael did not yawn because he did not have to focus solely on me or on interpreting my words while I stood at the whiteboard, talking and writing.

What I also found engaging about this first day, in hindsight, was that when I called an end to reading and independent note-making, students were engaged with my words because they had to reflect on what they had been trying to make sense of. In other words, during the debriefing of that day’s reading, they had an invested interest in

whether their understanding was fully or partially correct, complete, or even if what they attended to was deemed important by myself and their peers.

I did not recognize it on that first day, but the independent reading tasks created an engagement as a community of learners. As students became accustomed to reading their textbooks, the class discussions did actually become *class* discussions. Through reading the textbook, my students were developing a common language and they used it. When we debriefed any reading, it was the students who asked me direct questions, such as “Why did they drop the negative sign when they solved that equation?” or asked me to clarify “How did they find that number?” My students had developed an engagement with the ‘they’ of the textbooks, though I could not find that pronoun easily either in the textbook or in their writing. Again, in hindsight, my students were developing this community of learners which, at that point, was the ‘us’ in the classroom versus the ‘they’ who wrote the textbook. At the time, as I was engaged in our discussions and engaged with expanding on their questions and ideas, I did not think to ask my students who their ‘they’ was.

As the exploration progressed, there were varying degrees of engagement evolution. Of surprise to me were the quietest students. Before this exploration, I was concerned that I would never, for example, connect with Elisa. She was so quiet that you could forget she was in the classroom. However, as the reading exploration progressed, her voice and laughter could be heard in the room as her confidence emerged with her engagement. She was a motivated and independent learner, though fearful of mathematics. Being able to consider and organize information and then hear it again from multiple perspectives during class discussion, allowed her the opportunity to mesh or reinforce her ideas with ones being discussed. She also felt able to ask questions of myself, and her classmates. What was interesting was that when Elisa was away sick, she made notes without asking as she knew where we were in the textbook. On her return, she came in after school to show me so that I would know she was up to date.

Being engaged with the textbook was something that every student could do. It did not matter if they were a slow reader or an ELL student, they could find or express something from the textbook. Mathematics is not only written text, it is numbers, graphs and data, for example, and any one student could connect with some item or aspect of it. I learned by reading all my students written work that even the students whom I thought

were avoiding or completing minimal work were noting something. It was driven home to me, for example, through my struggling ELL learners, that listening and copying from the board was difficult. They could do the mathematical calculations, but the English did not make sense. Thus, it was easier for some of them to sleep when I taught traditionally. I have been told by counsellors and even by student guardians that mathematics would prove an easy mark for ELL students because it was not language dependent; the marks came from calculations and computations on their assessments. While there is definitely a language component to mathematics, it was neither viewed as nor understood as an increasing barrier to doing well in senior grades.

What was noteworthy for me was that my quiet students did not have to change their classroom natures to become more involved or engaged in mathematics. What did change through their engagement was their ability to be involved. My quiet ELL students did become engaged in different ways and at different times throughout the exploration. Antonio, for instance, wrote minimal text at the beginning, but wrote more and took greater chances when there were more explicit directives, such as “Note any mathematical terms that you want to remember or understand, and try to explain them”. He learned to search through any text I wrote and underline or circle key words and instructions. May and Kevin would do the same, but by the end of the exploration their written work was full of risk-taking. Kevin, as seen in my student analysis, compared the distance between Mars and the Earth in terms of the sun. His classroom engagement also changed as he began to leave his desk to ask me clarifying questions. He was a self-conscious student and did not do this before the exploration. May changed more so: she soon had a circle of friends with whom she was happy to share and talk. Her social confidence developed over the course of this exploration.

As with the ELL learners, my other quieter students became engaged to varying degrees and at varying times: Brianne did so first by remaining in the class and reading and talking to her friends about mathematics. She also stopped coming in at lunch to complain about mathematics, which proved a large part of allowing herself to engage in the course. Brianne and Landon were the first to become regular contributors to our class discussions. I was quite surprised the first time that Brianne put up her hand to state that the new step that differentiated simplifying rational expressions from solving rational equations on page 342 was, “What you do to one side, you do to the other”. The class had read the page independently and then noted one new or big idea. While

Brianne's proposal was not the same as other students, she had become comfortable and confident enough to volunteer her proposal, even after hearing other responses that were different from hers. She was able to engage in class discussions because she felt on a par with her classmates through reading the textbook and noticing an idea.

7.3 Communication

In the English classroom, or Social Studies, and even the Science classroom, students are accustomed to communicating their ideas. It is accepted that you will talk and share ideas in English, or read and peer edit another student's written work, for example. Students are not used to writing or discussing in mathematics classrooms and sharing, verbally in particular, can prove a risky event because mathematics is commonly viewed as predominantly right or wrong. You can have an opinion and offer it in English, but to share a wrong answer or faulty technique in mathematics is embarrassing and can be damaging to mathematical confidence.

The emerging themes discussed in this chapter were often entwined and developed coincidentally. For example, the increase in engagement of my students in class discussions led to a natural increase in oral communication of mathematical ideas. Of interest was that the increase in independent written note-making, which for my students developed into more general written mathematical communication, also led to an increase in speaking mathematically. As with the changes in engagement level being personal, the development of communication in this mathematics classroom for this reading exploration was also personal.

On the first day of this reading exploration, students tended to talk out loud and to each other. It was a different type of communication from their usual banter or questioning as they were now trying to find answers that were not simply computational. They were mainly asking questions about definitions and mathematical vocabulary, such as "What is an index?". Most students could read the words on this page, but could not make much meaning without knowing vocabulary from previous mathematics courses. This page did have key term definitions in the page margins, but these only concerned new vocabulary relevant to that chapter. The terms that concerned them were, however, not defined on this page, but in the glossary. Their initial struggle to try to remember past

terms was a mutual way for students to begin talking to one another as no-one could define older vocabulary.

The import of these first day discussions was large. First, I was not focusing the discussion: the students were talking about what they needed to know, not what I thought they should know. While there were several informal discussions happening concurrently, I was able to address the class as a whole both formally and informally, depending on the need. Secondly, it was the first day that Michael, for example, had spoken to his neighbours about mathematics. I heard him speak to the boy next to him and say, “I think you were in my math class last year. Do you remember what these were for?”. Another quiet student, Elisa, asked her neighbours questions, though she was not yet comfortable talking to me. My quiet ELL student, May, who did not normally copy notes from the board, once I spoke to her, was able to take time to translate mathematical vocabulary from the textbook into her native language.

While my quiet students were free to talk to their neighbours and ask me questions, the few who normally spoke in class continued as per normal. Their talk also focused on what they were reading and, again, what I noticed was that they did not have the technical language to communicate their ideas clearly. For example, Chloe asked, “Where did the little 2 come from?” in reference to an exponent.

The other two important ideas on communication that arose this day came from their written communication. After I collected and photocopied their work, I noticed that their written notes were overwhelmingly copied from the textbook. The students who literally copied the complete text that day had written notes that made sense because they were the textbooks words and examples. The ones who abbreviated their copied text often did not make so much written sense, such as Chloe’s or Antonio’s work. As will be illustrated, what did change as the exploration progressed was what, how or even if they copied. Their written mathematical communication did improve. And, perhaps, students are acclimatized to ‘taking’ notes in mathematics class. Even though I said ‘create’ notes, it could be that they were still more on ‘take’ = ‘copy’ mode.

What I also noticed from their written notes was what they had copied from the board during our class discussion. On day 1, I predominately organized this discussion, but my students were able to ask clarifying questions, such as why are there restrictions on

radicals. Although I have given traditional notes for many years, I realized this was the first time that I had actually taken time to view my students notes. I noted, for example, that Grace did not copy correctly:

Figure 7.5 Grace incorrectly copying from board

whereas other students, like Gabrielle, an ELL student, started out incorrectly copying, but fixed her mistake:

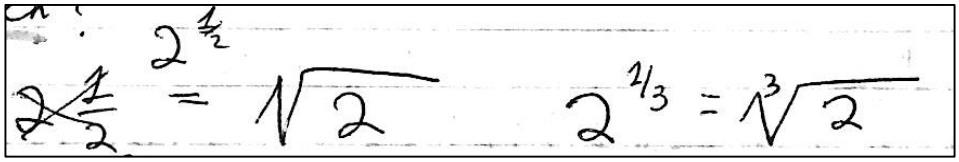


Figure 7.6 Gabrielle noticing and fixing incorrectly copied notes

Still further, Landon took his time thinking about and making notes, yet what he copied from the textbook and what he copied from board and recorded on the same side of a page were different:

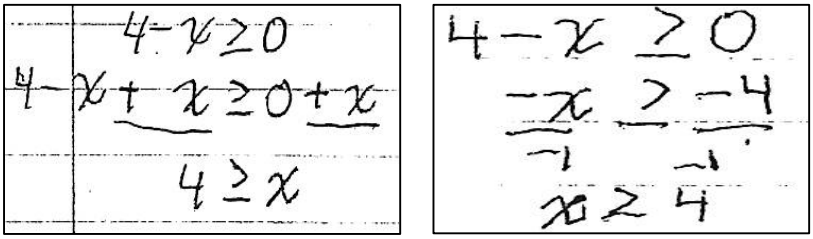


Figure 7.7 Landon correctly copying textbook and incorrectly copying from board

He did not copy that I circled the division by negative 1. When I collected their notes, it was always interesting to see how my notes were miscopied. Before this exploration, I thought that if my students only ever copied from the textbook that would be fine, as they were already copying what I wrote on the board. They would still be simply copying. Thus, it was interesting to find that students did copy incorrectly from the board.

Following this class discussion, my organization and structuring of future discussions became lessened class by class. As my students became more engaged and familiar with their textbooks, they had questions to ask and concerns to discuss and were less dependent on my telling them what to notice. Also, as time progressed, they were better able to communicate their questions, both orally and written.

It was the increase in the mathematical register during class discussions that surprised me. From the 'little 2' of day 1, their 'math-speak' developed. In hindsight, when they asked questions about what they had read, they had to communicate correctly to me, or I would not understand, or at least I pretended not to. Rather than say, multiply by $2x$, for example, they began to say, 'multiply by the lowest common denominator'. Sometimes they even defined what they said. For example, during the trigonometry unit, we were reading a passage together and I asked what they noticed. John quickly stated, "Key words of 'exact values'. Therefore, not science rounding." Not only had he noticed terminology that would impact the way he answered questions, he also put this into terms that were relevant to him. In addition, he was able to communicate this succinctly and in terms that the class had discussed and used previously.

7.4 Noticing

I happened to be writing part of my thesis during summer vacation. I was by the beach and during one of my breaks I watched a few fathers lead a group of about ten young children down to the beach. I could clearly hear them talking and, as they walked down, one of the fathers provided an overview of the area called 'beach'. When they stood on the 'beach', all ten children huddled around the fathers, who pointed and talked about what was around and what the beach was about. Soon enough, one small boy said, "Hey, dat cwab ith running way! (*sic*)". And he proceeded to squat over the crab to watch its slow progress. The young girl next to him asked him, "Why isn't that one moving?" When she received no response, she poked at another crab with a stick. When it still didn't move, she picked it up and began collecting dead crabs. As the fathers continued to talk and hold up things for the children to look at, a third child began throwing rocks into the water. Soon another began collecting and placing shiny shells in a pile, and then a little girl asked the group loudly, "Hey, why is there water squirting out of this hole?" One father said, "We'll get to that later". Undeterred, she began digging.

The mathematics textbook is like the beach; there is lots to attend to. For example, there are shiny, coloured photographs and diagrams like shells that attract attention; there are objects moving away from the textbook created by links to websites that move you outward, like the runaway crab; there are completed proofs that some view as objects like dead crabs; there are the green thinking prompts that are like holes in the sand that require further digging.

The above analogy can continue, but before it is left it is important to note that there are adults at the beach with the children. Some of their goals are to engage, inform and share with the children the beauty that is the beach. These parents direct the children's attention and help them understand the structure and organization of the beach, as well as teach safety skills and model methods of handling larger crabs so that they do not pinch you. All of this and more, while enjoying the subject of holiday.

It is well documented that children learn through what they notice, then they inquire about it and they experiment. As children approach high-school learning, their inquiring and experimenting phase can be limited or non-existent, particularly in traditionally taught subjects and classrooms. How does this textbook speak to noticing? The MHR textbook has many features to attract a student's attention: for example, coloured photographs of both mathematics being used outside the classroom and of interesting places, people and activities around Canada. The textbook also highlights key terms so that students will notice the important new vocabulary that they need to know. There are also bolded and differently coloured heading titles to notice as the student navigates the textbook. Overall, I feel there is too much to notice. As I mentioned in Section 4.5, concerning this textbook's structure, the readability of this textbook is diminished by the different factors vying for notice. What is also distracting after the textbook seeming to be structured to give students choice in what to notice, is the fact that every section begins with a "Focus on…" directive: that is, it is now telling the student what to notice.

Noticing prior to this exploration

Prior to this exploration, my students could only notice what they could infer from the notes I gave them, unless they chose to read and investigate outside of the mathematics classroom. As a traditional teacher of mathematics, I decided for them what was best to notice and organized these facts, examples and ideas in sectional notes that often

followed the organization of the textbook. The purpose of this was two-fold. If I followed the textbook structure, I could assign homework from the corresponding pages. Also, if I followed the textbook structure, I could hope that students might use the textbook for support if they needed help at home. I soon found that it was rare for students to take their textbooks home. Phones were used to snap images of the questions and answers.

As teachers of mathematics, we know that there are many different ways to, for example, teach factoring. However, we tend to teach the most efficient way or the way that in our experiences seemed to have worked best for our previous students. It does not work to teach all the different ways to factor at once, as students seem to think they are different and memorize each one as if it is to be used in different situations. They also cannot choose the best way as they are at a disadvantage. This technique is one of many tools and they are not sure of its use or ultimate purpose. Most students cannot notice what they do not really understand, so we notice and choose for them.

Noticing during the exploration

What was most interesting to me over this exploration was the varying levels of noticing that I noticed. For example, students would pay attention, or notice, different things on the same textbook page. Not only ideas, but also the way that the ideas were presented. For example, Michael and Amy focused solely on calculations for most of the exploration. Even when they defined a term, it was using a numerical example. In contrast, Grace and Gabrielle read the text and used words to explain numbers. While Elisa copied everything, her focus was primarily on definitions and mathematical terms. Chloe seemed to read everything, but she saw the words in the sentences as steps or a process and re-ordered them as such. Landon seemed to be the only one who noticed and considered the green thinking prompts (at least as judged by the notes they made).

What they had in common, that all tended to notice and record, seemed to be things that looked important, particularly algebraic definitions. Of the thirteen students who read and made notes on the introduction to absolute value, twelve directly copied from the textbook, Michael, Amy and Ryann's examples are below (left to right):

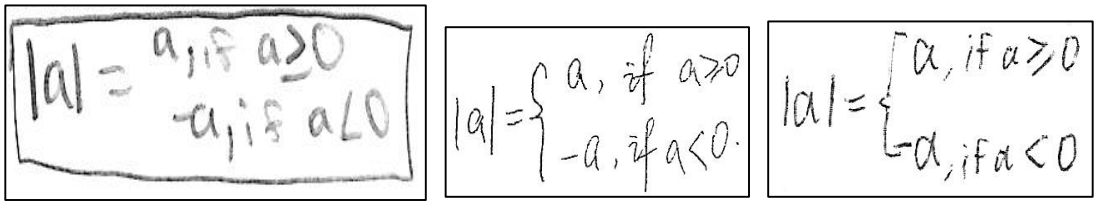


Figure 7.8 Students attending to and copying algebraic definitions

Brianne's was slightly different:

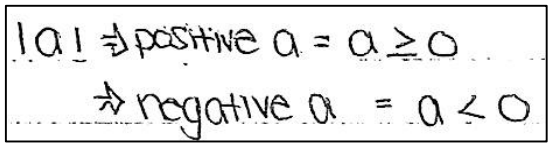


Figure 7.9 Adding words to copied algebraic definition

She had replaced both the implied positive symbol and the stated negative one with words. Brianne did notice this, and wrote what made sense to her.

What needs to be considered is that most often students notice what happens to be relevant, important or understandable to them at the time. Like reading, this depends on their background and familiarity, for example, to the reading. As I mentioned above in the Engagement theme (see §7.2), when our class was reading for the new or big idea on solving rational equations, Brianne noticed something that, for her, was the main new idea of this section. She was the only student who wrote her idea: even her friends who sat around her recorded something completely different. Whether Brianne wrote her choice because it was embedded in a two-sentence paragraph that read, "Working with rational equations is similar to working with rational expressions. A significant difference occurs because, in an equation, what you do to one side you must also do to the other side." and she noticed the cue of 'significant difference', is not known.

What cannot be known is if it was the context of the sentence that informed Brianne or her understanding that made her comfortable with this choice. It also cannot be known whether her friends felt that something further down the page had to be more important than something closer to the beginning. What was noticeable was that her friends did discuss and discount the other four bullet points and stood by their idea when it came time for class discussion. Also interesting is that other students, such as Michael and Landon, focused on the worked example and chose an idea related to the solving of that

equation. Differently, Michael wrote a numerical synopsis of his idea, rather than write the words “multiply both sides by the LCD”.

Overall, my students began to notice details as the exploration progressed. They were reading *all of* the mathematical text more closely. It *was* a surprise to me when Chloe noticed that a negative sign had been dropped, particularly as the written text did not mention that fact. The textbook assumed the students would remember that a reference angle is positive from the text sixteen pages previously. What students do with this noticing becomes the making of meaning.

7.5 Making meaning

In § 5.5, I described a lesson on solving for an angle using exact values. At this point in the exploration, my students had been reading the textbook and were more comfortable not only asking questions, but also asking ‘why’ questions. As a result, for this lesson I did not give notes on one way to solve. Rather, we collectively created notes, though they were notes in response to questions, not necessarily an organized ‘you can do it this way versus that way’ type of structure. What happened was that their personal meaning-making followed the noticing and they created written notes on what they noticed that made sense to them.

While I did not consider these as independent reading notes, they do reveal different ways that students make meaning. From this one small question, I noticed several different ways that my students *preferred* to consider the question and then solve it. Whether none of the methods they used were how I would choose to do or explain the question is irrelevant. All their methods made sense: there were no major mathematical inconsistencies or errors, and each would be able to score high marks if it were a test question. As importantly, I could not teach all these different ways between time constraints and confusing students with too many options.

For example, Michael reminded himself of SOH CAH TOA, even to the point of meta-commenting about ‘remember’ how to label sides of a right triangle, then applied this to special triangles and located the correct quadrant. He placed his right triangle in quadrant II as per his comment “because on negative side”.

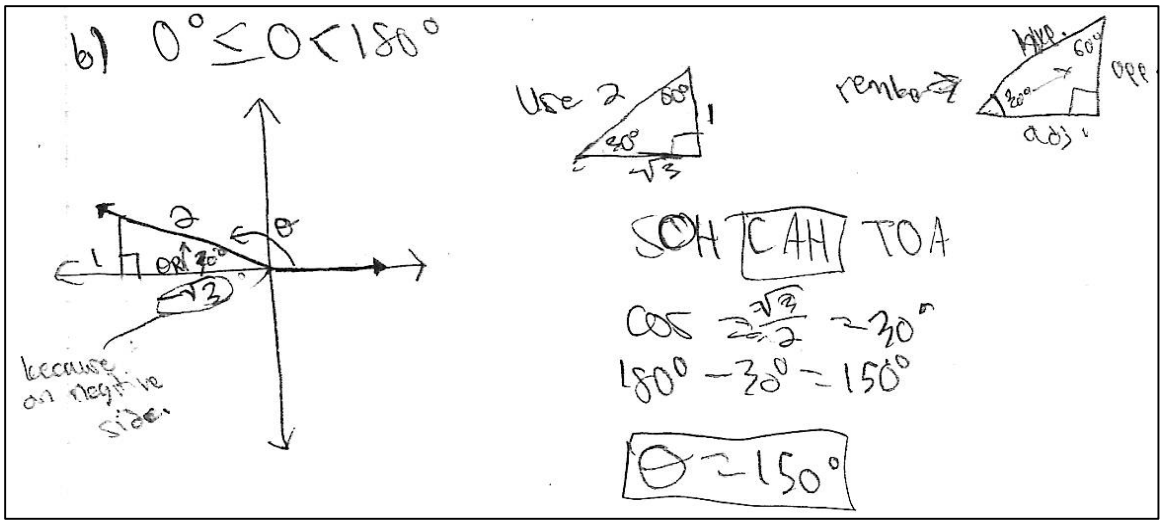


Figure 7.10 Michael explaining how to solve for an angle

Brianne miscopied the restriction, then explained the lack of a negative sign, which was missing from the textbook example. While it is not clear to me that she was certain why her special triangle was eventually moved to be in the second quadrant, her work clearly reflected her making sense of text and explaining to herself what she needed to do.

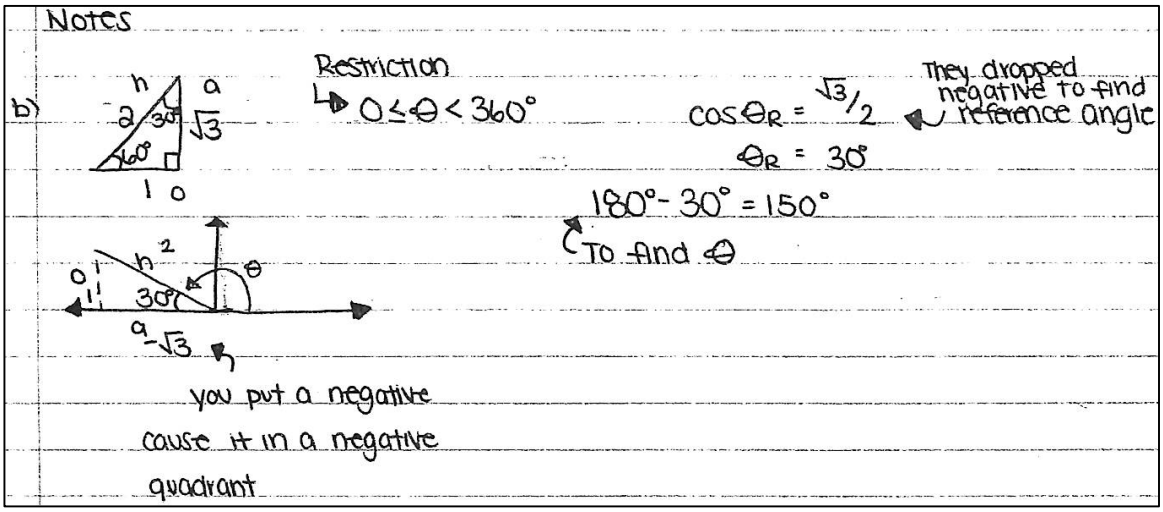


Figure 7.11 Brianne explaining how to solve for an angle

Brianne, too, meta-commented, though they feel more like steps to remember compared with Michael's reminders.

Kira used a combination of the two methods above, but started differently, and this is closer to the textbook explanation. She began by first locating the quadrant for the terminal arm through the restriction in combination with cosine being negative in

quadrants II and III. Grace went in a very different direction, by equating the numerator and denominator of the cosine ratio to the adjacent and hypotenuse of a triangle. She preferred to use the negative sign to direct herself to the correct quadrants, **then** looked at the restriction before she applied the side lengths of the special triangles. She next used her idea to complete the subsequent Your Turn question correctly.

It was a small class that day, but no one student was given my meaning; they all attacked the question and created personal notes from a choice of what made sense to them. According to Gerstein (2012), “The key to meaning-making is offering students choices to demonstrate their understanding of the content. As such, each learner should be given an option to demonstrate personalized learning in a way that is a best fit for him or her” (p. 1). I agree with most of this quotation, though I did not offer my students the choices – they questioned for them, and I think that is more to engagement than choosing. While the authors’ focus was on flipped classrooms, there is a resemblance to reading the textbook based on students considering the mathematics before structured discussions and classroom sharing.

The ultimate demonstration of meaning-making for this class was the final project. That Chloe chose metaphors to connect reading to cooking was personal, and her work was clear. It was akin to a Grade 11 version of Danika McKellar’s (2012) *Girls Got Curves.* That Grace chose to explain mathematics to others through the ‘Math party’ was also not surprising. Her project explained this chapter through the eyes of a student who required support and encouragement, more in the vein of *The Humongous Book of Calculus* (Kelley, 2006) that encouraged students and self-proclaimed to talk to those that did not speak mathematics. That Landon summarized the chapter succinctly on one sheet as he addressed the Big Ideas was also not surprising. Throughout this exploration, he focused on meaning and being able to have notes that explained the big ideas to his ‘future, idiot self’. That all students copied the algebraic representations that held the mathematical meaning for this chapter was perhaps also not surprising. All students of mathematics seem to look at these statements as things to commit to memory, as they seem quintessentially mathematical. However, they chose to write them and include them: I did not tell them to.

7.6 Mathematics as a literate subject

Literacy across the Curriculum is a focus and drive not only across BC, but also across North America as a whole. It essentially means that students should be acquiring literacy skills (traditionally the domain of the school subject English) in other subjects such as mathematics, science or art. Literacy skills are needed for reading and writing and, according to Google, include “such things as awareness of the sounds of language, awareness of print, and the relationship between letters and sounds. Other literacy skills include vocabulary, spelling, and comprehension” (Bainbridge, 2018).

My investigation of reading in the mathematics classroom was not directly concerned with literacy. While I did recognize that it was a provincial goal – and one that every high school in my district had as a school goal – it was not my focus. That it was a by-product of this exploration was not a surprise, but it was not a plan. I did not plan to make my students write or speak more or better, only to read more successfully. To this end, all the students in this classroom did attend to communication skills. As described throughout, even the reluctant students continued working to their strengths while trying to write and communicate through their weaknesses.

Prior to this study, of the four major language skills of reading, writing, listening and speaking, my students engaged predominantly in continual listening. As I was a traditional teacher, I would explain the current topic while writing notes on the board for my students to copy (not that I ever checked their notes!). Most classes were more than half-filled with my talk and writing of notes on the board. In hindsight, most students did not have time to read and interpret the notes that I wrote on the board, as they were trying to keep current with what I was writing. They could only copy. As I understood what I was writing, I could explain as I wrote. Though I did pause, ask questions and work to explain and encourage understanding, my students were not engaged in literacy. The textbook that my students used did not foster their literacy: it did not speak to pronunciation nor usage of the book’s vocabulary, nor ask comprehension questions, as examples. It merely stated things, one after the other.

Two years ago, when I initiated this exploration, the BC Ministry of Education had just launched the re-designed K–9 curriculum. My study actually has a better curricular fit today. While the new curriculum is not yet in place, mandated at the Grade 11 level,

there has been much talk in schools about the re-designed curriculum and the core competencies of communication, thinking, and personal and social elements. My students did a great deal of reading, writing, listening and speaking throughout this exploration.

Chapter 8. Considerations for Further Exploration

It should be important for the teachers in each discipline to educate their students to learn and communicate in that discipline (Frietag, 1997, p. 21).

One of the difficulties I have with cell phones upon my return to the classroom is not that they are distracting for my students (though they certainly can be), but that they are wonderful tools that take photos. This is a disappointment because students take their photos home, but not their textbooks. Some courses, such as English, require that the text travels with the student because the readings are longer. However, in mathematics class, it is easy to take snapshots of the homework questions and answers, and not use the textbook, which is customarily large and heavy.

It is not the fault of technology; it is the progression of traditional mathematics education. Mathematics teachers are indoctrinated into the 'best practice' of giving students the 'best examples and explanations' so that, to paraphrase the words of one of my students, they know so much that they go right to the questions. I have been such a teacher. I have scoured textbooks other than the ones my students use to find great examples to provide for my classes, and I have refined my lessons again and again so that I think I can get more in. However, I have also thought that my students would use *their* textbooks for back-up examples and explanations. Despite my recommendations to them, both in the classroom and when I tutored, they do no such thing. I have always wondered about this. If students do not use their mathematics textbooks, what is the point of all the work that goes into producing the next best mathematics textbook? Why do we not just have question booklets and teacher texts?

Yet I cannot imagine mathematics, and mathematics classrooms, without a textbook. This is not the traditionally trained and ingrained mathematical-me that is responding to this question. This is the reader-me answering. I cannot imagine when, for example, I do not understand either what my teacher said or why she completed an example in a certain way, that I cannot read and consider on my own. I believe that that is when much learning occurs. I equate 'telling', or giving examples, with introducing ideas. Learning occurs after, and reading, re-reading, examining, considering, noticing, contemplating

and so on are part of this. A teacher cannot tell you how to consider: that is individual, personal.

And, as can be argued, there are many YouTube support videos that students can watch and learn mathematics from. These do have their place, but what I have found through my students is that unless they are created by your teacher, they are not geared or synched to what happens in specific mathematics classrooms in different parts of specific countries; they are general and meant to target a topic. Time spent searching for the right one, or watching one that is too detailed or at the wrong level, is not productive or beneficial. Also, personal meaning-making does not necessarily happen when someone is talking at you.

As detailed throughout this thesis, reading is a transactional and generative activity where readers bring their backgrounds, comprehension and prior histories to the text to work on and create their new understanding. Reading invokes personal meaning-making. Reading should be part of the mathematics classroom, but offered in order to further mathematical understanding and learning, not to further reading for its own sake. To this end, this exploration involved students reading their mathematics textbooks in a senior mathematics class and creating personal written notes based on their interpretations of their readings. That they read their textbooks, and read them through the duration of this study, was significant.

In what follows, the thesis questions are revisited and the outcomes discussed. Next, the limitations of this exploration will be explained. Finally, the implications and considerations for further study will be explored, as well as what I have learned both as a researcher and as a mathematics teacher.

8.1 Responding to my Research Questions

1) ***What are my students judging as important from what they are reading in their mathematics textbooks?***

As was illustrated in the individual student analyses and in the cross-comparison of student notes, different students judged different mathematical elements as important. The range of their focus extended from judging numbers and worked examples as important to focusing upon written text and definitions.

This did not mean that they did not change their style of presentation or add other elements to their work over the course of this study. However, Michael, who predominantly focused on numbers from the beginning of the exploration, did continue to have them as his main focus. While he added text to his work approximately half-way through the exploration, it was mainly in subordination to his worked examples and predominantly as meta-comments. In contrast, the length of Landon's notes was comparable with Michael's, but Landon preferred to explain his ideas in words and have them followed or supported by minimal examples. Bryant, in contrast further, preferred not to include examples to document at all. He generally worked to define the process and algebraic definitions, as well as how they related to each other. He felt his mathematics homework was where numbers belonged. Bryant mainly included numbers in his notes when he had been absent.

There were those students, such as Chloe and Brianne, who judged the overall process to be important: their notes felt like the middle ground of the above. They focused on 'how to do' mathematics and their work was recognizable by 'steps' and directions. The imperatives embedded in the textbook sentences became their steps and textbook phrases such as 'You can only' became their orders. Notes such as this did remind me of notes that many teachers, myself included, would give students. They tended to follow the format of: 'Do this, then this, but watch out because you can only do that if this happens'.

What all my students judged as important seemed to come from things they could not explain, or what looked important, such as algebraic definitions. For example, all students but one copied the algebraic definition for absolute value, but none explained or could explain during discussion what it meant. My ELL students had routinely written algebraic definitions, even when there were none to copy from the textbook. It has been explained to me several times since this study that algebra is a large component of the mathematical background of international students, and working to understand them is important. The remainder of the class copied them because they looked important, and possibly what they thought mathematics really looked like.

Similarly, for their final project, all students but Michael copied all four algebraic versions for each of the linear and quadratic inequalities for all three sections. I did find this unexpected. As a student, I would not have because the only thing that changes, for

example, in $Ax + By + C < 0$ is the inequality sign. To write each one out speaks to memorization as, for example, you need to know that there are four different inequalities. I think this is important because my students were focusing on the specific change of one small sign. They may recognize that the left side and the right side do not vary, but they are not certain how to interpret or explain the variance of the sign. Perhaps they still could not read these signs after being exposed to them for several years and thus they had to equate a graph to each equation, as many did in their projects. Nor, perhaps, did they understand why the textbook wrote it four times, so it must be really important.

The other feature that all my students judged as important, even the ELL students who avoided writing in English, was vocabulary, though each student dealt with this in his or her written notes differently. Elisa, for instance, tended to start each set of notes with copied textbook definitions of key terms. Others, like Michael, would define terms with respect to numbers.

This exploration was eye-opening for me with respect to the struggles students have with the mathematical register. It was interesting for me to reflect on this exploration and re-read my field notes. On day 1, I was 'aghast' that I heard the phrase 'little two' in reference to an exponent. Yet, it should not have been a surprise to me as my students had not, until that point, had the opportunity to speak mathematically in my classroom. My assumption is that they probably have not had that opportunity in previous high school courses either. While all my students said and wrote that they felt that vocabulary was important, they had never had the chance to use it. I was again reminded of Landon asking me to do a question, but could not read to say 'three root seven'. Vocabulary and terminology are hard because students do not use them verbally. Using terminology does not mean making vocabulary cue cards or using them for a test. Students may hear the teacher speak mathematics and then see some of the terms in their homework questions, but they do not have the opportunity to put these terms into play.

I am reminded of my own children and their experiences in French Immersion. They not only listened to the teacher speak the language, they read, wrote and spoke it themselves. But there was a reason for them to do so. In my mathematics classroom, reading gave my students an on-going opportunity also to write and to speak the 'lingo' of mathematics (Hersh, 1997). Having a purpose to read, write and say mathematical terms and symbols changed the nature of our classroom discussions. Therefore, what

my students judged to be important in their mathematics textbooks was predominantly personal. What they all judged as important were algebraic definitions and mathematical terms and vocabulary.

2) How do they express and explain the above in their self-directed written notes?

The first question I had after Day 1 was, “what does it mean for my students to make notes?” Their high school mathematics experiences thus far had been to ‘take’ notes. This first day I had asked them to ‘create’ notes on what they felt was important, but did they ‘take’ notes from the textbook, rather than the more usual taking notes from the front board? My response to this would be yes, as on that first day the written notes of nearly every student were directly copied from the textbook. While they did judge different elements to be important to them, and they did copy to varying extent, what they wrote was predominately directly copied.

This did change to varying degrees over the course of the exploration. As evidenced by Chloe’s final project, she learned to read a chapter and summarize the big ideas, as well as present them in a very engaging and informative manner. At the other extreme, Serena remained a literal copier to the end: for her, it seemed that copying mathematical text was enough to take in information.

Every student had a specific way of expressing and explaining his or her self-directed written notes. By the end of this exploration it was possible to recognize student notes by what they wrote and how they wrote them. There were many writing features that my students used, but not all were used by everyone. For example, Landon wrote shorter texts that focused on answering the green thinking prompts and he used terminology in context. Kira also focused on terminology, but wrote lengthy detailed notes and her worked examples, while copied, were formatted as a double entry journal. On the right side of the page she wrote explanations to explain the details of the left side. Chloe also copied worked examples, but she created steps to direct her process and used meta-comments to reflect on certain aspects. Colour became one of Chloe’s signatures, whereas Kira only wrote in pencil. Grace was the student whose explanations were conversational. She worked to use terminology in context but would add a “You can tell this is” comment, as if she were explaining her work to someone.

Overall, there was a number of ways in which my students expressed and explained their written notes, and no one way was exactly the same as another. One of the difficulties in trying to format one way to evaluate and analyze every students' notes was that there was no one metric by which to measure their responses and their work. As well, change happened, but it happened at different times and in different ways for each student. The feature of their work was its personalization.

3) What are some of the impacts of the above on student activity within a mathematics classroom?

Reading the mathematics textbook seemed innocuous, but it had several impacts, both obvious and subtle. Without knowing, I had made a mathematical challenge out of reading our mathematics textbook, but my students never once complained. They accepted reading the textbook and happily set to task each time. As the exploration progressed, other teachers would walk in my classroom and wonder if my students were taking a test as they seemed so focused.

As described in Chapter 7, the engagement level of my students increased through this exploration. As their engagement increased, and their comfort with reading increased, their communication also increased, and this was with an improved use of the mathematical register. Their private writing affected their public speech. We became more of a community of learners rather than a troop with a leader. Our class discussions became my favourite part of the day. It took time, but as my students became accustomed to reading their textbooks, they began to notice different things and this allowed for many different access points into our conversations. They all began to contribute to class discussions, though the ELL students were still shy about using English. However, it did give them the opportunity to speak and try and use new language.

Some of the subtler impacts were equally as important. I have found, and it has been reinforced through this exploration because I was able to read their notes from the board as well, that many students have a difficult time processing written and spoken language at the same time, while trying to translate this into written language. It does become copying because a teacher talking and writing at the same time is challenging for many students to follow. If you add to this the ability of trying to make sense of new material, it can be overwhelming for many. Reading the textbook first gave students the opportunity

to both prepare their minds for new information, as well as create their own notes to which they could later add large or small details if they chose.

As mathematics teachers, we do not normally read our students' notes. This was impactful for me. I not only read their independently written notes, but I had a glimpse into what and how my students copied from the board. Whereas normally I read their computations on assessments, I was also reading their thoughts and considerations, as well as how they were interpreting and talking to themselves about their mathematics. It was eye-opening. I did get to know my students and their personalities and how they saw mathematics better through their notes, rather than only though how they did mathematics. From this, I could see where more or less was needed of an idea or topic. I could also see when and where confusion set in. Once anecdotal comments become part of report cards, their written notes can provide a wealth of information.

Ultimately, wanting to explore reading in a mathematics classroom positively impacted my students' literacy as the four components of language entered my mathematics classroom. My students were reading, writing, listening and speaking mathematics.

8.2 Limitations of this Study

The main limitations of this study related to the combination of a small class, student absences scattered across the ten weeks and several short weeks due to statutory holidays, professional development days and curriculum development days. As all of my class members participated in this reading study, this meant that the participation rate and thus sample collection was influenced by factors outside of the mathematics classroom. To minimize this limitation, notes were collected and considered from every student across the ten weeks of study. Further, the results presented in Chapter 6 include analysis on every student. The lengthier analyses are of students who were representative of the variety of change of this class, as well as those who demonstrated different areas of growth and focus. Essentially, there are notes analyzed from non-changers to engagers and from copiers to creators.

Conversely, the small class size was an advantage as it allowed me to listen more to individual students and to hear their thinking. While it would have been possible to hear some individual dialogues within a larger class, I was better able to hear the thinking of

all my students consistently in a smaller class. While I did not directly access their feelings about the exploration, as I could have through interviews for example, I could hear some of their thoughts about the process and their thinking live and unprompted as they engaged in different tasks.

My dual role of teacher and researcher held both advantages and disadvantages. The limitation was the consideration of influence. As a researcher of reading mathematics, I had to be cautious about influencing their reading and noticing. I was careful to monitor my actions and statements continually to attempt to remain neutral about my thoughts and spoken words with respect to reading and mathematics.

Keeping field notes helped minimize limitations, as it provided a means of continual reflection. For example, during class when I jotted down my thoughts, I was able to make my thinking and spoken words concrete and evident. In addition, there were classes where I gave lessons on reading strategies that focused attention on mathematics textbooks and enhanced the different ways of looking at and reading mathematics text. To minimize any influence of this, I did not consider these reading notes or lessons in my analysis of individual students' written work.

There were advantages of being a teacher/researcher. For example, I had control of the study, as I was always in class and able to note what happened or what was said on a daily basis. I was also able to collect data continually. Rather than rely on an outside interviewer or researcher and their timetable, I was able to monitor and adjust the exploration if and when needed. My teaching of reading mathematics became a feedback cycle that informed me in preparation for the next class.

8.3 What Did I Learn as a Mathematics Teacher: Further Considerations

As a mathematics teacher, there were many lessons for me through this exploration. It *was* research in its infancy, but, even then, impacts were felt. I know now that I cannot continue to be the only person in my mathematics classroom who interprets the textbook. It is clear my students can, though there is definitely room for growth and expansion on this research. Even from this short study, it is evident that students are

capable of, for example, taking an imperative textbook sentence like, “Simplify radicals and combine like terms.” (p. 276) and, as Grace did, change it to:

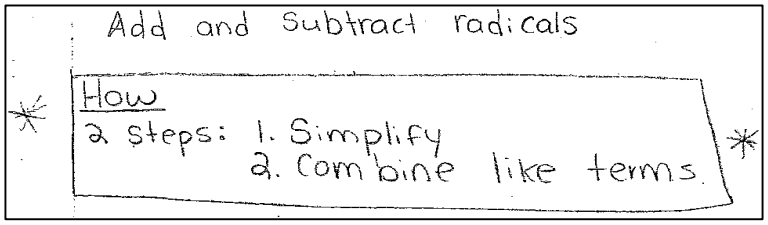


Figure 8.1 Grace modifying text to create steps

Teachers do this; students can too. This is a very simplistic example, but it is a common thing that mathematics teachers do and I have witnessed other mathematics teachers write those very same steps for their students to copy. We do not want to have our students dependent on our interpretations; that is learned helplessness.

What I also noticed is that the few reading interventions I employed, like *Clone an Author*, provided interesting data. I did exclude these notes directly, but what I found was that it was remarkable how they informed me about my students. These strategies provided nice pauses from direct reading and my students could use them further. Gabrielle, for example, used this for one of her independent readings where she wrote a question or a term on the top of a stickie with the answer below. While this was not the intended use of this strategy, it had fuelled her imagination to try other methods of recording her reading notes. Landon used stickies for the presentation of his final project. They allowed him to be succinct and enabled him to create a flow chart of his thoughts. These types of transactional reading strategies I would like to use again.

I would also like to provide more written feedback to my students other than the few times I was able to. While I did read all their notes, and I did discuss with them as a class what I noticed about what they wrote, I think that more direct, personal notes would also be helpful and it would be interesting to see whether or not that affects growth and change.

This exploration and research fills a hole. There is little data on reading in senior mathematics classrooms, and even less that I found in the literature on reading the textbook in a senior mathematics classroom. This research also highlights student

questioning, interpreting and considering. It moves toward incorporating mathematical habits of mind. For example, I would never have thought to read in Elisa's notes, "Why make it = 0". This was in reference to finding restrictions on rational expressions. She was questioning that if something cannot be equal to zero, why make it equal zero? Her mind was prepared.

As one of my final field-note entries I wrote:

I feel that they like this responsibility. They don't, for one, have to sit there and listen to me lecture *ad nauseam*. Not that I do that a lot, but there is a different responsibility implied by reading the textbook and interpreting meaning.

This exploration allowed me to move closer to the guide or facilitator role that is implicit in teaching. I engaged differently in the classroom during this exploration as my position changed, and not only in being able to move. I was also able to use my understanding and knowledge of mathematics in response to my students' needs. In this way, my classroom shifted toward a more student-centered environment.

While it may seem a simple idea to have students read their mathematics textbook and make notes afterwards, it is a teaching strategy that any teacher, progressive or traditional, can employ. It is often the simple, manageable changes that reap the largest rewards. In this case it was demonstrated that the implementation of reading in the mathematics classroom moved toward creating responsible, autonomous, engaged and meaning-making students who can hold a mathematics textbook and indicate pages that they read and say, "I can believe I have taught myself some mathematics".

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Appendix A – Textbook Scans

1) Pages 273 – 276. Part of 'Link the Ideas' from Section 5.1 on Radical Expressions

4. Write an addition and subtraction statement using only mixed radicals for each calculation in step 3b). A mixed radical is the product of a monomial and a radical. In $r\sqrt[n]{x}$, r is the coefficient, n is the index, and x is the radicand.

Reflect and Respond

5. Develop a general equation that represents the addition of radicals. Compare your equation and method with a classmate's. Identify any rules for using your equation.

6. Use integral values of a to verify that $\sqrt{a} + \sqrt{a} + \sqrt{a} + \sqrt{a} = 4\sqrt{a}$.

Link the Ideas

Like Radicals

Radicals with the same radicand and index are called like radicals.

Pairs of Like Radicals	Pairs of Unlike Radicals
$5\sqrt{7}$ and $-\sqrt{7}$	$2\sqrt{5}$ and $2\sqrt{3}$
$\frac{2}{3}\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$	$\sqrt[4]{5a}$ and $\sqrt[5]{5a}$

How are like radicals similar to like terms?

When adding and subtracting radicals, only like radicals can be combined. You may need to convert radicals to a different form (mixed or entire) before identifying like radicals.

Restrictions on Variables

If a radical represents a real number and has an even index, the radicand must be non-negative.

The radical $\sqrt{4 - x}$ has an even index. So, $4 - x$ must be greater than or equal to zero.

$$\begin{aligned} 4 - x &\geq 0 \\ 4 - x + x &\geq 0 + x \\ 4 &\geq x \end{aligned}$$

Isolate the variable by applying algebraic operations to both sides of the inequality symbol.

The radical $\sqrt{4 - x}$ is only defined as a real number if x is less than or equal to four. You can check this by substituting values for x that are greater than four, equal to four, and less than four.

Example 1

Convert Mixed Radicals to Entire Radicals

Express each mixed radical in entire radical form. Identify the values of the variable for which the radical represents a real number.

a) $7\sqrt{2}$ **b)** $a^4\sqrt{a}$ **c)** $5b\sqrt[3]{3b^2}$

Solution

a) Write the coefficient 7 as a square root: $7 = \sqrt{7^2}$.
Then, multiply the radicands of the square roots.

$$\begin{aligned} 7\sqrt{2} &= \sqrt{7^2}(\sqrt{2}) \\ &= \sqrt{7^2(2)} \\ &= \sqrt{49(2)} \\ &= \sqrt{98} \end{aligned}$$

How could you verify the answer?

b) Express the coefficient a^4 as a square root: $a^4 = \sqrt{(a^4)^2}$.
Multiply the radicals.

$$\begin{aligned} a^4\sqrt{a} &= \sqrt{(a^4)^2}(\sqrt{a}) \\ &= \sqrt{(a^4)^2(a)} \\ &= \sqrt{a^8(a)} \\ &= \sqrt{a^9} \end{aligned}$$

For the radical in the original expression to be a real number, the radicand must be non-negative. Therefore, a is greater than or equal to zero.

c) Write the entire coefficient, $5b$, as a cube root.

$$\begin{aligned} 5b &= \sqrt[3]{(5b)^3} \\ &= \sqrt[3]{5^3b^3} \end{aligned}$$

Multiply the radicands of the cube roots.

$$\begin{aligned} 5b\sqrt[3]{3b^2} &= \left(\sqrt[3]{5^3b^3}\right)\left(\sqrt[3]{3b^2}\right) \\ &= \sqrt[3]{5^3b^3(3b^2)} \\ &= \sqrt[3]{375b^5} \end{aligned}$$

Since the index of the radical is an odd number, the variable, b, can be any real number.

Why can a radical with an odd index have a radicand that is positive, negative, or zero?

Your Turn

Convert each mixed radical to an entire radical. State the values of the variable for which the radical is a real number.

a) $4\sqrt{3}$ **b)** $j^3\sqrt{j}$ **c)** $2k^2(\sqrt[3]{4k})$

Radicals in Simplest Form

A radical is in simplest form if the following are true.

- The radicand does not contain a fraction or any factor which may be removed.
- The radical is not part of the denominator of a fraction.

For example, $\sqrt{18}$ is not in simplest form because 18 has a square factor of 9, which can be removed. $\sqrt{18}$ is equivalent to the simplified form $3\sqrt{2}$.

Example 2

Express Entire Radicals as Mixed Radicals

Convert each entire radical to a mixed radical in simplest form.

a) $\sqrt{200}$ **b)** $\sqrt[4]{c^9}$ **c)** $\sqrt{48y^5}$

Solution

a) Method 1: Use the Greatest Perfect-Square Factor

The following perfect squares are factors of 200: 1, 4, 25, and 100. Write $\sqrt{200}$ as a product using the greatest perfect-square factor.

$$\begin{aligned}\sqrt{200} &= \sqrt{100(2)}\\ &= \sqrt{100}(\sqrt{2})\\ &= 10\sqrt{2}\end{aligned}$$

Method 2: Use Prime Factorization

Express the radicand as a product of prime factors. The index is two. So, combine pairs of identical factors.

$$\begin{aligned}\sqrt{200} &= \sqrt{2(2)(2)(5)(5)}\\ &= \sqrt{2^2(2)(5^2)}\\ &= 2(5)\sqrt{2}\\ &= 10\sqrt{2}\end{aligned}$$

Did You Know?

The radical symbol represents only the *positive* square root. So, even though $(-10)^2 = 100$, $\sqrt{100} \neq \pm 10$. $\sqrt{100} = +10$ $-\sqrt{100} = -10$ In general, $\sqrt{x^2} = x$ only when x is positive.

b) Method 1: Use Prime Factorization

$$\begin{aligned}\sqrt[4]{c^9} &= \sqrt[4]{c(c)(c)(c)(c)(c)(c)(c)}\\ &= \sqrt[4]{c^4(c^4)(c)}\\ &= c(c)\sqrt[4]{c}\\ &= c^2(\sqrt[4]{c})\end{aligned}$$

What number tells you how many identical factors to combine?

Method 2: Use Powers

$$\begin{aligned}\sqrt[4]{c^9} &= c^{\frac{9}{4}}\\ &= c^{\frac{8}{4}+\frac{1}{4}}\\ &= c^{\frac{8}{4}}\left(c^{\frac{1}{4}}\right)\\ &= c^2\left(c^{\frac{1}{4}}\right)\\ &= c^2(\sqrt[4]{c})\end{aligned}$$

How will you decide what fractions to use for the sum?

For the radical to represent a real number, $c \geq 0$ because the index is an even number.

c) $\sqrt{48y^5}$

Determine the greatest perfect-square factors for the numerical and variable parts.

$$\begin{aligned}\sqrt{48y^5} &= \sqrt{16(3)(y^4)(y)}\\ &= 4y^2\sqrt{3y}\end{aligned}$$

How can you determine the values of the variables for which the radical is defined as a real number?

Your Turn

Express each entire radical as a mixed radical in simplest form. Identify any restrictions on the values for the variables.

a) $\sqrt{52}$ **b)** $\sqrt[4]{m^7}$ **c)** $\sqrt{63n^7p^4}$

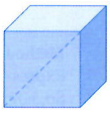
Did You Know?

A bentwood box is made from a single piece of wood. Heat and moisture make the wood pliable, so it can be bent into a box. First Nations peoples of the Pacific Northwest have traditionally produced these boxes for storage, and some are decorated as symbols of wealth.

Formline Revolution Bentwood Chest
by Corey Moraes, Tsimshian artist

Example 3

Compare and Order Radicals



Five bentwood boxes, each in the shape of a cube have the following diagonal lengths, in centimetres.

$4(13)^{\frac{1}{2}}$ $\quad 8\sqrt{3}$ $\quad 14$ $\quad \sqrt{202}$ $\quad 10\sqrt{2}$

Order the diagonal lengths from least to greatest without using a calculator.

Solution

Express the diagonal lengths as entire radicals.

$$\begin{aligned} 4(13)^{\frac{1}{2}} &= 4\sqrt{13} \\ &= \sqrt{4^2}(\sqrt{13}) \\ &= \sqrt{4^2(13)} \\ &= \sqrt{16(13)} \\ &= \sqrt{208} \end{aligned}$$

$$\begin{aligned} 8\sqrt{3} &= \sqrt{8^2}(\sqrt{3}) \\ &= \sqrt{8^2(3)} \\ &= \sqrt{64(3)} \\ &= \sqrt{192} \end{aligned}$$

$$\begin{aligned} 14 &= \sqrt{14^2} \\ &= \sqrt{196} \end{aligned}$$

$\sqrt{202}$ is already written as an entire radical.

$$\begin{aligned} 10\sqrt{2} &= \sqrt{10^2}(\sqrt{2}) \\ &= \sqrt{100(2)} \\ &= \sqrt{200} \end{aligned}$$

Compare the five radicands and order the numbers.
$\sqrt{192} < \sqrt{196} < \sqrt{200} < \sqrt{202} < \sqrt{208}$

The diagonal lengths from least to greatest are $8\sqrt{3}$, 14, $10\sqrt{2}$, $\sqrt{202}$, and $4(13)^{\frac{1}{2}}$.

Your Turn

Order the following numbers from least to greatest:
5, $3\sqrt{3}$, $2\sqrt{6}$, $\sqrt{23}$

Example 4

Add and Subtract Radicals

Simplify radicals and combine like terms.

a) $\sqrt{50} + 3\sqrt{2}$
b) $-\sqrt{27} + 3\sqrt{5} - \sqrt{80} - 2\sqrt{12}$
c) $\sqrt{4c} - 4\sqrt{9c}$, $c \geq 0$

2) Pages 143- 144 from Section 3.1 on Quadratic functions; noting parallel structure of imperative text instructions.

Investigate Graphs of Quadratic Functions in Vertex Form

Part A: Compare the Graphs of $f(x) = x^2$ and $f(x) = ax^2$, $a \neq 0$

Materials

- grid paper or graphing technology

1. a) Graph the following functions on the same set of coordinate axes, with or without technology.

$f(x) = x^2$ $\quad$ $f(x) = -x^2$
$f(x) = 2x^2$ $\quad$ $f(x) = -2x^2$
$f(x) = \frac{1}{2}x^2$ $\quad$ $f(x) = -\frac{1}{2}x^2$

b) Describe how the graph of each function compares to the graph of $f(x) = x^2$, using terms such as *narrower*, *wider*, and *reflection*.

c) What relationship do you observe between the parameter, a, and the shape of the corresponding graph?

2. a) Using a variety of values of a, write several of your own functions of the form $f(x) = ax^2$. Include both positive and negative values.

b) Predict how the graphs of these functions will compare to the graph of $f(x) = x^2$. Test your prediction.

Reflect and Respond

3. Develop a rule that describes how the value of a in $f(x) = ax^2$ changes the graph of $f(x) = x^2$ when a is

a) a positive number greater than 1

b) a positive number less than 1

c) a negative number

Part B: Compare the Graphs of $f(x) = x^2$ and $f(x) = x^2 + q$

4. a) Graph the following functions on the same set of coordinate axes, with or without technology.

$f(x) = x^2$
$f(x) = x^2 + 4$
$f(x) = x^2 - 3$

b) Describe how the graph of each function compares to the graph of $f(x) = x^2$.

c) What relationship do you observe between the parameter, q, and the location of the corresponding graph?

5. a) Using a variety of values of q, write several of your own functions of the form $f(x) = x^2 + q$. Include both positive and negative values.

b) Predict how these functions will compare to $f(x) = x^2$. Test your prediction.

Reflect and Respond

6. Develop a rule that describes how the value of q in $f(x) = x^2 + q$ changes the graph of $f(x) = x^2$ when q is

a) a positive number $\quad$ **b)** a negative number

Part C: Compare the Graphs of $f(x) = x^2$ and $f(x) = (x - p)^2$

7. a) Graph the following functions on the same set of coordinate axes, with or without technology.

$f(x) = x^2$
$f(x) = (x - 2)^2$
$f(x) = (x + 1)^2$

b) Describe how the graph of each function compares to the graph of $f(x) = x^2$.

c) What relationship do you observe between the parameter, p, and the location of the corresponding graph?

8. a) Using a variety of values of p, write several of your own functions of the form $f(x) = (x - p)^2$. Include both positive and negative values.

b) Predict how these functions will compare to $f(x) = x^2$. Test your prediction.

Reflect and Respond

9. Develop a rule that describes how the value of p in $f(x) = (x - p)^2$ changes the graph of $f(x) = x^2$ when p is

a) a positive number $\quad$ **b)** a negative number

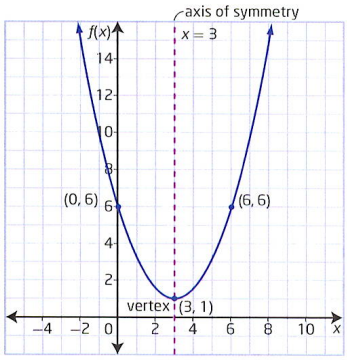
3) Readability of text – noting white space and visual rest at bottom of page

The y-coordinate of the vertex is called the **minimum value** if the parabola opens upward or the **maximum value** if the parabola opens downward

The parabola is symmetric about a line called the **axis of symmetry**. This line divides the function graph into two parts so that the graph on one side is the mirror image of the graph on the other side. This means that if you know a point on one side of the parabola, you can determine a corresponding point on the other side based on the axis of symmetry.

The axis of symmetry intersects the parabola at the vertex.

The x-coordinate of the vertex corresponds to the equation of the axis of symmetry.



minimum value (of a function)

- the least value in the range of a function
- for a quadratic function that opens upward, the y-coordinate of the vertex

maximum value (of a function)

- the greatest value in the range of a function
- for a quadratic function that opens downward, the y-coordinate of the vertex

axis of symmetry

- a line through the vertex that divides the graph of a quadratic function into two congruent halves
- the x-coordinate of the vertex defines the equation of the axis of symmetry

Quadratic functions written in **vertex form**, $f(x) = a(x - p)^2 + q$, are useful when graphing the function. The vertex form tells you the location of the vertex (p, q) as well as the shape of the parabola and the direction of the opening.

You can examine the parameters a, p, and q to determine information about the graph.

vertex form (of a quadratic function)

- the form $y = a(x - p)^2 + q$, or $f(x) = a(x - p)^2 + q$, where a, p, and q are constants and $a \neq 0$

4) Readability of text – noting placement of white space and condensing of text

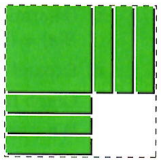
Materials

- algebra tiles

Part B: Completing the Square

The quadratic function developed in step 3 in Part A is in standard form. The function used in step 5 is in vertex form. These quadratic functions are equivalent and can provide different information. You can convert from vertex to standard form by expanding the vertex form. How can you convert from standard to vertex form?

7. a) Select algebra tiles to represent the expression $x^2 + 6x$. Arrange them into an incomplete square as shown.



b) What tiles must you add to complete the square?

c) What trinomial represents the new completed square?

d) How can you rewrite this trinomial in factored form as the square of a binomial?

8. a) Repeat the activity in step 7 using each expression in the list. Record your results in an organized fashion. Include a diagram of the tiles for each expression.

$x^2 + 2x$
$x^2 + 4x$
$x^2 + 8x$
$x^2 + 10x$

What tiles must you add to each expression to make a complete square?

b) Continue to model expressions until you can clearly describe the pattern that emerges. What relationship is there between the original expression and the tiles necessary to complete the square? Explain.

9. Repeat the activity, but this time model expressions that have a negative x-term, such as $x^2 - 2x$, $x^2 - 4x$, $x^2 - 6x$, and so on.

10. a) Without using algebra tiles, predict what value you need to add to the expression $x^2 + 32x$ to represent it as a completed square. What trinomial represents this completed square?

b) How can you rewrite the trinomial in factored form as the square of a binomial?

Reflect and Respond

11. a) How are the tiles you need to complete each square related to the original expression?

b) Does it matter whether the x-term in the original expression is positive or negative? Explain.

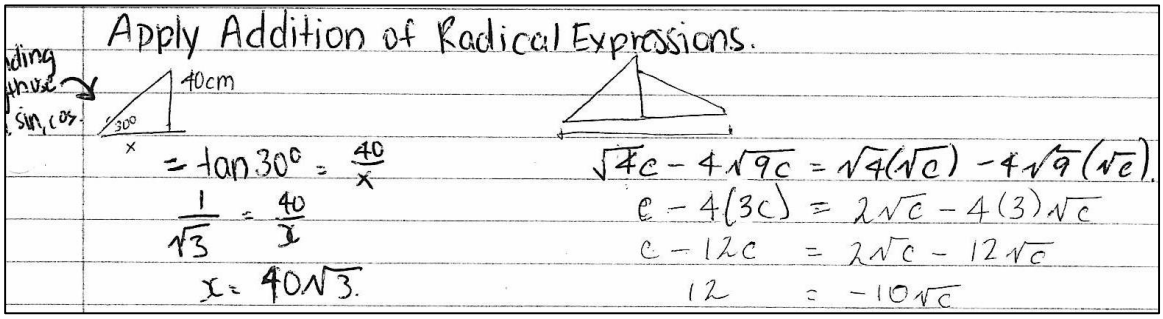
c) Is it possible to complete the square for an expression with an x-term with an odd coefficient? Explain your thinking.

12. The expressions $x^2 + \blacksquare x + \blacktriangle$ and $(x + \bullet)^2$ both represent the same perfect square. Describe how the missing values are related to each other.

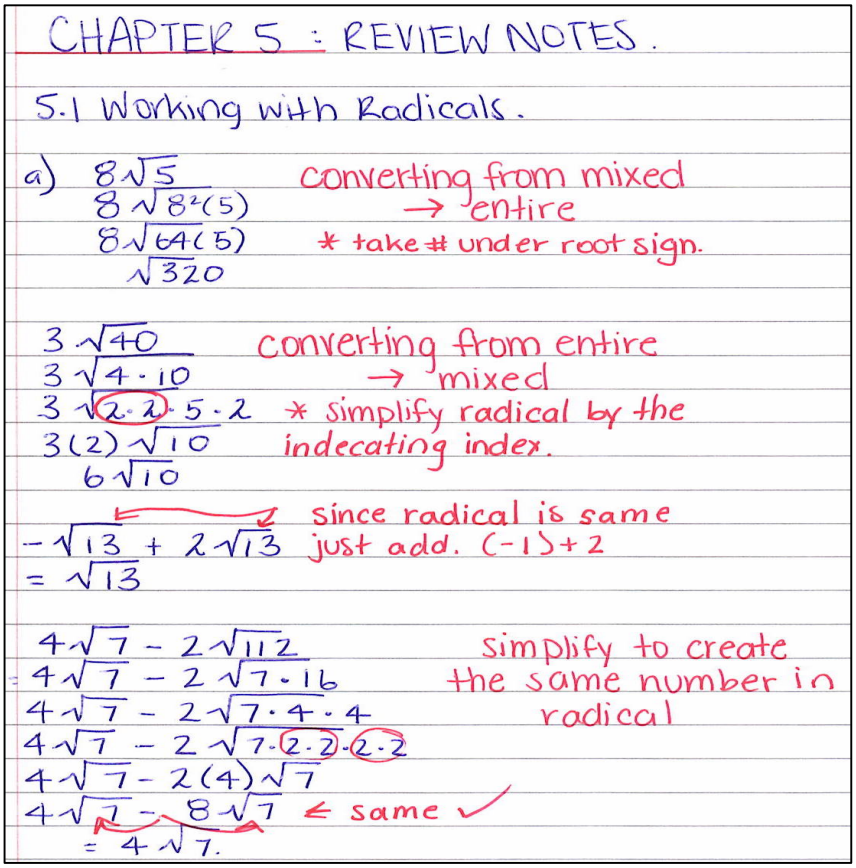
Appendix B – Student Supporting Work

CHLOE

1) Notes Chloe completed for homework from the wrong page. She predominately copied these notes, and meshed examples from different pages. There is no sense-making here, particularly in light of the fact that we had not yet studied trigonometry and exact values, as well as addition and subtraction of radicals.



2) Sample page from her review notes.



3) Chloe's now standard use of colour and meta-comments. Here she reminds herself of details that she previously made mistakes on, such as order of operations.

Absolute Value : is writtan as $|a|$ it can mean

$$|a| = \begin{cases} a \text{ if } a \geq 0 \\ -a, \text{ if } a < 0 \end{cases}$$

Positive Number = $|+5| = 5$ absolute value of a + number is always a + number

Zero = $|0| = 0$ zero is always zero

Negative Number = $|-5| = -(-5)$ absolute value of a negative number is the negative number, resulting in a postive.

two neg. signs make a positive.

Ex. a) $|9| = 9$ since $|a| = a \geq 0$

b) $|-12| = -(-12)$ because - makes a positive

$= 12$

* ABSOLUTE VALUES : Should be treated like (bracetes)

– Do the operations inside the absolute value.

Both operations must be done before.

a) $|4| - |-6| = 4 - 6$

$= -2$

STEPS!!

b) $5 - 3|2-7| = 5 - 3|-5|$

$= 5 - 3(5)$

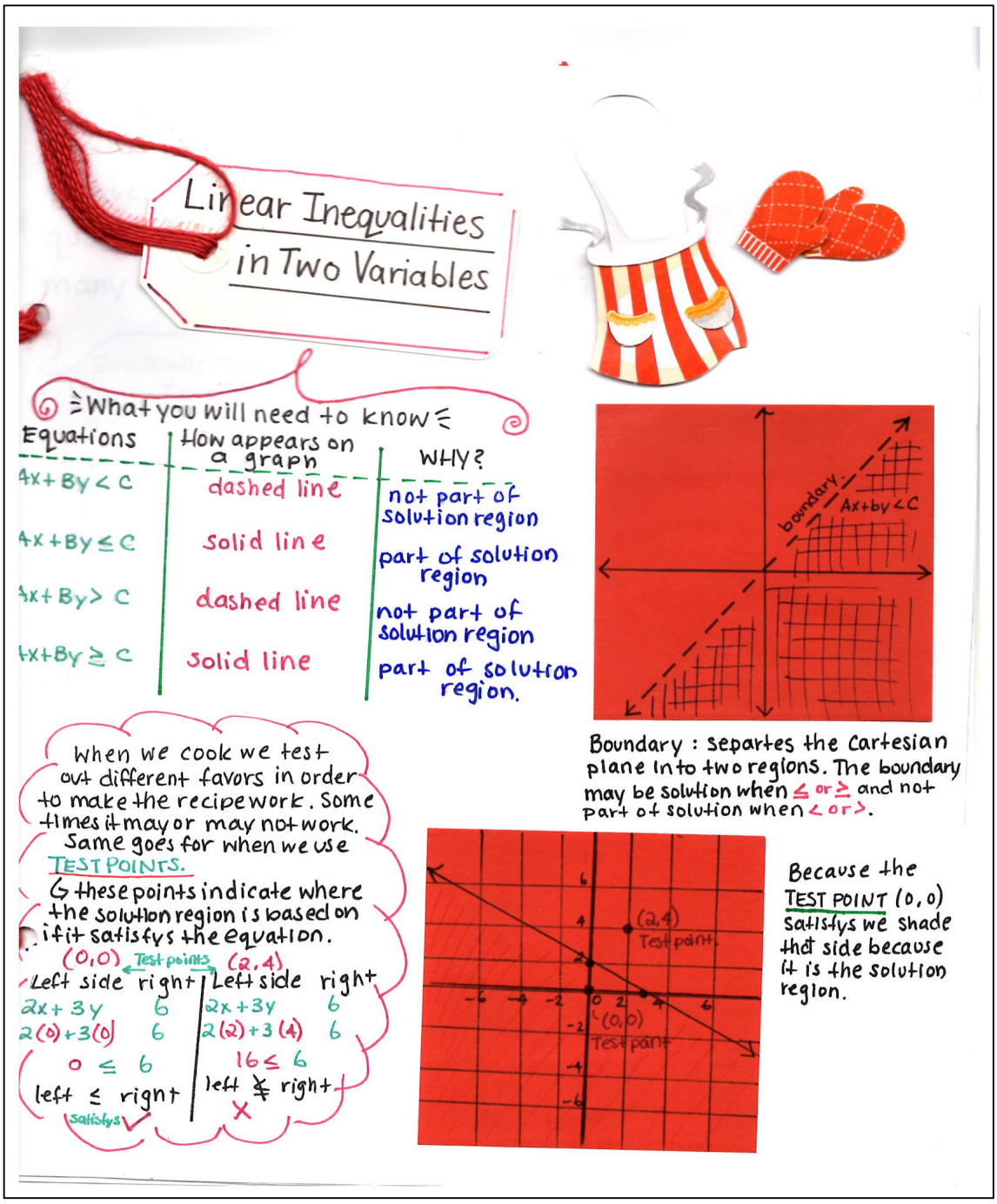
$= 5 - 15$

$= -10.$

① Evalute the expressions inside the absolute value.

② taking the absolute value of the resulting expression.

4) Page 2 of Chloe's final project that was a Cookbook. Notice her use of the pronoun *we.* This is a clear departure from her tone of authority and use of the pronoun *you.* In addition, she is using analogies to explain mathematics.



Linear Inequalities in Two Variables

What you will need to know

Equations	How appears on a graph	WHY?
$4x+By<C$	dashed line	not part of solution region
$4x+By\leq C$	solid line	part of solution region
$4x+By>C$	dashed line	not part of solution region
$4x+By\geq C$	solid line	part of solution region.

Boundary : separtes the cartesian plane into two regions. The boundary may be solution when ≤ or ≥ and not part of solution when < or >.

When we cook we test out different favors in order to make the recipe work. Some times it may or may not work. Same goes for when we use TEST POINTS.

these points indicate where the solution region is based on if it satisfys the equation.

(0,0) Test points (2,4)

Left side	right	Left side	right
$2x+3y$	6	$2x+3y$	6
$2(0)+3(0)$	6	$2(2)+3(4)$	6
$0\leq 6$		$16\leq 6$	
left ≤ right		left ≰ right	
satisfys ✓		✗	

Because the TEST POINT (0,0) satisfys we shade that side because it is the solution region.

BRIANNE

5) Compared to page 275 of the textbook (see Appendix A1), Brianne's notes are a near replica.

Express Entire Radicals as mixed radicals

METHOD 1 : THE GREATEST PERFECT-SQUARE FACTOR

Example $\sqrt{200}$ The factors for 200 is 1 4 25 & 100

$\sqrt{200} = \sqrt{100(2)}$ It's 100 cause it's a factor!

$= \sqrt{100(\sqrt{2})}$?

$= 10\sqrt{2}$

METHOD 2 : PRIME FACTORIZATION

Example $\sqrt{200}$ Express the radicand as a product of prime factors

$\sqrt{200} = \sqrt{2(2)(2)(5)(5)}$

$= \sqrt{2^2(2)(5^2)}$

$= 2(5)\sqrt{2}$

$= 10\sqrt{2}$

METHOD 1 : Use Prime Factorization

Example $\sqrt[4]{c^9}$

$\sqrt[4]{c^9} = \sqrt[4]{c(c)(c)(c)(c)(c)(c)(c)(c)}$

$= \sqrt[4]{c^4(c^4)(c)}$

$= c(c)\sqrt[4]{c}$

$= c^2(\sqrt[4]{c})$

Example METHOD 2: Use powers

$\sqrt[4]{c^9}$

$\sqrt[4]{c^9} = c^{9/4}$

$= c^{8/4 + 1/4}$

$= c^2(c^{1/4})$

$= c^2(\sqrt[4]{c})$

* For the radical to represent as a real number $c \geq 0$

MICHAEL

6) Day 1 notes – Numeric examples directly copied from textbook.

Simplifying Radicals — March 29

eg. 273–274

Express each mixed radical in entire radical form.

Ex. $7\sqrt{2}$

① Write outside number as a square root

② Multiply

$\sqrt{7^2}(\sqrt{2})$

$= \sqrt{49}(2)$

$= \sqrt{98}$

$a^4\sqrt{a}$

$\sqrt{(a^4)^2}(\sqrt{a})$

$= \sqrt{a^8}(a)$

$= \sqrt{a^9}$

$5b\sqrt[3]{3b^2}$

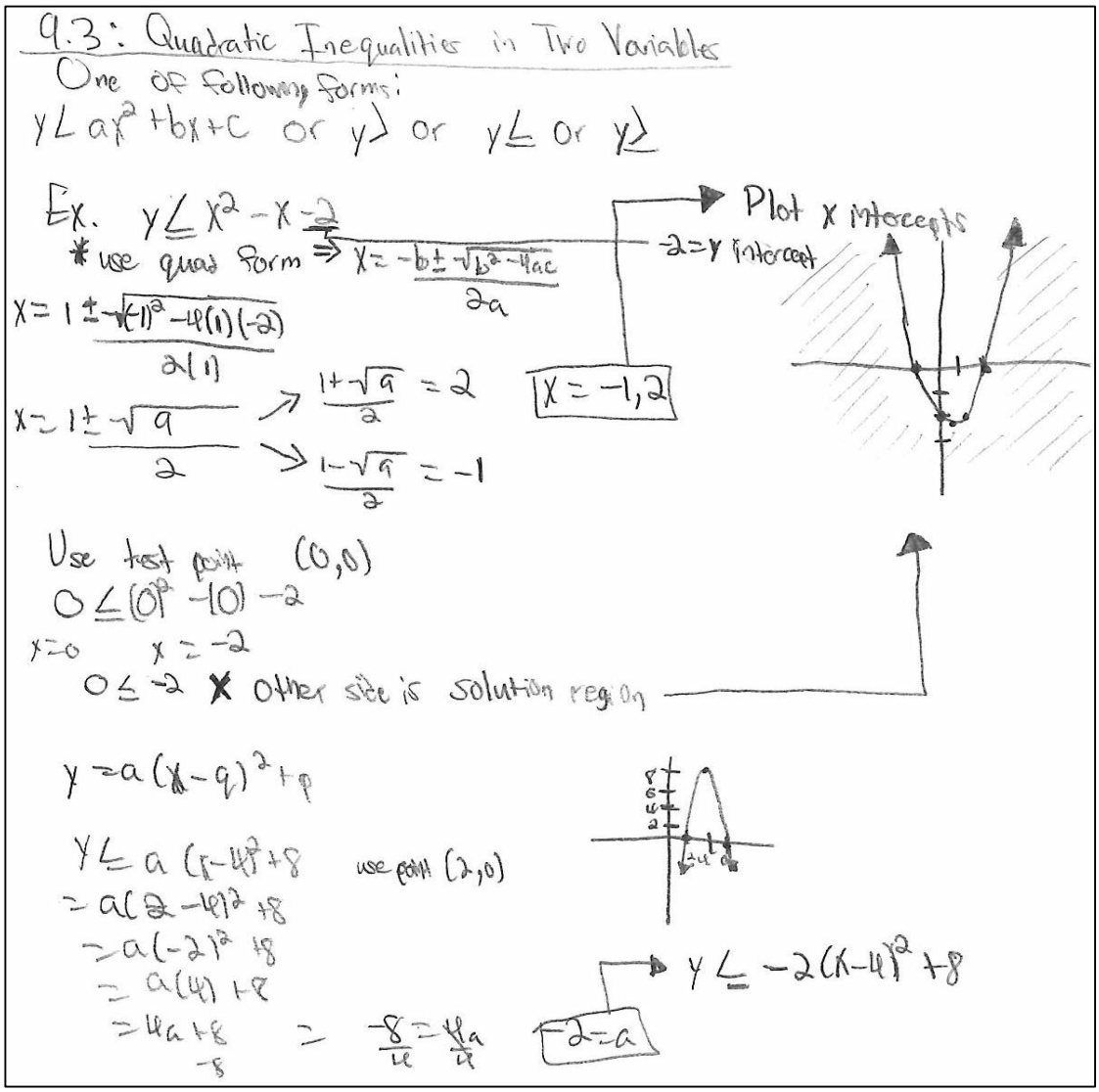
$(\sqrt[3]{5^3b^3})(\sqrt[3]{3b^2})$

$= \sqrt[3]{5^3b^3(3b^2)} \rightarrow (5^3 = 125 \times 3 = 375)$

$= \sqrt[3]{375b^5}$

* If cubed cube root it.

7) A sample page of Michael's final project.



9.3: Quadratic Inequalities in Two Variables

One of following forms:

$y < ax^2 + bx + c$ or $y >$ or $y \leq$ or $y \geq$

Ex. $y \leq x^2 - x - 2$

* use quad form $\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{1 \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{1 \pm \sqrt{9}}{2}$ → $\frac{1+\sqrt{9}}{2} = 2$, $\frac{1-\sqrt{9}}{2} = -1$

$\boxed{x = -1, 2}$

Plot x intercepts

$-2 = y$ intercept

Use test point $(0,0)$

$0 \leq (0)^2 - (0) - 2$

$x = 0 \quad x = -2$

$0 \leq -2$ ✗ other side is solution region

$y = a(x-q)^2 + p$

$y \leq a(x-4)^2 + 8$ use point $(2,0)$

$= a(2-4)^2 + 8$

$= a(-2)^2 + 8$

$= a(4) + 8$

$= 4a + 8$

-8

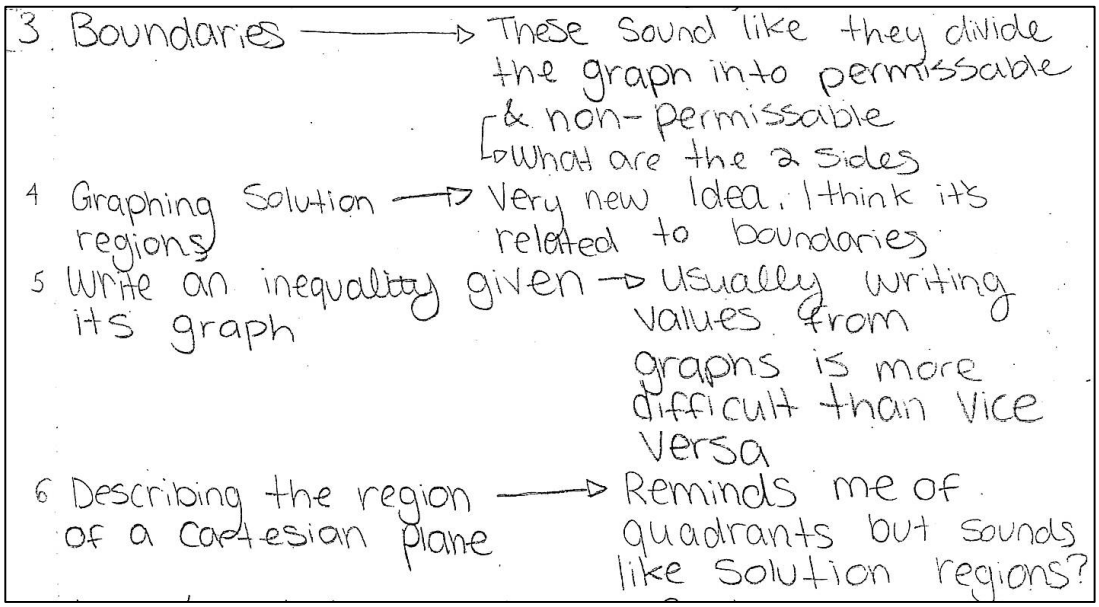
$= \frac{-8}{4} = \frac{4a}{4}$

$\boxed{-2 = a}$

$y \leq -2(x-4)^2 + 8$

GRACE

8) Part of Grace's list of new ideas from Chapter 9. Here Grace is connecting general ideas from the chapter.



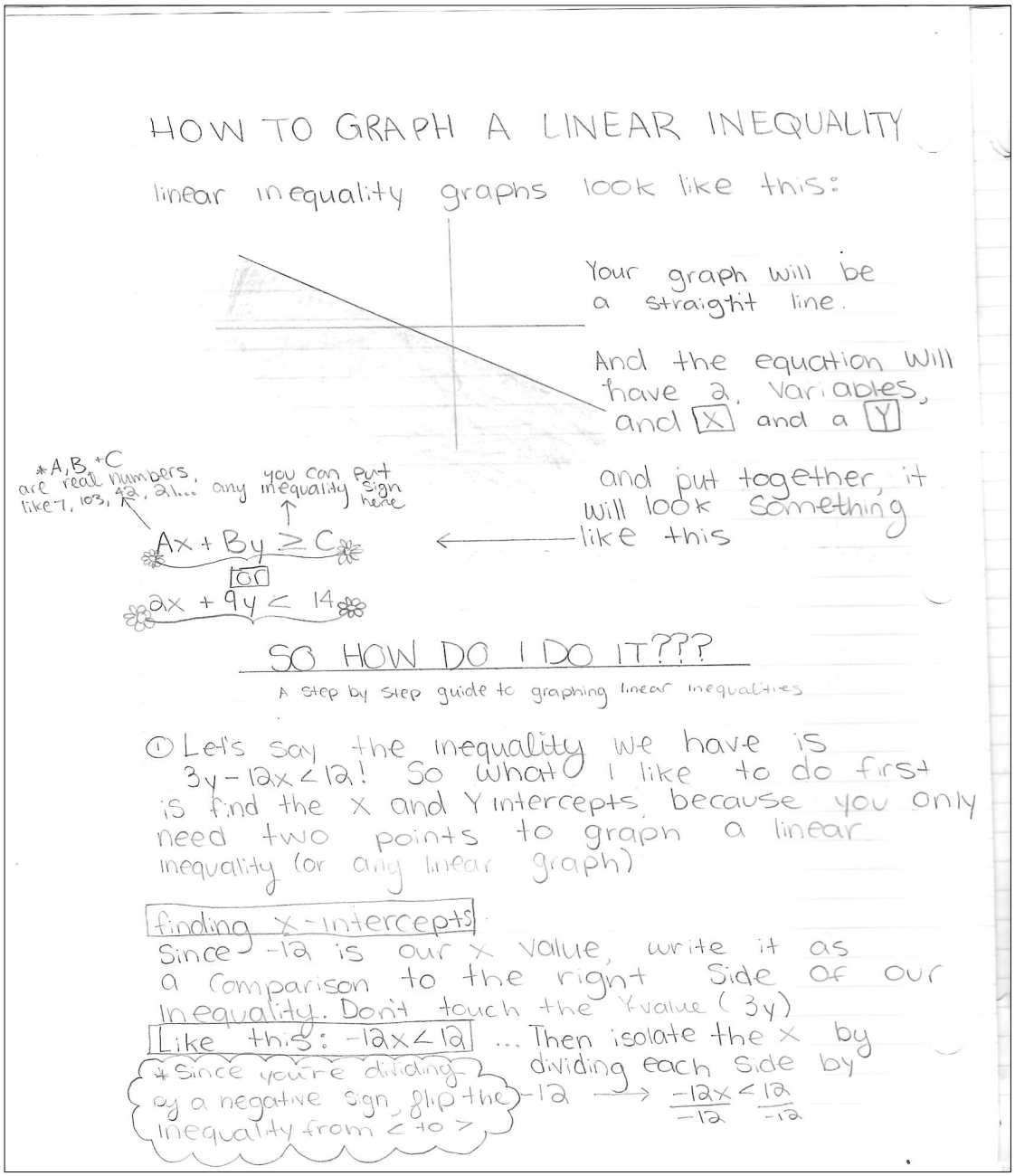
3. Boundaries → These sound like they divide the graph into permissable & non-permissable
→ What are the 2 sides

4 Graphing Solution regions → Very new idea. I think it's related to boundaries

5 Write an inequality given its graph → Usually writing values from graphs is more difficult than vice versa

6 Describing the region of a Cartesian plane → Reminds me of quadrants but sounds like solution regions?

9) The second page from Grace's 'Math Party 2016' book. While her notes and mathematical work continued to be written in pencil, she added colour and accent with stickers, highlighters, and some coloured pencil on the graphs. The page below is completely done in pencil.



HOW TO GRAPH A LINEAR INEQUALITY

linear inequality graphs look like this:

Your graph will be a straight line.

And the equation will have 2 variables, and $\boxed{X}$ and a $\boxed{Y}$

and put together, it will look something like this

*A, B + C are real numbers, like 7, 103, 42, 21... any inequality sign here

$Ax + By \geq C$

$\boxed{\text{or}}$

$2x + 9y < 14$

SO HOW DO I DO IT???

A step by step guide to graphing linear inequalities

① Let's say the inequality we have is $3y - 12x < 12$! So what I like to do first is find the X and Y intercepts, because you only need two points to graph a linear inequality (or any linear graph)

$\boxed{\text{finding X-intercepts}}$

Since -12 is our X value, write it as a comparison to the right side of our inequality. Don't touch the Y value ($3y$)

$\boxed{\text{Like this: } -12x < 12}$ ... Then isolate the X by dividing each side by -12 $\longrightarrow \frac{-12x}{-12} < \frac{12}{-12}$

*Since you're dividing by a negative sign, flip the inequality from < to >

LANDON

10) Day 1 notes.

Simplifying Radicals

p. 273 - 274

Like Radicals

Radicals with the same radicand and index are called like radicals

Pairs of Like Radicals	Pairs of unlike rad
$5\sqrt{7}$ and $-\sqrt{7}$	$2\sqrt{5}$ and $2\sqrt{3}$
$\frac{2}{3}\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$	$\sqrt[4]{5a}$ and $\sqrt[5]{5a}$

index

$\sqrt[n]{x}$ $\sqrt[n]{x}$ radicand

$\sqrt{4-x} \rightarrow$ index $= 2$

- The radical $\sqrt{4-x}$ has an even index. So, $4-x$ must be greater than or equal to zero

$4-x \geq 0$

$4 - x + x \geq 0 + x$

$4 \geq x$

$\boxed{\frac{\sqrt{4-5}}{\sqrt{-1}} = -E-}$

11) Landon – Focus on numerics (first three copied; last three created) and answering the textbook's green prompt questions to further his understanding of 'like terms'.

p. 276 – Ex 4 – Add and Subtract Radicals

1) Simplify and combine like terms

$\sqrt{50} + 3\sqrt{2} = \sqrt{25(2)} + 3\sqrt{2}$

$= 5\sqrt{2} + 3\sqrt{2}$ → These are like terms

$= 8\sqrt{2}$

$-\sqrt{27} + 3\sqrt{5} - \sqrt{80} - 2\sqrt{12}$

$= -\sqrt{3\cdot3\cdot3} + 3\sqrt{5} - \sqrt{2\cdot2\cdot2\cdot2\cdot5} - 2\sqrt{2\cdot2\cdot3}$

$= -3\sqrt{3} + 3\sqrt{5} - 4\sqrt{5} - 4\sqrt{3}$

$= -7\sqrt{3} - \sqrt{5}$ Radicals with like terms combine

$\sqrt{4c} - 4\sqrt{9c} = \sqrt{4}\sqrt{c} - 4\sqrt{9}\sqrt{c}$

$= 2\sqrt{c} - 4\cdot3\sqrt{c}$

$= -10\sqrt{c}$

Examples

a) $2\sqrt{7} + 13\sqrt{7}$ → $\sqrt{7}$ is the like term

$15\sqrt{7}$

b) $\sqrt{24} - \sqrt{6}$

$\sqrt{6\cdot4} - \sqrt{6}$

$\sqrt{3\cdot2\cdot2\cdot2} - \sqrt{6}$

$2\sqrt{6} - 1\sqrt{6}$ → $1\sqrt{6}$

$\sqrt{6}$

$2 - 1 = 1$

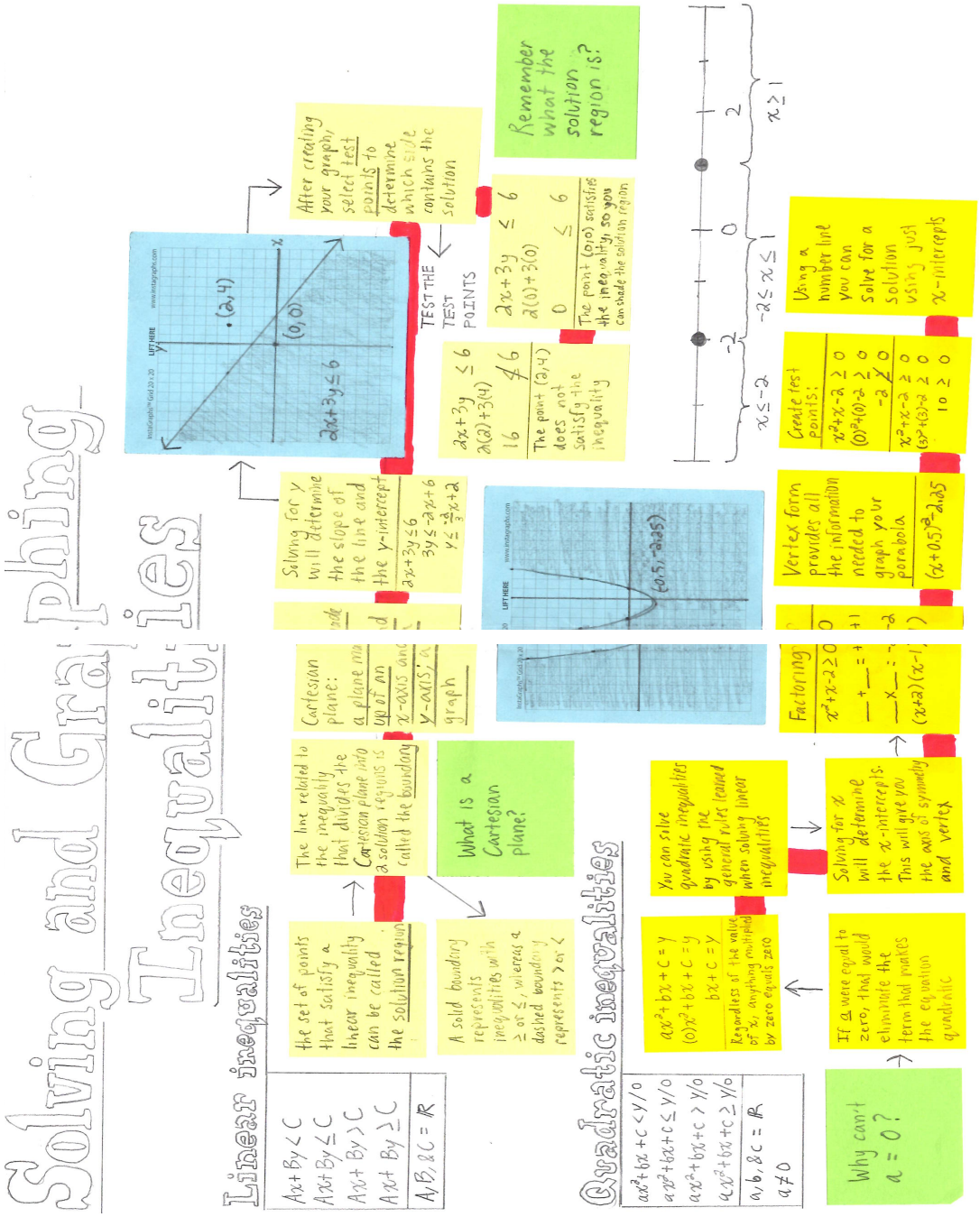
c) $\sqrt{20x} - 3\sqrt{45x}$

$\sqrt{2\cdot2\cdot5\cdot x} - 3\sqrt{3\cdot3\cdot5\cdot x}$

$2\sqrt{5x} - 9\sqrt{5x}$

$-7\sqrt{5x}$

12) Landon's use of stickie notes for his final project.



13) Elisa – Reminding herself to think.

Restrictions

Ex. $\sqrt{5x}$ $\quad x \geq 0$ $\quad$ think $\quad \frac{5x}{5} \geq \frac{0}{5} \quad x \geq 0$

Ex. $\sqrt{x-5}$ $\quad x \geq 5$ $\quad$ think $\quad x-5 \geq 0+5 \quad x \geq 5$

Ex. $\sqrt{3a-7}$ $\quad 3a-7 \geq 0 \rightarrow \frac{3a}{3} \geq \frac{7}{3} \rightarrow a \geq 7/3$

Ex. $\sqrt{-2a+1}$ $\quad -2a+1 \geq 0-1 \quad \frac{-2a}{-2} \geq \frac{-1}{-2} \quad a \leq 1/2$

flip sign

Ex. $\sqrt{x^2} = x \in \mathbb{R}$ (no restrictions)

Ex. $\sqrt[3]{w} = \sqrt[3]{-8} = \sqrt[3]{-2 \cdot -2 \cdot -2} = -2$ (no restrictions)

14) Elisa's creation of imperative steps:

Simplify $= \frac{1}{x^2} + \frac{5}{xy}$

make denominator the same — Multiply Both $= \frac{1}{x^2} \times \left(\frac{y}{y}\right) + \frac{5}{xy}\left(\frac{x}{x}\right)$

Have the same denominator $= \frac{y}{x^2y} + \frac{5x}{x^2y}$

Add the numerator $= \frac{y+5x}{x^2y}$

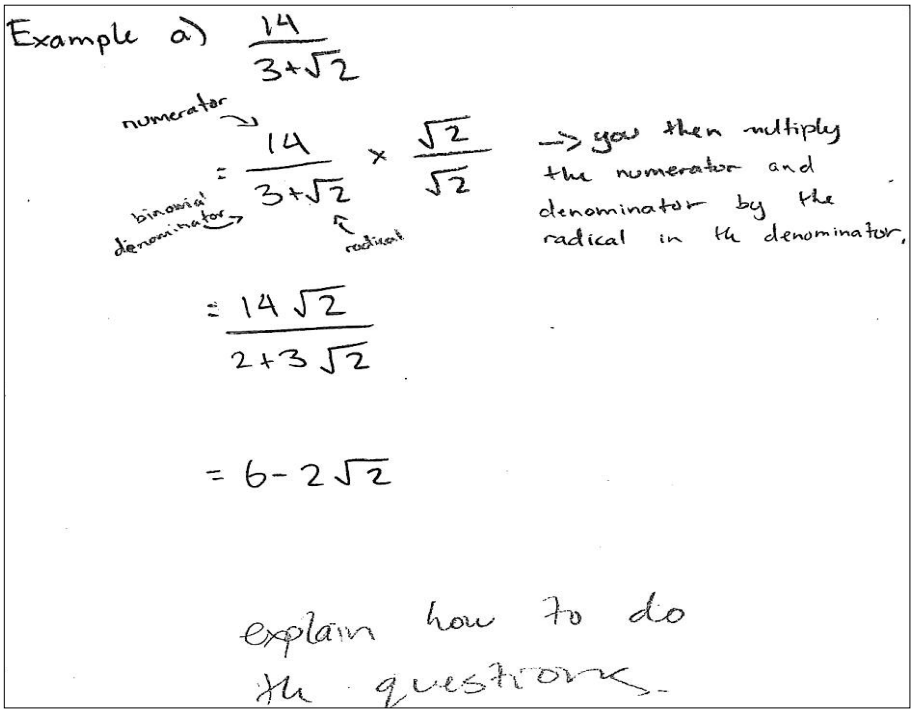
Simplify $= \frac{x+5}{x-3} - \frac{x-2}{x+1}$

Multiply both to make denominator the same $= \frac{x+5}{x-3}\left(\frac{x+1}{x+1}\right) - \frac{x-2}{x+1}\left(\frac{x-3}{x-3}\right)$

Have the denominator be the same. $= \frac{(x^2+6x+5)-(x^2-5x+6)}{(x-3)(x+1)}$

Subtract numerator $= \frac{11x-1}{(x-3)(x+1)}$

15) Serena: An example of studenting - creating meta-comments and work (incorrect) after I spoke with her about expanding on her thoughts.



Example a) $\frac{14}{3+\sqrt{2}}$

numerator

$= \frac{14}{3+\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ → you then multiply the numerator and denominator by the radical in the denominator.

binomial denominator

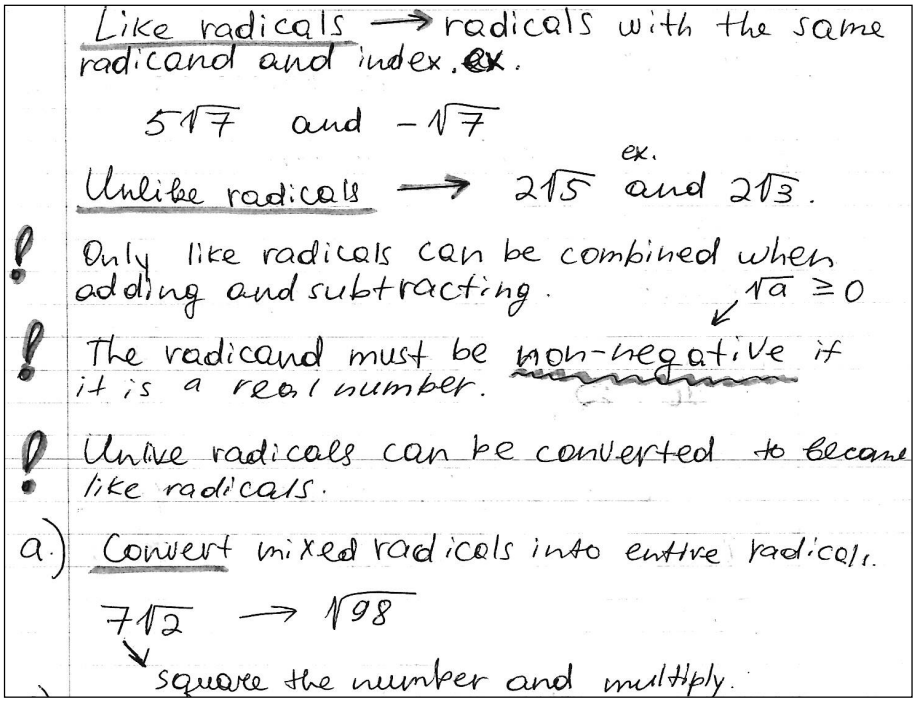
radical

$= \frac{14\sqrt{2}}{2+3\sqrt{2}}$

$= 6-2\sqrt{2}$

explain how to do the questions.

16) Gabrielle – An excerpt from her day 1 notes.



Like radicals → radicals with the same radicand and index. ex.

$5\sqrt{7}$ and $-\sqrt{7}$

Unlike radicals → ex. $2\sqrt{5}$ and $2\sqrt{3}$.

! Only like radicals can be combined when adding and subtracting.

! The radicand must be non-negative if it is a real number. $\sqrt{a} \geq 0$

! Unlike radicals can be converted to become like radicals.

a.) Convert mixed radicals into entire radicals.

$7\sqrt{2} \rightarrow \sqrt{98}$

square the number and multiply.

17) Gabrielle – Beginning of Trigonometry dialogue.

– Hi! I'm sorry for the confusing picture, I will try to explain it over the phone, my cell died
– Hi. I am confused. You better explain well!
– so, this pictures shows an angle in standard position. There are several key points that you need to remember about it.
– Wate a second, what is this standard angle? Is it somehow related to graphics?
– No. But you can think about it that way. This is a Cartesian plane, it is used to show an angle in standard position. The Cartesian plane consists of the x-axis and the y-axis, just like in the graphics section. The x-axis and the y-axis divide the Cartesian plane into 4 Quadrants (pieces). The top right piece (quadrant)

18) Gabrielle dialogue – relating old, graphics knowledge to new information.

– Do you remember asking me, wether this topic is related to graphics?
– Is it?
– As in parabolas, the Standard angle has a vertex, but this vertex should always be at the beginning / the origin of the Cartesian plane, otherwise it is not an angle in the Standard position.
– So, this must be the 2-nd rule?
– Yes, it is. Before I said that an angle in the Standard position has an initial arm, remember?
– Yes, what are you trying to tell me?
– An angle in standard position also has a terminal arm. This is the line on the picture that is connected to the initial arm to create an angle.

19) Antonio - Answering directed reading prompt.

2) This Passage is about Simplifying rational expressions
I learned about rational expressions, rational expressional can be written as quotient of tw
poly nomials.
Rational expression can be reduced by diving numerator and denominator by non-zero common
factor.
You can check by putting in values to the variable.
Rational expression is sometimes not defined when the denominator is zero

3)
a) simplifying is easy forme because I always use the formula for Simplifying.
I know the diffrence between rational expressions and those that are not.

b) I didn't know that an expression is not defined when is denomia tor is zero.
I didn't know when to use subsitution.

c) I hope to remember subsitution.
Subsitution is putting in values for check

20) May – Rather than create notes, she created lists of terms she would need to know.

mathematice 函算
terminology 术语
establishes 建立
algebra 代数学
coefficient 系数
variables 变量

21) John – As in class, his trigonometry dialogue had formulae.

: Ohhhhh

: Yes, and it seems that there are formulas to find the reference angles, and it varies from quadrant to quadrant.

: Ohhh

. If our θ° is below 90°, then that is its reference angle, if its located in the second quadrant, use: $180^{\circ} - \theta$ to calculate the reference angle. Respectively, use $\theta - 180^{\circ}$ for the third, and $360^{\circ} - \theta$ for the fourth. And thats pretty much it.